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Research Article

Global Exponential Robust Stability of Static Interval Neural Networks with Time Delay in the Leakage Term

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The stability of a class of static interval neural networks with time delay in the leakage term is investigated. By using the method of M-matrix and the technique of delay differential inequality, we obtain some sufficient conditions ensuring the global exponential robust stability of the networks. The results in this paper extend the corresponding conclusions without leakage delay. An example is given to illustrate the effectiveness of the obtained results.

1. Introduction

Recently, neural networks have been widely studied because of their successful applications in different areas, such as pattern recognition, image processing, detection of moving objects, and optimization problems. The stability of the neural networks with time delay, upon which these applications largely depend, has been extensively studied (see [1–10]). However, to the best of our knowledge, there has been very little existing work on neural networks, especially, on static neural networks with time delay in the leakage term [11–18]. This is due to some theoretical and technical difficulties [13]. So, the main purpose of this paper is to study the stability of the static interval neural networks with time delay in the leakage term. By using the properties of M-matrix and delay differential inequality, we obtain some sufficient conditions ensuring the global exponential robust stability. Our results extend the corresponding conclusions without leakage delay.

2. Model Description and Preliminaries

In this section, we list all the notations which will be frequently used throughout the paper and give a few definitions, lemmas, and assumptions.

Notations. Let R be the set of real number, and let R^n and $R^{m\times n}$ be the space of n-dimensional real vectors and $m\times n$ real matrices, separately. E denotes an $n\times n$ unit matrix.

$N \triangleq \{1,2,\ldots,n\}$. For $A,B \in R^{m\times n}$ or $A,B\in R^n$, the notation $A \geq B$ ($A > B$) means that each pair of corresponding elements of A and B satisfies the inequality "$\geq$ ($>$)." $|\cdot|$ denotes the Euclidean norm. For any $u\in R$, $\operatorname{sgn}(u)$ is the sign function of u.

$C[X,Y]$ denotes the space of continuous mappings from the topological space X to the topological space Y. Particularly, let $C \triangleq C([-\tau,0],R^n)$ denote the family of all continuous R^n-valued function ϕ defined on $[-\tau,0]$ with the norm $\|\phi\| = \sup_{-\tau\leq s\leq 0}|\phi(s)|$.

For $x\in R^n$, $\varphi\in C$, we define $[x]^+ = (|x_1|,\ldots,|x_n|)^T$, $[\varphi(t)]_\tau = ([\varphi_1(t)]_\tau,\ldots,[\varphi_n(t)]_\tau)^T$, $[\varphi_i(t)]_\tau = \sup_{-\tau\leq s\leq 0}\{\varphi_i(t+s)\}$, $i\in N$, and $[\varphi(t)]_\tau^+ \triangleq [[\varphi(t)]^+]_\tau$. $D^+\varphi(t)$ denotes the upper-right-hand derivative of $\varphi(t)$ at time t.

Consider the following interval static neural network model with leakage delay:

$$\begin{aligned}\frac{dx_i(t)}{dt} &= -a_i(\lambda)\,x_i(t-\sigma)\\ &\quad + f_i\left(\sum_{j=1}^{n}\int_{-\tau(\lambda)}^{0} x_j(t+\theta)\,dw_{ij}(\theta,\lambda)+I_i\right),\\ &\qquad t\geq 0,\\ x_i(t) &= \phi_i(t),\quad -r\leq t\leq 0,\end{aligned} \tag{1}$$

where $i, j = 1, 2, \ldots, n$, x_i and I_i denote the state and the external inputs of the ith neuron, separately. The integer n corresponds to the number of units in a neural network, and $f_i(\cdot)$ denotes the signal propagation function of the ith unit. $\lambda \in \Lambda \subset R$ is a parameter, and $a_i(\lambda)$ represents the rate with which ith neuron will reset its potential to the resting state in isolation when disconnected from the network and external inputs. $w_{ij}(\theta, \lambda)$ is nondecreasing bounded variation functions on $[-\tau(\lambda), 0]$, and $\int_{-\tau(\lambda)}^{0} x_j(t+\theta) dw_{ij}(\theta, \lambda)$ is a Lebesgue-Stieltjes integration. There exist positive constants τ, $\underline{a}_i$, $\overline{a}_i$, and w_{ij}^* such that for any $\lambda \in \Lambda$, $0 < \underline{a}_i \le a_i(\lambda) \le \overline{a}_i$, $0 \le \tau(\lambda) \le \tau$, and $|\int_{-\tau(\lambda)}^{0} dw_{ij}(\theta, \lambda)| \le w_{ij}^* < \infty$. $\sigma \ge 0$ represents the leakage delay, $r = \max\{\sigma, \tau\}$. $\phi(s) = (\phi_1(s), \ldots, \phi_n(s))^T$, where $\phi_i(s)$ is derivative on $[-r, 0]$, and $[\dot{\phi}_i]_r^+ \le m_i$, $i \in N$.

Definition 1. The equilibrium point x^* of system (1) is said to be globally exponentially stable if there exist a positive constant γ and a vector $M > 0$ such that

$$[x(t) - x^*]^+ \le M e^{-\gamma(t-t_0)}, \quad t \ge t_0. \tag{2}$$

Definition 2. System (1) is said to be globally exponentially robustly stable if its equilibrium point x^* is globally exponentially stable for any $\tau(\lambda) \in [0, \tau]$ and $a_i(\lambda) \in [\underline{a}_i, \overline{a}_i]$.

Definition 3 (see [19]). Let the matrix $D = (d_{ij})_{n\times n}$ with $d_{ii} > 0$ and $d_{ij} \le 0, i \ne j, i, j = 1, 2, \ldots, n$. Then each of the following conditions is equivalent to the statement "D is a nonsingular M-matrix."

(1) All the leading principle minors of D are positive.

(2) The diagonal elements of D are all positive, and there exists a positive vector d such that $Dd > 0$ or $D^T d > 0$.

Lemma 4 (see [20]). *Let $a < b \le +\infty$, and $v(t) \in C[[a, b], R^n]$ satisfies*

$$\begin{gathered} D^+ v(t) \le P v(t) + Q[v(t)]_\tau, \quad t \in [a, b), \\ v(a+s) \in PC, \quad s \in [-\tau, 0], \end{gathered} \tag{3}$$

where $P = (p_{ij})_{n\times n}$, $p_{ij} \ge 0$ for $i \ne j$, $Q = (q_{ij})_{n\times n} \ge 0$, and $J = (J_1, \ldots, J_n)^T \ge 0$, $i, j = 1, 2, \ldots, n$. Suppose that there exist a scalar $\lambda > 0$ and a vector $z = (z_1, z_2, \ldots, z_n)^T > 0$ such that

$$\left[\lambda E + P + Q e^{\lambda\tau}\right] z < 0. \tag{4}$$

If the initial condition satisfies

$$\begin{gathered} v(t) \le M z e^{-\lambda(t-a)} - (P+Q)^{-1} J, \\ M \ge 0, \quad t \in [a-\tau, a], \end{gathered} \tag{5}$$

then $v(t) \le M z e^{-\lambda(t-a)} - (P+Q)^{-1} J$ for $t \in [a, b)$.

For the model (1), we introduce the following assumptions.

(A_1) The signal propagation functions $f_i(\cdot)$ are Lipschitz continuous; that is, there are positive constants k_i, $i \in N$ such that for all $s_1, s_2 \in R$

$$|f_i(s_1) - f_i(s_2)| \le k_i |s_1 - s_2|. \tag{6}$$

(A_2) Let $-(U + V)$ be a nonsingular M-matrix, where

$$\begin{gathered} U = (u_{ij})_{n\times n}, \qquad V = (v_{ij})_{n\times n}, \\ u_{ij} = 0, \quad i \ne j, \qquad u_{ii} = -\underline{a}_i, \\ v_{ii} = \sigma\overline{a}_i^2 + (1 + \sigma\overline{a}_i) k_i w_{ii}^*, \\ v_{ij} = (1 + \sigma\overline{a}_i) k_i w_{ij}^*, \quad i \ne j. \end{gathered} \tag{7}$$

3. Main Results

Theorem 5. *Suppose that the conditions (A_1) and (A_2) hold, and then system* (1) *has at least one equilibrium point.*

Proof. From (A_2) and Definition 3, we know there exists a positive vector $d = (d_1, \ldots, d_n)^T$ such that $(U + V)d < 0$. That is

$$-\underline{a}_i d_i + \sigma\overline{a}_i^2 d_i + (1 + \sigma\overline{a}_i) k_i \sum_{j=1}^{n} w_{ij}^* d_j < 0, \qquad i = 1, \ldots, n. \tag{8}$$

From (8) we can get

$$k_i \sum_{j=1}^{n} w_{ij}^* d_j < \frac{\underline{a}_i - \sigma\overline{a}_i^2}{1 + \sigma\overline{a}_i} d_i \le \underline{a}_i d_i, \qquad i = 1, \ldots, n. \tag{9}$$

Combining with Definition 3, we know $\underline{A} - KW$ is a nonsingular M-matrix, where $\underline{A} = \operatorname{diag}(\underline{a}_1, \ldots, \underline{a}_n)$, $K = \operatorname{diag}(k_1, \ldots, k_n)$, and $W = (w_{ij}^*)_{n\times n}$.

In a similar way of proof for the literature [7], by the theory of topological degree and homotopy invariance theorem, the existence of the equilibrium point of system (1) can be proved. Suppose that x^* is an equilibrium point of system (1), let $y_i(t) = x_i(t) - x_i^*$, $\varphi_i(t) = \phi_i(t) - x_i^*$, and then system (1) becomes

$$\begin{aligned} \frac{dy_i(t)}{dt} &= -a_i(\lambda) y_i(t-\sigma) \\ &\quad + f_i\left(\sum_{j=1}^{n} \int_{-\tau(\lambda)}^{0} \left(y_j(t+\theta) + x_j^*\right) dw_{ij}(\theta, \lambda) + I_i\right) \\ &\quad - f_i\left(\sum_{j=1}^{n} \int_{-\tau(\lambda)}^{0} x_j^* dw_{ij}(\theta, \lambda) + I_i\right), \quad t \ge 0, \\ y_i(t) &= \varphi_i(t), \quad -r \le t \le 0. \end{aligned} \tag{10}$$

It is clear that the stability of zero solution to system (10) is equivalent to the stability of the equilibrium point x^* of system (1). So we only consider the stability of zero solution to system (10). □

Theorem 6. *Assume that the conditions* (A_1) *and* (A_2) *are satisfied, and then the zero solution to system* (10) *is globally exponentially robustly stable.*

Proof. From the Middle Value theorem, we obtain

$$-y_i(t-\sigma) = -y_i(t) + \sigma \dot{y}_i(t-(1-\alpha)\sigma), \quad i = 1, 2, \ldots, n, \tag{11}$$

where $0 < \alpha < 1$. Then from (10) we get

$$\begin{aligned} \frac{dy_i(t)}{dt} = & -a_i(\lambda) y_i(t) \\ & + \sigma a_i(\lambda) \dot{y}_i(t-(1-\alpha)\sigma) \\ & + f_i\left(\sum_{j=1}^{n} \int_{-\tau(\lambda)}^{0} \left(y_j(t+\theta) + x_j^*\right) dw_{ij}(\theta,\lambda) + I_i\right) \\ & - f_i\left(\sum_{j=1}^{n} \int_{-\tau(\lambda)}^{0} x_j^* dw_{ij}(\theta,\lambda) + I_i\right). \end{aligned} \tag{12}$$

Case 1. If $0 \le t < (1-\alpha)\sigma$, then $\dot{y}_i(t-(1-\alpha)\sigma) = \dot{\varphi}_i(t-(1-\alpha)\sigma)$.

Let $V_i(t) = |y_i(t)|$, $i \in N$. Then from (A_1), we have

$$\begin{aligned} & D^+ V_i(t) \\ & = \operatorname{sgn}(y_i(t)) \dot{y}_i(t) \\ & = \operatorname{sgn}(y_i(t)) \Bigg(-a_i(\lambda) y_i(t) + \sigma a_i(\lambda) \dot{y}_i(t-(1-\alpha)\sigma) \\ & \qquad + f_i\left(\sum_{j=1}^{n} \int_{-\tau(\lambda)}^{0} \left(y_j(t+\theta) + x_j^*\right) dw_{ij}(\theta,\lambda) + I_i\right) \\ & \qquad - f_i\left(\sum_{j=1}^{n} \int_{-\tau(\lambda)}^{0} x_j^* dw_{ij}(\theta,\lambda) + I_i\right)\Bigg) \\ & \le -a_i(\lambda) |y_i(t)| + \sigma a_i(\lambda) |\dot{y}_i(t-(1-\alpha)\sigma)| \\ & \quad + k_i \sum_{j=1}^{n} \int_{-\tau(\lambda)}^{0} |y_j(t+\theta)| dw_{ij}(\theta,\lambda) \\ & \le -\underline{a}_i |y_i(t)| + \sigma \overline{a}_i m_i + k_i \sum_{j=1}^{n} w_{ij}^* [y_j(t)]_\tau^+ \\ & = \sum_{j=1}^{n} p_{ij} V_j(t) + \sum_{j=1}^{n} q_{ij} [V_j(t)]_\tau + l_i, \end{aligned} \tag{13}$$

where $p_{ii} = -\underline{a}_i$, $p_{ij} = 0$, $i \ne j$, $q_{ij} = k_i w_{ij}^*$, and $l_i = \sigma \overline{a}_i m_i$.

From (A_2) and Definition 3, we know that there exists a vector $d > 0$ such that $(U+V)d < 0$. That is

$$-\underline{a}_i d_i + \sigma \overline{a}_i d_i + (1 + \sigma \overline{a}_i) k_i \sum_{j=1}^{n} w_{ij}^* d_j < 0. \tag{14}$$

From (14), we can get

$$\sum_{j=1}^{n} w_{ij}^* d_j \le d_i \frac{\underline{a}_i - \sigma \overline{a}_i^2}{1 + \sigma \overline{a}_i} \le d_i \underline{a}_i. \tag{15}$$

Hence, $-(P+Q)$ is a nonsingular M-matrix. Thus, there exists a vector $z_0 > 0$ such that $(P+Q)z_0 < 0$. By using the continuity, we know there exists at least one constant $\lambda_0 > 0$ such that

$$\left(\lambda_0 E + P + Q e^{\lambda_0 \tau}\right) z_0 < 0. \tag{16}$$

Since $\varphi \in C[[-r,0], R^n]$, then, for $z_0 > 0$, there exists a constant $M \ge 0$ such that

$$v(t) = [y(t)]^+ \le M z_0 e^{-\lambda_0 t} - (P+Q)^{-1} L, \quad t \in [-r, 0], \tag{17}$$

where $L = (l_1, \ldots, l_n)^T$. Then from (13), (16), (17), and Lemma 4, we get

$$v(t) = [y(t)]^+ \le M z_0 e^{-\lambda_0 t} - (P+Q)^{-1} L, \quad t \in [0, (1-\alpha)\sigma]. \tag{18}$$

Case 2. If $t \ge (1-\alpha)\sigma$, then from (10) we have

$$\begin{aligned} & \dot{y}_i(t-(1-\alpha)\sigma) \\ & = -a_i(\lambda) y_i(t-(2-\alpha)\sigma) \\ & \quad + f_i\left(\sum_{j=1}^{n} \int_{-\tau(\lambda)}^{0} \left(y_j(t-(1-\alpha)\sigma+\theta) + x_j^*\right) dw_{ij}(\theta,\lambda) + I_i\right) \\ & \quad - f_i\left(\sum_{j=1}^{n} \int_{-\tau(\lambda)}^{0} x_j^* dw_{ij}(\theta,\lambda) + I_i\right). \end{aligned} \tag{19}$$

Substituting (19) into (12), we get

$$\begin{aligned}\frac{dy_i(t)}{dt} &= -a_i(\lambda)\, y_i(t) + \sigma a_i(\lambda)\\ &\quad\times\Bigg[-a_i(\lambda)\, y_i(t-(2-\alpha)\sigma)\\ &\quad + f_i\Bigg(\sum_{j=1}^{n}\int_{-\tau(\lambda)}^{0}\big(y_j(t-(1-\alpha)\sigma+\theta)\\ &\quad + x_j^*\big)\, dw_{ij}(\theta,\lambda) + I_i\Bigg)\\ &\quad - f_i\Bigg(\sum_{j=1}^{n}\int_{-\tau(\lambda)}^{0} x_j^* dw_{ij}(\theta,\lambda) + I_i\Bigg)\Bigg]\\ &\quad + f_i\Bigg(\sum_{j=1}^{n}\int_{-\tau(\lambda)}^{0}\big(y_j(t+\theta) + x_j^*\big)\, dw_{ij}(\theta,\lambda) + I_i\Bigg)\\ &\quad - f_i\Bigg(\sum_{j=1}^{n}\int_{-\tau(\lambda)}^{0} x_j^* dw_{ij}(\theta,\lambda) + I_i\Bigg).\end{aligned} \tag{20}$$

Let $V_i(t) = |y_i(t)|$, $i \in N$. Then, from (A_1) and (A_2), we have

$$\begin{aligned} D^+V_i(t) &= \operatorname{sgn}(y_i(t))\, \dot{y}_i(t)\\ &= \operatorname{sgn}(y_i(t))\Bigg\{-a_i(\lambda)\, y_i(t) + \sigma a_i(\lambda)\\ &\quad\times\Bigg[-a_i(\lambda)\, y_i(t-(2-\alpha)\sigma)\\ &\quad + f_i\Bigg(\sum_{j=1}^{n}\int_{-\tau(\lambda)}^{0}\big(y_j(t-(1-\alpha)\sigma+\theta)\\ &\quad + x_j^*\big)\, dw_{ij}(\theta,\lambda) + I_i\Bigg)\\ &\quad - f_i\Bigg(\sum_{j=1}^{n}\int_{-\tau(\lambda)}^{0} x_j^* dw_{ij}(\theta,\lambda) + I_i\Bigg)\Bigg]\\ &\quad + f_i\Bigg(\sum_{j=1}^{n}\int_{-\tau(\lambda)}^{0}\big(y_j(t+\theta)\\ &\quad + x_j^*\big)\, dw_{ij}(\theta,\lambda) + I_i\Bigg)\\ &\quad - f_i\Bigg(\sum_{j=1}^{n}\int_{-\tau(\lambda)}^{0} x_j^* dw_{ij}(\theta,\lambda) + I_i\Bigg)\Bigg\}\\ &\le -a_i(\lambda)\,|y_i(t)| + a_i^2(\lambda)\,\sigma[y_i(t)]^+_{r+(1-\alpha)\sigma}\\ &\quad + a_i(\lambda)\,\sigma k_i\sum_{j=1}^{n} w_{ij}^*[y_j(t)]^+_{r+(1-\alpha)\sigma}\\ &\quad + k_i\sum_{j=1}^{n} w_{ij}^*[y_j(t)]^+_{r+(1-\alpha)\sigma}\\ &\le -\underline{a}_i\,|y_i(t)| + \sigma\overline{a}_i^2[y_i(t)]^+_{r+(1-\alpha)\sigma}\\ &\quad + (\sigma\overline{a}_i + 1)\, k_i\sum_{j=1}^{n} w_{ij}^*[y_j(t)]^+_{r+(1-\alpha)\sigma}\\ &= \sum_{j=1}^{n} u_{ij}V_j(t) + \sum_{j=1}^{n} v_{ij}[V_j(t)]_{r+(1-\alpha)\sigma}.\end{aligned} \tag{21}$$

Since $-(U+V)$ is a nonsingular M-matrix, there exists a vector $z > 0$ such that $(U+V)z < 0$. By using the continuity, we know there exists at least one constant $\gamma > 0$ such that

$$\left(\gamma E + P + Qe^{\gamma(r+\sigma)}\right) z < 0. \tag{22}$$

From (14) and (15), we know $y(t)$is bounded on $[-r, (1-\alpha)\sigma]$. So there exists a vector $\widehat{\eta} = \eta(1,\ldots,1)^T$ such that $[y(t)]^+ \le \widehat{\eta}$, $t \in [-r, (1-\alpha)\sigma]$. Then we can get

$$\begin{gathered} v_i(t) = |y_i(t)| \le \widehat{\eta} z e^{-\gamma(t-(1-\alpha)\sigma)},\\ t \in [-r, (1-\alpha)\sigma], \quad i \in N. \end{gathered} \tag{23}$$

Then from (21), (22), (23), and Lemma 4 with $J = 0$, we have

$$\begin{gathered} v_i(t) = |y_i(t)| \le \widehat{\eta} z e^{-\gamma(t-(1-\alpha)\sigma)},\\ t \in [(1-\alpha)\sigma, \infty], \quad i \in N. \end{gathered} \tag{24}$$

Let $M = z\widehat{\eta}e^{\gamma\sigma}$, we get

$$[x(t) - x^*]^+ \le Me^{-\gamma t}, \quad t \ge 0. \tag{25}$$

Thus, the equilibrium $x^*$of system (1) is globally exponentially robustly stable. □

Remark 7. If $\sigma = 0$, the system (1) becomes the static interval neural networks without time delay in the leakage term. So, this paper includes the results of Han et al. (2011) as a special case.

Remark 8. If $\sigma = 0$,

$$\begin{gathered} w_{ij}(\theta,\lambda) = \begin{cases} w_{ij}, & \theta = 0,\\ 0, & -\tau \le \theta < 0, \end{cases}\\ \tau(\lambda) = \tau, \qquad a_i(\lambda) = a, \qquad b_{ij} = \tau w_{ij}, \end{gathered} \tag{26}$$

then system (1) becomes the following static neural network model:

$$\begin{gathered} \frac{dx_i(t)}{dt} = -a_i x_i(t) + f_i\left(\sum_{j=1}^{n} b_{ij}x_j(t) + I_i\right), \quad t \ge 0,\\ x_i(t) = \phi_i(t), \quad -r \le t \le 0. \end{gathered} \tag{27}$$

If $\sigma = 0$ and

$$w_{ij}(\theta,\lambda) = \begin{cases} \sum_{k=0}^{m} w_{ij}^{k}(\lambda), & \theta = \tau_0 = 0, \\ \sum_{k=1}^{m} w_{ij}^{k}(\lambda), & -\tau_1 \le \theta < 0, \\ \sum_{k=2}^{m} w_{ij}^{k}(\lambda), & -\tau_2 \le \theta < -\tau_1, \\ \vdots \\ w_{ij}^{m}(\lambda), & -\tau_m \le \theta < -\tau_{m-1}, \\ 0, & -\tau \le \theta < \tau_m, \end{cases} \tag{28}$$

where $-\tau < -\tau_m < \cdots < -\tau_1 < \tau_0 = 0$, then system (1) becomes a class of static neural network models with discrete time delays as follows:

$$\begin{aligned} \frac{dx_i(t)}{dt} &= -a_i(\lambda)\,x_i(t) \\ &\quad + f_i\left(\sum_{k=0}^{m}\sum_{j=1}^{n} w_{ij}^{k}(\lambda)\,x_j(t-\tau_k) + I_i\right), \\ &\qquad t \ge 0, \\ x_i(t) &= \phi_i(t), \quad -r \le t \le 0. \end{aligned} \tag{29}$$

If $\sigma = 0$ and $w_{ij}(\theta,\lambda) \in C^1[-\tau,0]$, then system (1) becomes the following static model with continuous time delays:

$$\begin{aligned} \frac{dx_i(t)}{dt} &= -a_i(\lambda)\,x_i(t) \\ &\quad + f_i\left(\sum_{j=1}^{n}\int_{-\tau(\lambda)}^{0} x_j(t+\theta)\,w_{ij}'(\theta,\lambda)\,d\theta + I_i\right), \\ &\qquad t \ge 0, \\ x_i(t) &= \phi_i(t), \quad -r \le t \le 0. \end{aligned} \tag{30}$$

From (29) and (30), we can see that S-type distributed time delay contains discrete time delays and continuous time delays as two special cases, so our results generalized the results of the related literature [1, 3, 10].

4. Example

Consider the following system:

$$\begin{aligned} \frac{dx_1(t)}{dt} &= -(2+\lambda)\,x_1\left(t-\frac{1}{10}\right) \\ &\quad + \sin\left(\sum_{j=1}^{2}\int_{-\tau(\lambda)}^{0} x_j(t+\theta)\,dw_{ij}(\theta,\lambda) + 1\right), \\ \frac{dx_2(t)}{dt} &= -(2+\lambda)\,x_2\left(t-\frac{1}{10}\right) \\ &\quad + \cos\left(\sum_{j=1}^{2}\int_{-\tau(\lambda)}^{0} x_j(t+\theta)\,dw_{ij}(\theta,\lambda) + 2\right), \\ x_i(t) &= \phi_i(t), \quad -r \le t \le 0, \end{aligned} \tag{31}$$

where $\lambda \in [0,1]$, $|\int_{-\tau(\lambda)}^{0} dw_{11}(\theta,\lambda)| \le 1/3$, $|\int_{-\tau(\lambda)}^{0} dw_{12}(\theta,\lambda)| \le 1/2$, $|\int_{-\tau(\lambda)}^{0} dw_{21}(\theta,\lambda)| \le 1/2$, and $|\int_{-\tau(\lambda)}^{0} dw_{22}(\theta,\lambda)| \le 1/4$, $k_1 = k_2 = 1$. It can be obtained that

$$U = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}, \qquad V = \begin{pmatrix} \frac{4}{3} & \frac{13}{20} \\ \frac{13}{20} & \frac{49}{40} \end{pmatrix}. \tag{32}$$

Thus

$$-(U+V) = -\begin{pmatrix} -\frac{2}{3} & \frac{13}{20} \\ \frac{13}{20} & -\frac{31}{40} \end{pmatrix} \tag{33}$$

is a M-matrix. From Theorem 6, the equilibrium point of system (27) is globally exponentially robust stable.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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