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*Research Article*

# Unique Coincidence and Fixed Point Theorem for $g$-Weakly C-Contractive Mappings in Partial Metric Spaces

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We prove that every map satisfying the $g$-weakly C-contractive inequality in partial metric space has a unique coincidence point. Our results generalize several well-known existing results in the literature.

## 1. Introduction and Preliminaries

The Banach contraction principle is the source of metric fixed point theory. This principle had been extended by many authors in different directions (see [1]).

Chatterjea [2] introduced the following contraction which has been named later as C-contraction.

*Definition 1* (see [2]). Let $(X, d)$ be a metric space and $f : X \to X$ a mapping. Then $f$ is called a C-contraction if there exists $k \in [0, 1/2)$ such that

$$d(fx, fy) \le k(d(x, fy) + d(fx, y)) \tag{1}$$

holds for all $x, y \in X$.

Under this kind of contractive inequality, Chatterjea [2] established the following fixed point result.

**Theorem 2** (see [2]). *Every C-contraction in a complete metric space has a unique fixed point.*

As a generalization of C-contractive mapping, Choudhury [3] introduced the concept of weakly C-contractive mapping and proved that every weakly C-contractive mapping in a complete metric space has a unique fixed point.

*Definition 3* (see [3]). Let $(X, d)$ be a metric space and $f : X \to X$ a mapping. Then $f$ is called a weakly C-contractive if $f$ satisfies

$$\begin{aligned} d(fx, fy) \le {} & \frac{1}{2}(d(x, fy) + d(fx, y)) \\ & - \phi(d(x, fy), d(fx, y)) \end{aligned} \tag{2}$$

for all $x, y \in X$, where $\phi : [0, \infty) \times [0, \infty) \to [0, \infty)$ is a continuous mapping such that $\phi(t, s) = 0$ if and only if $t = s = 0$.

Under this kind of contraction, Choudhury [3] established the following fixed point result.

**Theorem 4** (see [3, Theorem 2.1]). *Every weakly C-contraction in a complete metric space has a unique fixed point.*

Recently, Harjani et al. [4] studied some fixed point results for weakly C-contractive mappings in a complete metric space endowed with a partial order. Moreover, Shatanawi [5] proved some fixed point and coupled fixed point theorems for a nonlinear weakly C-contraction type mapping in metric and ordered metric spaces.

In another aspect, the notion of a partial metric space has been introduced by Matthews [6] in 1994 as a generalization of the usual metric in such a way that each object does not necessarily have to have a zero distance from itself. A motivation behind introducing the concept of a partial metric was to obtain appropriate mathematical models in the theory of computation and, in particular, to give a modified version of the Banach contraction principle (see, e.g., [7, 8]). Subsequently, several authors studied the problem of existence and uniqueness of a fixed point for mappings satisfying different contractive conditions on partial metric spaces (e.g., [9–13]).

We recall some definitions and properties of partial metric spaces.

*Definition 5.* A *partial metric* on a nonempty set $X$ is a function $p : X \times X \to \mathbb{R}^+$ such that for all $x, y, z \in X$,

(p1) $x = y \Leftrightarrow p(x, x) = p(x, y) = p(y, y)$;

(p2) $p(x, x) \leq p(x, y)$;

(p3) $p(x, y) = p(y, x)$;

(p4) $p(x, z) \leq p(x, y) + p(y, z) - p(y, y)$.

A partial metric space is a pair $(X, p)$ such that $X$ is nonempty set and $p$ is a partial metric on $X$.

From the above definition, if $p(x, y) = 0$, then $x = y$. But if $x = y$, $p(x, y)$ may not be 0 in general. A famous example of a partial metric space is the pair $(\mathbb{R}^+, p)$, where $p : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ is defined as $p(x, y) = \max\{x, y\}$. For some more examples of partial metric spaces, we refer to [8, 12].

Each partial metric $p$ on $X$ generates a $T_0$ topology $\tau_p$ on $X$ which has as a base the family of open $p$-balls: $\{B_p(x, \epsilon) : x \in X, \epsilon > 0\}$, where $B_p(x, \epsilon) = \{y \in X : p(x, y) < p(x, x) + \epsilon\}$ for all $x \in X$ and $\epsilon > 0$. A sequence $\{x_n\}$ in $X$ converges to a point $x \in X$, with respect to $\tau_p$ if and only if $p\,(x, x) = \lim_{n\to\infty} p(x, x_n)$. A sequence $\{x_n\}$ in $X$ is called Cauchy sequence if $\lim_{n,m\to\infty} p(x_n, x_m)$ exists and is finite.

*Definition 6* (see [6, 13]). Let $(X, p)$ be a partial metric space. Then,

(i) a sequence $\{x_n\}$ in a partial metric space $(X, p)$ converges to a point $x \in X$ if and only if $p(x, x) = \lim_{n\to\infty} p(x, x_n)$;

(ii) a sequence $\{x_n\}$ in a partial metric space $(X, p)$ is called a Cauchy sequence if there exists (and is finite) $\lim_{n,m\to\infty} p(x_n, x_m)$;

(iii) a partial metric space $(X, p)$ is said to be complete if every Cauchy sequence $\{x_n\}$ in $X$ converges to a point $x \in X$; that is, $p(x, x) = \lim_{n,m\to\infty} p(x_n, x_m)$.

If $p$ is a partial metric on $X$, then the function $p^s : X \times X \to \mathbb{R}^+$ given by

$$p^s(x, y) = 2p(x, y) - p(x, x) - p(y, y) \tag{3}$$

is a metric on $X$.

**Lemma 7** (see [6, 13]). *Let $(X, p)$ be a partial metric space. Then,*

(a) *$\{x_n\}$ is a Cauchy sequence in $(X, p)$ if and only if it is a Cauchy sequence in the metric space $(X, p^s)$;*

(b) *$(X, p)$ is complete if and only if the metric space $(X, p^s)$ is complete. Furthermore, $\lim_{n\to\infty} p^s(x_n, x) = 0$ if and only if*

$$p(x, x) = \lim_{n\to\infty} p(x_n, x) = \lim_{n,m\to\infty} p(x_n, x_m). \tag{4}$$

Moreover, Bhaskar and Lakshmikantham [14] presented coupled fixed point theorems for contractions in partially ordered metric spaces. This concept attracted many mathematician and for more related work on coupled fixed and coincidence points results we refer the readers to recent works in [4, 9, 10, 15–19].

*Definition 8.* Let $f, g : X \to X$ be two mappings. One says that $x \in X$ is a coincidence point of $f$ and $g$ if $f(x) = g(x)$.

In this paper, we extend the concept of a weakly C-contractive mapping to the context of partial metric space and define $g$-weakly C-contractive map. Moreover, we prove that every $g$-weakly C-contractive mapping in a complete partial metric space has a unique coincidence point. Our result generalizes several well-known results in the literature.

## 2. Unique Coincidence and Fixed Point Theorem

*Definition 9.* Let $(X, p)$ be a partial metric space and $g : X \to X$ a map. Then, the mapping $f : X \to X$ is said to be $g$-weakly C-contractive if

$$\begin{aligned} p(fx, fy) \leq{} & \frac{1}{2}\left(p(gx, fy) + p(fx, gy)\right) \\ & - \phi\left(p(gx, fy), p(fx, gy)\right) \end{aligned} \tag{5}$$

for all $x, y, u, v \in X$, where $\phi : [0, \infty) \times [0, \infty) \to [0, \infty)$ is a continuous mapping such that $\phi(t, s) = 0$ if and only if $t = s = 0$.

Now we state and prove our main result.

**Theorem 10.** *Let $(X, p)$ be a complete partial metric space and $f : X \to X$ a $g$-weakly C-contraction mapping. Suppose that $f(X) \subset g(X)$. Then, $f$ and $g$ have a unique coincidence point in $X$.*

*Proof.* Let $x_0 \in X$ be arbitrary point in $X$. Since $f(X) \subset g(X)$, we can construct sequence $\{gx_n\}$ in $X$ as

$$gx_{n+1} = fx_n, \quad \forall n \geq 0. \tag{6}$$

Set $\delta_n = p(gx_n, gx_{n+1})$.

If there exists $n \in \mathbb{N}$ such that $\delta_n = 0$, then by (p1) and (p2) we have $gx_n = gx_{n+1} = fx_n$. Hence, $f$ and $g$ have a coincidence point in $X$. Now assume that $\delta_n \neq 0$ for all $n \geq 0$. Thus by (5), we have

$$\begin{aligned} p(gx_{n+1}, gx_{n+2}) &= p(fx_n, fx_{n+1}) \\ &\leq \frac{1}{2}\left(p(gx_n, fx_{n+1}) + p(fx_n, gx_{n+1})\right) \\ &\quad - \phi\left(p(gx_n, fx_{n+1}), p(fx_n, gx_{n+1})\right) \\ &= \frac{1}{2}\left(p(gx_n, gx_{n+2}) + p(gx_{n+1}, gx_{n+1})\right) \\ &\quad - \phi\left(p(gx_n, gx_{n+2}), p(gx_{n+1}, gx_{n+1})\right). \end{aligned} \tag{7}$$

By property (p4), we have

$$\begin{aligned} & p(gx_n, gx_{n+2}) + p(gx_{n+1}, gx_{n+1}) \\ & \quad \leq p(gx_n, gx_{n+1}) + p(gx_{n+1}, gx_{n+2}). \end{aligned} \tag{8}$$

Thus from (7), we have

$$\begin{aligned} p(gx_{n+1}, gx_{n+2}) &\le \frac{1}{2}(p(gx_n, gx_{n+2}) + p(gx_{n+1}, gx_{n+1})) \\ &\quad - \phi(p(gx_n, gx_{n+2}), p(gx_{n+1}, gx_{n+1})) \end{aligned} \tag{9}$$

$$\begin{aligned} &\le \frac{1}{2}(p(gx_n, gx_{n+1}) + p(gx_{n+1}, gx_{n+2})) \\ &\quad - \phi(p(gx_n, gx_{n+2}), p(gx_{n+1}, gx_{n+1})) \\ &\le \max\{p(gx_n, gx_{n+1}), p(gx_{n+1}, gx_{n+2})\} \\ &\quad - \phi(p(gx_n, gx_{n+2}), p(gx_{n+1}, gx_{n+1})). \end{aligned} \tag{10}$$

From (9), we have either $p(gx_n, gx_{n+2}) \neq 0$ or $p(gx_{n+1}, gx_{n+1}) \neq 0$ since $p(gx_n, gx_{n+1}) \neq 0$ for all $n \in \mathbb{N}$.

If

$$\max\{p(gx_n, gx_{n+1}), p(gx_{n+1}, gx_{n+2})\} = p(gx_{n+1}, gx_{n+2}), \tag{11}$$

then since $\phi(p(gx_n, gx_{n+2}), p(gx_{n+1}, gx_{n+1})) > 0$ and by (10), we have

$$\begin{aligned} p(gx_{n+1}, gx_{n+2}) &\le p(gx_{n+1}, gx_{n+2}) \\ &\quad - \phi(p(gx_n, gx_{n+2}), p(gx_{n+1}, gx_{n+1})) \\ &< p(gx_{n+1}, gx_{n+2}), \end{aligned} \tag{12}$$

which is a contradiction. Thus, we have

$$\max\{p(gx_n, gx_{n+1}), p(gx_{n+1}, gx_{n+2})\} = p(gx_n, gx_{n+1}) \tag{13}$$

and therefore

$$\begin{aligned} p(gx_{n+1}, gx_{n+2}) &\le p(gx_n, gx_{n+1}) \\ &\quad - \phi(p(gx_n, gx_{n+2}), p(gx_{n+1}, gx_{n+1})) \\ &\le p(gx_n, gx_{n+1}). \end{aligned} \tag{14}$$

By the above inequalities, we have that $\{\delta_n\} = \{p(gx_n, gx_{n+1})\}$ is a non increasing sequence of positive real numbers. Therefore, there is some $\delta \ge 0$ such that

$$\lim_{n\to\infty} p(gx_n, gx_{n+1}) = \delta. \tag{15}$$

Then taking the limit as $n \to \infty$ in (10), we have

$$\delta \le \delta - \lim_{n\to\infty} \phi(p(gx_n, gx_{n+2}), p(gx_{n+1}, gx_{n+1})) \le \delta. \tag{16}$$

Then,

$$\delta - \lim_{n\to\infty} \phi(p(gx_n, gx_{n+2}), p(gx_{n+1}, gx_{n+1})) = \delta \tag{17}$$

and therefore

$$\lim_{n\to\infty} \phi(p(gx_n, gx_{n+2}), p(gx_{n+1}, gx_{n+1})) = 0. \tag{18}$$

By continuity of $\phi$, we conclude that

$$\begin{gathered} \lim_{n\to\infty} p(gx_n, gx_{n+2}) = 0, \\ \lim_{n\to\infty} p(gx_{n+1}, gx_{n+1}) = 0. \end{gathered} \tag{19}$$

Letting $n \to \infty$ in (9) and (15), (19), and the continuity of $\phi$, we conclude that $\delta = 0$. Thus,

$$\lim_{n\to\infty} p(gx_n, gx_{n+1}) = 0. \tag{20}$$

Next, we will prove that

$$\lim_{n,m\to\infty} p(gx_n, gx_m) = 0. \tag{21}$$

Suppose the contrary; that is,

$$\lim_{n,m\to\infty} p(gx_n, gx_m) \neq 0. \tag{22}$$

Then there exists an $\epsilon > 0$ for which we can find subsequences $\{gx_{n(k)}\}$, $\{gx_{m(k)}\}$ of $\{gx_n\}$ such that $n(k)$ is the smallest integer for which

$$n(k) > m(k) \ge k, \qquad p(gx_{n(k)}, gx_{m(k)}) \ge \epsilon. \tag{23}$$

This means that

$$p(gx_{n(k)-1}, gx_{m(k)}) < \epsilon. \tag{24}$$

From the above two inequalities and (p4), we have

$$\begin{aligned} \epsilon &\le p(gx_{n(k)}, gx_{m(k)}) \\ &\le p(gx_{n(k)}, gx_{n(k)-1}) + p(gx_{n(k)-1}, gx_{m(k)}) \\ &\quad - p(gx_{n(k)-1}, gx_{n(k)-1}) \\ &\le p(gx_{n(k)}, gx_{n(k)-1}) + p(gx_{n(k)-1}, gx_{m(k)}) \\ &< \epsilon + p(gx_{n(k)}, gx_{n(k)-1}). \end{aligned} \tag{25}$$

Letting $k \to \infty$ and using (20), we get

$$\lim_{k\to\infty} p(gx_{n(k)}, gx_{m(k)}) = \epsilon. \tag{26}$$

By (p3) and (p4), we have

$$\begin{aligned} &p(gx_{n(k)}, gx_{m(k)}) \\ &\quad\le p(gx_{n(k)}, gx_{n(k)+1}) + p(gx_{n(k)+1}, gx_{m(k)}) \\ &\qquad - p(gx_{n(k)+1}, gx_{n(k)+1}) \\ &\quad\le p(gx_{n(k)}, gx_{n(k)+1}) + p(gx_{n(k)+1}, gx_{m(k)}) \\ &\quad\le p(gx_{n(k)}, gx_{n(k)+1}) + p(gx_{n(k)+1}, gx_{m(k)+1}) \\ &\qquad + p(gx_{m(k)+1}, gx_{m(k)}) - p(gx_{m(k)+1}, gx_{m(k)+1}) \\ &\quad\le p(gx_{n(k)}, gx_{n(k)+1}) + p(gx_{n(k)+1}, gx_{m(k)+1}) \\ &\qquad + p(gx_{m(k)+1}, gx_{m(k)}) \\ &\quad\le 2p(gx_{n(k)}, gx_{n(k)+1}) + p(gx_{n(k)}, gx_{m(k)+1}) \\ &\qquad + p(gx_{m(k)+1}, gx_{m(k)}) - p(gx_{n(k)}, gx_{n(k)}) \\ &\quad\le 2p(gx_{n(k)}, gx_{n(k)+1}) + p(gx_{n(k)}, gx_{m(k)+1}) \\ &\qquad + p(gx_{m(k)+1}, gx_{m(k)}) \\ &\quad\le 2p(gx_{n(k)}, gx_{n(k)+1}) + p(gx_{n(k)}, gx_{m(k)}) \\ &\qquad + 2p(gx_{m(k)+1}, gx_{m(k)}) - p(gx_{m(k)}, gx_{m(k)}) \\ &\quad\le 2p(gx_{n(k)}, gx_{n(k)+1}) + p(gx_{n(k)}, gx_{m(k)}) \\ &\qquad + 2p(gx_{m(k)+1}, gx_{m(k)}). \end{aligned} \tag{27}$$

Letting $k \to +\infty$ in the above inequalities and using (20) and (26), we have

$$\begin{aligned} \lim_{k\to+\infty} p(gx_{n(k)}, gx_{m(k)}) &= \lim_{k\to+\infty} p(gx_{n(k)+1}, gx_{m(k)}) \\ &= \lim_{k\to+\infty} p(gx_{n(k)+1}, gx_{m(k)+1}) \\ &= \lim_{k\to+\infty} p(gx_{n(k)}, gx_{m(k)+1}) = \epsilon. \end{aligned} \tag{28}$$

Therefore, from (5), we have

$$\begin{aligned} &p(gx_{m(k)+1}, gx_{n(k)+1}) \\ &\quad= p(fx_{m(k)}, fx_{n(k)}) \\ &\quad\le \frac{1}{2}\left(p(gx_{m(k)}, fx_{n(k)}) + p(fx_{m(k)}, gx_{n(k)})\right) \\ &\qquad - \phi\left(p(gx_{m(k)}, fx_{n(k)}), p(fx_{m(k)}, gx_{n(k)})\right) \\ &\quad= \frac{1}{2}\left(p(gx_{m(k)}, gx_{n(k)+1}) + p(gx_{m(k)+1}, gx_{n(k)})\right) \\ &\qquad - \phi\left(p(gx_{m(k)}, gx_{n(k)+1}), p(gx_{m(k)+1}, gx_{n(k)})\right). \end{aligned} \tag{29}$$

Letting $k \to +\infty$ in the above and using (28) and the continuity of $\phi$, we conclude that

$$\epsilon \le \epsilon - \phi(\epsilon, \epsilon) < \epsilon \tag{30}$$

which is a contradiction. Thus, we have

$$\lim_{n,m\to+\infty} p(gx_n, gx_m) = 0. \tag{31}$$

Therefore, $\{gx_n\}$ is a Cauchy sequence in the complete partial metric space $(X, p)$.

By Lemma 7, we have that

$$\lim_{n,m\to+\infty} p^s(gx_n, gx_m) = 0. \tag{32}$$

Thus, $\{gx_n\}$ is a Cauchy sequence in the complete metric space $(X, p^s)$. Hence, by Lemma 7, $\{gx_n\}$ is a Cauchy sequence in the complete metric space $(X, p)$. Again, by Lemma 7, there exists $x \in X$ such that

$$\lim_{n\to+\infty} p^s(gx_n, gx) = 0 \tag{33}$$

which implies that

$$p(gx, gx) = \lim_{n\to+\infty} p(gx_n, gx) = \lim_{n,m\to+\infty} p(gx_n, gx_m). \tag{34}$$

Next, we prove that $\lim_{n\to+\infty} p(fx, gx_{n+1}) = p(fx, gx)$.

Letting $n \to +\infty$ in

$$p(fx, gx_{n+1}) \le p(fx, gx) + p(gx, gx_{n+1}) - p(gx, gx), \tag{35}$$

we have

$$\lim_{n\to+\infty} p(fx, gx_{n+1}) \le p(fx, gx). \tag{36}$$

Also, letting $n \to +\infty$ in

$$\begin{aligned} p(fx, gx) \le p(fx, gx_{n+1}) + p(gx_{n+1}, gx) \\ - p(gx_{n+1}, gx_{n+1}), \end{aligned} \tag{37}$$

we have

$$p(fx, gx) \le \lim_{n\to+\infty} p(fx, gx_{n+1}). \tag{38}$$

From (36) and (38), we have

$$\lim_{n\to+\infty} p(fx, gx_{n+1}) = p(fx, gx). \tag{39}$$

Now, we prove that $fx = gx$. By (5), we have

$$\begin{aligned} p(fx, gx_{n+1}) &= p(fx, fx_n) \\ &\le \frac{1}{2}\left(p(gx, fx_n) + p(fx, gx_n)\right) \\ &\quad - \phi\left(p(gx, fx_n), p(fx, gx_n)\right) \\ &= \frac{1}{2}\left(p(gx, gx_{n+1}) + p(fx, gx_n)\right) \\ &\quad - \phi\left(p(gx, gx_{n+1}), p(fx, gx_n)\right). \end{aligned} \tag{40}$$

Letting $n \to +\infty$ in the above and using (39) and the continuity of $\phi$, we conclude that

$$p(fx, gx) \le \frac{1}{2} p(fx, gx) - \phi(0, p(fx, gx)) \le \frac{1}{2} p(fx, gx). \tag{41}$$

Hence, $p(fx, gx) = 0$. By (p1) and (p2), we have $fx = gx$. Thus, $x$ is a coincidence point of $f$ and $g$.

To prove the uniqueness of the coincidence point of $f$ and $g$, suppose that $y$ is another coincidence point of $f$ and $g$. From (5), we have

$$p(gx, gy) = p(fx, fy) \le \frac{1}{2}(p(x, y) + p(x, y)) - \phi(p(x, y), p(x, y)). \tag{42}$$

Therefore, we have $\phi(p(x, y), p(x, y)) = 0$. Hence, $p(x, y) = 0$. By (p1) and (p2), we have $x = y$.

Thus, $f$ and $g$ have a unique coincidence point. □

As an immediate consequence of the above theorem, we have the following fixed point result.

**Corollary 11.** *Let $(X, p)$ be a complete partial metric space and $f : X \to X$ a weakly C-contraction mapping. That is, $T$ satisfies*

$$p(fx, fy) \le \frac{1}{2}(p(x, fy) + p(fx, y)) - \phi(p(x, fy), p(fx, y)) \tag{43}$$

*for all $x, y \in X$, where $\phi : [0, \infty) \times [0, \infty) \to [0, \infty)$ is a continuous mapping such that $\phi(t, s) = 0$ if and only if $t = s = 0$.*

*Then, there exists a unique $x \in X$ such that $fx = x$.*

**Corollary 12.** *Let $(X, p)$ be a complete partial metric space. Suppose that the mapping $f : X \to X$ satisfies the following contractive condition:*

$$p(fx, fy) \le kp(fx, y) + lp(x, fy), \tag{44}$$

*for all $x, y, u, v \in X$, where $k, l$ are nonnegative constants with $k + 2l < 1$. Then, $f$ has a unique fixed point.*

*Proof.* Take $\phi(t, s) = ((1/2) - l)t + ((1/2) - k)s$, where $k, l$ are nonnegative constants with $k + 2l < 1$. □

**Corollary 13** (see [10, Corollary 2.7]). *Let $(X, p)$ be a complete partial metric space. Suppose that the mapping $f : X \to X$ satisfies the following contractive condition:*

$$p(fx, fy) \le \frac{k}{2}(p(fx, y) + p(x, fy)), \tag{45}$$

*for all $x, y \in X$, where $0 \le k < 2/3$. Then, $f$ has a unique fixed point.*

*Proof.* Take $\phi(t, s) = ((1/2) - (k/2))\ (t + s)$, where $0 \le k < 2/3$. □

## Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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