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*Research Article*

# Finite-Time $H_\infty$ Control for a Class of Discrete-Time Markov Jump Systems with Actuator Saturation via Dynamic Antiwindup Design

**Junjie Zhao,[1] Jing Wang,[2] and Bo Li[1]**

[1] *School of Automation, Nanjing University of Science and Technology, Nanjing 210094, China*
[2] *School of Electrical and Information Engineering, Anhui University of Technology, Ma'anshan 243002, China*

Correspondence should be addressed to Bo Li; ggarfieldhero80@gmail.com

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We deal with the finite-time control problem for discrete-time Markov jump systems subject to saturating actuators. A finite-state Markovian process is given to govern the transition of the jumping parameters. A controller designed for unconstrained systems combined with a dynamic antiwindup compensator is given to guarantee that the resulting system is mean-square locally asymptotically finite-time stabilizable. The proposed conditions allow us to find dynamic anti-windup compensator which stabilize the closed-loop systems in the finite-time sense. All these conditions can be expressed in the form of linear matrix inequalities and therefore are numerically tractable, as shown in the example included in the paper.

## 1. Introduction

It is well known that more and more attention has been paid to the study of actuator saturation due to its practical and theoretical importance. Therefore, various approaches were investigated to handle systems with actuator saturation and dynamic antiwindup approach which is one of the most effective ways to deal with it. To this end, a great number of results have been reported in the literature; see, for example, [1, 2]. Furthermore, the stabilization problem of singular Markovian jump systems with discontinuities and saturation inputs was presented in [3]. Via dynamic antiwindup fuzzy design, the robust stabilization problem of state delayed T-S fuzzy systems with input saturation was proposed in [4].

On the other hand, Markov jump is frequently encountered in many practical systems. Therefore, the study of Markov jump systems has been a hot research topic due to its importance, and many results have been proposed based on various control techniques, such as robust control [5–9], $H_\infty$ control [10, 11], Passivity-based control [12–14], fuzzy dissipative control [15], and neural networks control [14, 16]. Furthermore, observer based finite-time $H_\infty$ control problem of discrete-time Markov jump systems was studied [17].

As it is well known, when dealing with the stability of s system, a distinction should have been made between classical Lyapunov stability and finite-time stability (FTS). Conversely, a system is said to be finite-time stable if, once we fix a time-interval, its state does not exceed some bounds during this time-interval. Some results on FTS have been carried out; see, [18, 19]. Furthermore, finite-time $H_\infty$ filtering problem of time-delay stochastic jump systems with unbiased estimation was proposed in [20]. By applying dynamic observer-based state feedback and the Lyapunov-Krasovskii functional approach, the finite-time $H_\infty$ control problem for time-delay nonlinear jump systems was addressed in the work of He and Liu [21]. However, to the best of our knowledge, the problem of finite-time stabilization of discrete-time stochastic systems subject to actuator saturation has not been fully investigated and it is the main purpose of our study.

In this paper, the attention is focused on the finite-time $H_\infty$ control problem of discrete-time Markov jump systems with actuator saturation based on dynamic anti-windup approach. A controller designed for unconstrained systems combined with a dynamic antiwindup compensator is given to ensure the stochastic finite-time boundedness and stochastic finite-time stabilization of the resulting closed-loop system for all admissible disturbances. The desired compensator can be designed via solving a convex optimization problem. Finally, a numerical example is employed to show the effectiveness of the proposed method.

*Notation 1.* Throughout the paper, for symmetric matrices $X$ and $Y$, the notation $X \geq Y$ (resp., $X > Y$) means that the matrix $X - Y$ is positive semidefinite (resp., positive definite). $I$ is the identity matrix with appropriate dimension. The notation $N^T$ represents the transpose of the matrix $N$; $\lambda_{\max}(M)$ (resp., $\lambda_{\min}(M)$) means the largest (resp., smallest) eigenvalue of the matrix $M$; $(\Omega, \mathcal{F}, \mathcal{P})$ is a probability space; $\Omega$ is the sample space; $\mathcal{F}$ is the $\sigma$-algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$;$\mathcal{E}\{\cdot\}$ denotes the expectation operator with respect to some probability measure $\mathcal{P}$. Matrices, if not explicitly stated, are assumed to have compatible dimensions. The symbol $*$ is used to denote a matrix which can be inferred by symmetry. $He\{A\} = A^T + A$.

## 2. Preliminaries and Problem Description

Consider the following discrete-time Markov jump system $(\Sigma)$ in the probability space $(\Omega, \mathcal{F}, \mathcal{P})$:

$$\begin{aligned} x_p(k+1) &= A_p(r(k))\, x_p(k) + B_{p,u}(r(k))\, \mathrm{sat}(u(k)) \\ &\quad + B_{p,w}(r(k))\, w(k), \\ y(k) &= C_{p,y}(r(k))\, x_p(k) + D_{p,yw}(r(k))\, w(k), \\ z(k) &= C_{p,z}(r(k))\, x_p(k) + D_{p,zu}(r(k))\, \mathrm{sat}(u(k)) \\ &\quad + D_{p,zw}(r(k))\, w(k), \end{aligned} \tag{1}$$

where $x_p(k) \in \mathbb{R}^{n_p}$ is the state vector, $u(k) \in \mathbb{R}^{n_u}$ is the control input, and $\mathrm{sat}(u(k)) \in \mathbb{R}^{n_u}$ is the saturated control input. $w(k) \in L_2^p[0 \ \ +\infty)$ is the external disturbances, $y(k) \in \mathbb{R}^{n_y}$ is the measurement output, and $z(k) \in \mathbb{R}^{n_z}$ is the performance output. $\{r(k)\}$ is a discrete-time Markov process and takes values from a finite set $S = \{1, 2, \ldots, \mathcal{N}\}$ with transition probabilities given by

$$\Pr\left(r_{k+1} = j \mid r_k = i\right) = \pi_{ij}, \tag{2}$$

where $\pi_{ij} \geq 0$, for $\forall j, i \in S$, and $\sum_{j \in S} \pi_{ij} = 1$. Moreover, the transition rates matrix of the system $(\Sigma)$ is defined by

$$\begin{bmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1\mathcal{N}} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2\mathcal{N}} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{\mathcal{N}1} & \pi_{\mathcal{N}2} & \cdots & \pi_{\mathcal{N}\mathcal{N}} \end{bmatrix}. \tag{3}$$

The plant inputs are supposed to be bounded as follows:

$$-u_{0(k)} \leq u_{(k)} \leq u_{0(k)}, \quad u_{0(k)} > 0,\ k = 1, \ldots, m. \tag{4}$$

For the system $(\Sigma)$, to simplify the notation, we denote $A_{pi} = A_p(r(k))$ for each $r(k) = i \in S$, and the other symbols are similarly denoted. Assume that a linear controller is designed for any $r(k) = i \in S$; then,

$$\begin{aligned} x_c(k+1) &= A_{ci} x_c(k) + B_{cy,i}\, y(k) + B_{cw,i}\, w(k) + v_1, \\ y_c(k) &= C_{ci} x_c(k) + D_{cy,i}\, y(k) + D_{cw,i}\, w(k) + v_2, \end{aligned} \tag{5}$$

where $x_c(k) \in \mathbb{R}^{n_c}$ is the controller state and $y_c(k) \in \mathbb{R}^{n_u}$ is the controller output; $v_1$ and $v_2$ will be used for antiwindup augmentation. In absence of actuator saturation, the unconstrained closed-loop is formed by setting the following:

$$u = y_c, \qquad v_1 = 0, \qquad v_2 = 0. \tag{6}$$

*Assumption 1.* The unconstrained closed-loop system (1)–(5) is well posed and internally stable.

In the presence of actuator saturation, the relation between $u$ and $y_c$ is that $u = \mathrm{sat}(y_c)$. To minimize performance degradation caused by saturation, the following antiwindup compensator is designed for the closed-loop systems:

$$\begin{aligned} x_{\mathrm{aw}}(k+1) &= A_{\mathrm{aw},i} x_{\mathrm{aw}}(k) + B_{\mathrm{aw},i}\, \psi\left(y_c(k)\right), \\ v(k) &= C_{\mathrm{aw},i} x_{\mathrm{aw}}(k) + D_{\mathrm{aw},i}\, \psi\left(y_c(k)\right), \end{aligned} \tag{7}$$

where $\psi(y_c(k)) = \mathrm{sat}(y_c(k)) - y_c(k)$. The resulting nonlinear closed-loop system (1), (5), (7) is depicted in Figure 1 and can be represented in the following compact form:

$$\begin{aligned} x(k+1) &= A_i x(k) + B_{qi}\, \psi\left(y_c(k)\right) + B_{wi}\, w(k), \\ y_c(k) &= K_i x(k) + K_{\phi,i}\, \psi\left(y_c(k)\right) + K_{wi}\, w(k), \\ z(k) &= C_{zi} x(k) + D_{zq,i}\, \psi\left(y_c(k)\right) + D_{zw,i}\, w(k), \end{aligned} \tag{8}$$

where

$$\begin{aligned} A_i &= \begin{bmatrix} A_{pi} + B_{pu,i} D_{cy,i} C_{py,i} & B_{pu,i} C_{ci} & B_{pu,i} I_2 C_{aw,i} \\ B_{cy,i} C_{py,i} & A_{ci} & I_1 C_{aw,i} \\ 0 & 0 & A_{aw,i} \end{bmatrix}, \\ B_{qi} &= \begin{bmatrix} B_{pu,i} I_2 D_{aw,i} + B_{pu,i} \\ I_1 D_{aw,i} \\ B_{aw,i} \end{bmatrix}, \\ B_{wi} &= \begin{bmatrix} B_{pw,i} + B_{pu,i} D_{cy,i} D_{p,yw,i} + B_{pu,i} D_{cw,i} \\ B_{cy,i} D_{p,yw,i} + B_{cw,i} \\ 0 \end{bmatrix}, \\ C_{zi} &= \left[C_{pz,i} + D_{p,zu,i} D_{ci} C_{py,i} \quad D_{p,zu,i} \quad I_2 C_{aw,i}\right], \\ D_{zq,i} &= I_2 D_{aw,i} + D_{p,zu,i}, \\ D_{zw,i} &= D_{p,zw,i} + D_{p,zu,i} D_{cw,i} + D_{p,zu,i} D_{ci} D_{p,yw,i}, \\ K_i &= \left[D_{cy,i} C_{py,i} \quad C_{ci} \quad I_2 C_{aw,i}\right], \\ K_{\phi,i} &= I_2 D_{aw,i}, \qquad K_{w,i} = D_{cw,i} + D_{p,yw,i}, \\ I_1 &= \left[I \quad 0\right], \qquad I_2 = \left[0 \quad I\right]. \end{aligned} \tag{9}$$

For this system, we introduce the following definitions and assumption.

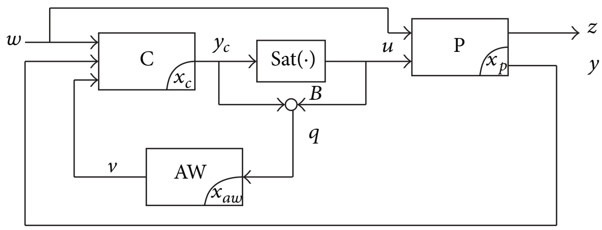


Figure 1: The closed-loop systems with input saturation.

*Assumption 2* (see [17]). The external disturbance $w(k)$ is varying and satisfies the following constraint condition:

$$\sum_{k=0}^{T} w(k)^T w(k) \le d, \quad d \ge 0. \tag{10}$$

*Definition 3* (see [17]). The resulting closed-loop system (8) is stochastic finite-time stable (SFTB) with respect to $(\delta_x, \epsilon, R_i, N, d)$ with $0 < \delta_x < \epsilon, R_i > 0$, and $N \in Z_{k\ge 0}$, if

$$E\left\{x^T(0) R_i x(0)\right\} \le \delta_x^2 \Longrightarrow E\left\{x^T(k) R_i x(k)\right\} < \epsilon^2, \quad \forall k \in \{1, 2, \ldots, N\}. \tag{11}$$

*Definition 4* (see [17]). The resulting closed-loop system (8) is said to be stochastic $H_\infty$ finite-time stable with respect to $(\delta_x, \epsilon, R_i, N, \gamma, d)$ with $0 < \delta_x < \epsilon$, $R_i > 0$, $\gamma > 0$, and $N \in Z_{k\ge 0}$, if the system (8) is SFTB with respect to $(\delta_x, \epsilon, R_i, N, \gamma, d)$, and under the zero-initial condition, the output $z(k)$ satisfies

$$E\left\{\sum_{j=0}^{N} z^T(j) z(j)\right\} \le \gamma^2 E\left\{\sum_{j=0}^{N} w^T(j) w(j)\right\}, \tag{12}$$

for any nonzero $w(k)$ which satisfies (10), where $\gamma$ is a prescribed positive scalar.

## 3. Main Results

In this section, we investigate the stabilization analysis of the unconstrained systems and the antiwindup controller design of the resulting closed-loop system. Some sufficient conditions in terms of LMI are given. Before presenting the main results, we give some lemmas as follows.

**Lemma 5** (see [4]). *For the closed-loop systems* (8) *with the matrix $K_i$, the appropriate matrix $L_i \in \mathbb{R}^{m\times n}$ is given, if $x(k)$ is in the set $D(u_o)$, where $D(u_o)$ is defined as follows:*

$$D(u_o) = \left\{x(k) \in \mathbb{R}^n;\ -u_{0(k)} \le \left(K_{i(k)} + L_{i(k)}\right) x(t) \le u_{0(k)},\ u_{0(k)} > 0,\ k = 1, \ldots, m\right\}, \tag{13}$$

*then for any diagonal positive matrix $T \in \mathbb{R}^{m\times m}$, one has the following:*

$$\psi(u(k))^T T\left(\psi(u(k)) - L_i x(k) + K_{\phi,i}\psi\left(y_c(k)\right) + K_{wi} w(k)\right) \le 0. \tag{14}$$

**Lemma 6** (see [12]). *For the given symmetric matrix $S \in \mathbb{R}^{(n+m)\times(n+m)}$,*

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix}, \tag{15}$$

*where $S_{11} \in \mathbb{R}^{n\times n}$, $S_{12} \in \mathbb{R}^{n\times m}$, and $S_{22} \in \mathbb{R}^{m\times m}$, the following conditions are equivalent:*

(1) $S < 0$;

(2) $S_{11} < 0,\ S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;

(3) $S_{22} < 0,\ S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

*3.1. Design of Controller.* In this section, we design the controller for the unconstrained systems with $v_1 = 0$ and $v_2 = 0$. Combining system (1) with controller (5), we have

$$\begin{aligned} x(k+1) &= A_i x(k) + B_{wi} w(k), \\ z(k) &= C_{zi} x(k) + D_{zw,i} w(k), \end{aligned} \tag{16}$$

where

$$\begin{aligned} A_i &= \begin{bmatrix} A_{pi} + B_{pu,i} D_{cy,i} C_{py,i} & B_{pu,i} C_{ci} \\ B_{cy,i} C_{py,i} & A_{ci} \end{bmatrix}, \\ B_{wi} &= \begin{bmatrix} B_{pw,i} + B_{pu,i} D_{cy,i} D_{p,yw,i} + B_{pu,i} D_{cw,i} \\ B_{cy,i} D_{p,yw,i} + B_{cw,i} \end{bmatrix}. \end{aligned} \tag{17}$$

**Theorem 7.** *For each $r(k) = i \in S$, the unconstrained system (16) is SFTB with respect to $(\delta_x, \epsilon, R_i, N, d)$ with $0 < \delta_x < \epsilon$, if there exist scalars $\mu \ge 0$, $\sigma_1 \ge 0$, $\sigma_2 \ge 0$, and the given $\lambda > 0$, two sets of mode-dependent symmetric positive-defined matrices $\{X_i, i \in S\}$ and $\{Q_i, i \in S\}$, such that the following conditions hold:*

$$\begin{bmatrix} -\mu\lambda I & 0 & L_{1i}^T \\ * & -Q_i & L_{2i}^T \\ * & * & -W \end{bmatrix} < 0, \tag{18}$$

$$\begin{bmatrix} \sigma_2 d^2 - \mu^{-N}\epsilon^2 & * \\ \delta_x & -\sigma_1 \end{bmatrix} < 0, \tag{19}$$

$$\lambda X_i < I, \tag{20}$$

$$\sigma_1 R_i^{-1} < X_i < R_i^{-1}, \tag{21}$$

$$0 < Q_i < \sigma_2 I, \tag{22}$$

*where*

$$W = \operatorname{diag}\{X_1,\ X_2,\ \ldots,\ X_n\},$$
$$\overline{L}_{1i}^T = \left[\sqrt{\pi_{i1}}A_i^T \quad \sqrt{\pi_{i2}}A_i^T \quad \cdots \quad \sqrt{\pi_{in}}A_i^T\right], \tag{23}$$
$$L_{2i}^T = \left[\sqrt{\pi_{i1}}B_{wi}^T \quad \sqrt{\pi_{i2}}B_{wi}^T \quad \cdots \quad \sqrt{\pi_{in}}B_{wi}^T\right].$$

*Proof.* Define the following Lyapunov function for each $\delta(t) = i \in S$:

$$V(k) = x(k)^T P_i x(k). \tag{24}$$

It is readily obtained that

$$\begin{aligned} E\{V(k+1)\} &= E\left\{\sum_{j=1}^{n}\pi_{ij}x(k+1)^T P_j x(k+1)\right\} \\ &= \xi(k)^T[L_{1i}\ \ L_{2i}]^T\,\overline{W}\,[L_{1i}\ \ L_{2i}]\,\xi(k), \end{aligned} \tag{25}$$

where

$$\begin{aligned} \xi(k) &= \left[x(k)^T \quad w(k)^T\right], \\ \overline{W} &= \operatorname{diag}\{P_1,\ P_2,\ \ldots,\ P_h\}. \end{aligned} \tag{26}$$

By using of Schur complement lemma to (18), and note that $P_i^{-1} = X_i$ and $\lambda X_i < I$, we derive $\lambda I < P_i$; then, we have

$$\xi(k)^T\begin{bmatrix}-\mu\lambda I & 0 \\ * & -Q_i\end{bmatrix}\xi(k) + \xi(k)^T[L_{1i}\ \ L_{2i}]^T\overline{W}[L_{1i}\ \ L_{2i}]\xi(k) < 0. \tag{27}$$

It follows that

$$E\{V(k+1)\} < \mu x(k)^T P_i x(k) + w(k)^T Q_i w(k). \tag{28}$$

It is shown that

$$E\{V(k+1)\} < \mu V(k) + \sup_{(i\in S)}\{\lambda_{\max}(Q_i)\}\, w(k)^T w(k). \tag{29}$$

Then we have

$$\begin{aligned} &E\{V(k+1)\} \\ &\quad < \mu E\{V(k)\} + \sup_{(i\in S)}\{\lambda_{\max}(Q_i)\}\, E\left\{w(k)^T w(k)\right\}. \end{aligned} \tag{30}$$

Since $\mu \geq 1$, it is easily found that

$$\begin{aligned} E\{V(k+1)\} &< \mu E\{V(0)\} + \sup_{(i\in S)}\{\lambda_{\max}(Q_i)\} \\ &\quad \times E\left\{\sum_{j=0}^{k-1}\mu^{k-j-1}w(j)^T w(j)\right\} \\ &\leq \mu^k E\{V(0)\} + \sup_{(i\in S)}\{\lambda_{\max}(Q_i)\}\,\mu^k d^2. \end{aligned} \tag{31}$$

Letting

$$\overline{P}_i = R_i^{-1/2} P_i R_i^{-1/2}, \tag{32}$$

and noting that

$$E\left\{x^T(0) R_i x(0)\right\} \leq \delta_x^2, \tag{33}$$

it can be verified that

$$\begin{aligned} E\{V(0)\} &= E\left\{x^T(0) P_i x(0)\right\} \\ &= E\left\{x^T(0) R_i^{1/2}\overline{P}_i R_{i^{1/2}} x(0)\right\} \\ &\leq \sup_{i\in S}\left\{\lambda_{\max}\left(\overline{P}_i\right)\right\} E\left\{x^T(0) R_i x(0)\right\} \\ &\leq \sup_{i\in S}\left\{\lambda_{\max}\left(\overline{P}_i\right)\right\}\delta_x^2. \end{aligned} \tag{34}$$

Similarly, for all $i \in S$, we can obtain

$$\begin{aligned} E\{V(k)\} &= E\left\{x^T(k) P_i x(k)\right\} \\ &= E\left\{x^T(k) R_i^{1/2}\overline{P}_i R_{i^{1/2}} x(k)\right\} \\ &\geq \inf_{i\in S}\{\lambda_{\min}\}\left(\overline{P}_i\right) E\left\{x^T(k) R_i x(k)\right\}. \end{aligned} \tag{35}$$

Then, it is not difficult to find that

$$E\left\{x^T(k) R_i x(k)\right\} < \frac{\sup_{i\in S}\left\{\lambda_{\max}\left(\overline{P}_i\right)\right\}\mu^k\delta_x^2 + \sup_{(i\in S)}\{\lambda_{\max}(Q_i)\}\mu^k d^2}{\inf_{i\in S}\{\lambda_{\min}\}\left(\overline{P}_i\right)}, \tag{36}$$

which implies that

$$\frac{\sup_{i\in S}\left\{\lambda_{\max}\left(\overline{P}_i\right)\right\}\mu^k\delta_x^2 + \sup_{(i\in S)}\{\lambda_{\max}(Q_i)\}\mu^k d^2}{\inf_{i\in S}\{\lambda_{\min}\}\left(\overline{P}_i\right)} < \epsilon^2. \tag{37}$$

Then, one can obtain that

$$\begin{aligned} &\sup_{i\in S}\left\{\lambda_{\max}\left(\overline{P}_i\right)\right\}\delta_x^2 + \sup_{(i\in S)}\{\lambda_{\max}(Q_i)\}\, d^2 \\ &\quad < \inf_{i\in S}\{\lambda_{\min}\}\left(\overline{P}_i\right)\mu^{-N}\epsilon^2. \end{aligned} \tag{38}$$

Setting

$$\begin{aligned} X_i &= P_i^{-1}, \\ \sigma_1 R_i^{-1} &< X_i < R_i^{-1}, \\ 0 &< Q_i < \sigma_2 I, \end{aligned} \tag{39}$$

it is easy to see that

$$\sigma_1^{-1}\delta_x^2 + \sigma_2 d^2 < \mu^{-N}\epsilon^2. \tag{40}$$

It is obvious that (40) is equivalent to (19).
This completes the proof. □

*3.2. Design of Dynamic Antiwindup Compensator*

**Theorem 8.** *For each $r(k) = i \in S$, with antiwindup compensator (7), such that the resulting closed-loop system (10) is SFTB with respect to $(\delta_x, \epsilon, R_i, N, d)$ with $0 < \delta_x < \epsilon$, if there exist scalars $\mu \geq 0$, $\sigma_1 \geq 0$, and $\sigma_2 \geq 0$, three sets of mode-dependent symmetric positive-defined matrices $\{X_i, i \in S\}$, $\{Q_i, i \in S\}$ and diag positive-defined matrices $\{S_i, i \in S\}$, and two sets of mode-dependent matrices $\{Y_i, i \in S\}$ and $\{\overline{L}_i = L_i X_i, \overline{A}_{aw,i} = A_{aw,i}X_i, \overline{C}_{aw,i} = C_{aw,i}X_i, \overline{B}_{aw,i} = B_{aw,i}S_i, \overline{D}_{aw,i} = D_{aw,i}S_i \ i \in S\}$, such that the following conditions hold:*

$$\begin{bmatrix} -\mu X_i & 0 & \overline{L}_i^T & \overline{L}_{1i}^T \\ * & -Q_i & K_{wi}^T & L_{2i}^T \\ * & * & -2S_i - He\left(\overline{K}_{\varphi,i}\right) & \overline{L}_{3i}^T \\ * & * & * & -W \end{bmatrix} < 0, \quad (41)$$

$$\begin{bmatrix} \sigma_2 d^2 - \mu^{-N}\epsilon^2 & * \\ \delta_x & -\sigma_1 \end{bmatrix} < 0, \quad (42)$$

$$\begin{bmatrix} X_i & * \\ \overline{K}_i + \overline{L}_i & u_{0(k)}^2 \end{bmatrix} > 0, \quad k = 1, \ldots, m, \quad (43)$$

$$\sigma_1 R_i^{-1} < X_i < R_i^{-1}, \quad (44)$$

$$0 < Q_i < \sigma_2 I, \quad (45)$$

*where*

$$\begin{gathered} W = \operatorname{diag}\{X_1, X_2, \ldots, X_n\}, \\ \overline{L}_{1i}^T = \left[\sqrt{\pi_{i1}}\overline{A}_i^T \ \ \sqrt{\pi_{i2}}\overline{A}_i^T \ \ \cdots \ \ \sqrt{\pi_{in}}\overline{A}_i^T\right], \\ L_{2i}^T = \left[\sqrt{\pi_{i1}}B_{wi}^T \ \ \sqrt{\pi_{i2}}B_{wi}^T \ \ \cdots \ \ \sqrt{\pi_{in}}B_{wi}^T\right], \\ L_{3i}^T = \left[\sqrt{\pi_{i1}}\overline{B}_{qi}^T \ \ \sqrt{\pi_{i2}}\overline{B}_{qi}^T \ \ \cdots \ \ \sqrt{\pi_{in}}\overline{B}_{qi}^T\right], \end{gathered} \quad (46)$$

*with*

$$\overline{A}_i = \begin{bmatrix} A_{pi}X_i + B_{pu,i}D_{cy,i}C_{py,i}X_i & B_{pu,i}C_{ci}X_i & B_{pu,i}I_2\overline{C}_{aw,i} \\ B_{cy,i}C_{py,i}X_i & A_{ci}X_i & I_1\overline{C}_{aw,i} \\ 0 & 0 & \overline{A}_{aw,i} \end{bmatrix},$$

$$\overline{B}_{qi} = \begin{bmatrix} B_{pu,i}I_2\overline{D}_{aw,i} + B_{pu,i}X_i \\ I_1\overline{D}_{aw,i} \\ \overline{B}_{aw,i} \end{bmatrix},$$

$$\overline{K}_i = \left[D_{cy,i}C_{py,i}X_i \ \ C_{ci}X_i \ \ I_2\overline{C}_{aw,i}\right], \qquad \overline{K}_{\phi,i} = I_2\overline{D}_{aw,i}. \quad (47)$$

*Proof.* Define the following Lyapunov function for each $\delta(t) = i \in S$:

$$V(k) = x(k)^T P_i x(k). \quad (48)$$

It is readily obtained that

$$\begin{aligned} E\{V(k+1)\} &= E\left\{\sum_{j=1}^{n}\pi_{ij}x(k+1)^T P_j x(k+1)\right\} \\ &= \xi(k)^T\left[L_{1i} \ \ L_{2i} \ \ L_{3i}\right]^T\overline{W}\left[L_{1i} \ \ L_{2i} \ \ L_{3i}\right]\xi(k), \end{aligned} \quad (49)$$

where

$$\begin{gathered} \xi(k) = \left[x(k)^T \ \ w(k)^T \ \ \psi(k)^T\right], \\ \overline{W} = \operatorname{diag}\{P_1, P_2, \ldots, P_h\}, \\ L_{1i}^T = \left[\sqrt{\pi_{i1}}A_i^T \ \ \sqrt{\pi_{i2}}A_i^T \ \ \cdots \ \ \sqrt{\pi_{in}}A_i^T\right], \\ L_{2i}^T = \left[\sqrt{\pi_{i1}}B_{wi}^T \ \ \sqrt{\pi_{i2}}B_{wi}^T \ \ \cdots \ \ \sqrt{\pi_{in}}B_{wi}^T\right], \\ L_{3i}^T = \left[\sqrt{\pi_{i1}}B_{qi}^T \ \ \sqrt{\pi_{i2}}B_{qi}^T \ \ \cdots \ \ \sqrt{\pi_{in}}B_{qi}^T\right]. \end{gathered} \quad (50)$$

Then, by pre- and postmultiplying (41) by $\operatorname{diag}\{P_i, I, T_i, I\}$ with $P_i = X_i^{-1}$, $T_i = S_i^{-1}$, we have

$$\begin{bmatrix} -\mu P_i & 0 & L_i^T T_i & L_{1i}^T \\ * & -Q_i & K_{wi}^T & L_{2i}^T \\ * & * & -2T_i - He\left(T_i K_{\phi,i}\right) & L_{3i}^T \\ * & * & * & -W \end{bmatrix} < 0. \quad (51)$$

By using of Schur complement lemma, we derive

$$\begin{aligned} &\xi(k)^T\begin{bmatrix} -\mu P_i & 0 & L_i^T T_i \\ * & -Q_i & K_{wi}^T \\ * & * & -2T_i - He\left(T_i K_{\phi,i}\right) \end{bmatrix}\xi(k) \\ &\quad + \xi(k)^T\left[L_{1i} \ \ L_{2i} \ \ L_{3i}\right]^T\overline{W}\left[L_{1i} \ \ L_{2i} \ \ L_{3i}\right]\xi(k) < 0. \end{aligned} \quad (52)$$

It follows that

$$\begin{aligned} E\{V(k+1)\} &< \mu x(k)^T P_i x(k) + w(k)^T Q_i w(k) \\ &\quad + \psi(k)^T\left(2T_i + He\left\{T_i K_{\phi,i}\right\}\right)\psi(k) \\ &\quad - 2\psi(k)^T T_i L_i x(k) + 2\psi(k)^T T_i K_{wi} w(k). \end{aligned} \quad (53)$$

Since $\psi(k)^T(2T_i + He\{T_i K_{\phi,i}\})\psi(k) - 2\psi(k)^T T_i L_i x(k) + 2\psi(k)^T T_i K_{wi} w(k) \leq 0$, we get

$$E\{V(k+1)\} < \mu x(k)^T P_i x(k) + w(k)^T Q_i w(k). \quad (54)$$

It is shown that

$$E\{V(k+1)\} < \mu V(k) + \sup_{(i\in S)}\{\lambda_{\max}(Q_i)\}\, w(k)^T w(k). \tag{55}$$

Then, we have

$$\begin{aligned} E\{V(k+1)\} &< \mu E\{V(k)\} \\ &\quad + \sup_{(i\in S)}\{\lambda_{\max}(Q_i)\}\, E\{w(k)^T w(k)\}. \end{aligned} \tag{56}$$

Since $\mu \geq 1$, it is easily found that

$$\begin{aligned} E\{V(k+1)\} &< \mu E\{V(0)\} + \sup_{(i\in S)}\{\lambda_{\max}(Q_i)\} \\ &\quad \times E\left\{\sum_{j=0}^{k-1}\mu^{k-j-1} w(j)^T w(j)\right\} \\ &\leq \mu^k E\{V(0)\} + \sup_{(i\in S)}\{\lambda_{\max}(Q_i)\}\,\mu^k d^2. \end{aligned} \tag{57}$$

The following proof is similar to the process of Theorem 7. Based on Lemma 5, it is easy to obtain that

$$\begin{bmatrix} P_i & * \\ K_i + L_i & u_{0(k)}^2 \end{bmatrix} > 0, \quad k = 1,\ldots,m, \tag{58}$$

then pre- and post-multiply (58) by $\operatorname{diag}\{X_i, I\}$ which implies (43). This completes the proof. □

**Theorem 9.** *For each $r(k) = i \in S$, with antiwindup compensator* (7)*, such that the resulting closed-loop system* (10) *is said to be Stochastic $H_\infty$ finite-time stable via state feedback with respect to $(\delta_x, \epsilon, R_i, N, \gamma, d)$, if there exist three scalars $\mu \geq 0$, $\sigma_1 \geq 0$, and $\gamma \geq 0$, two sets of mode-dependent symmetric positive-defined matrices $\{X_i, i \in S\}$ and* diag *matrices $\{S_i, i \in S\}$, and two sets of mode-dependent matrices $\{Y_i, i \in S\}$ and $\{\overline{L}_i = L_i X_i, i \in S\}$, such that the following conditions hold:*

$$\begin{bmatrix} -\mu X_i & 0 & \overline{L}_i^T & \overline{L}_{1i}^T & \overline{C}_{zi}^T \\ * & -\gamma^2\mu^{-N} I & K_{wi}^T & L_{2i}^T & D_{zw,i}^T \\ * & * & -2S_i - He\left(\overline{K}_{\phi,i}\right) & \overline{L}_{3i}^T & \overline{D}_{zq,i}^T \\ * & * & * & -W & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \tag{59}$$

$$\begin{bmatrix} \mu^{-N}\left(d^2\gamma^2 - \epsilon^2\right) & * \\ \delta_x & -\sigma_1 \end{bmatrix} < 0, \tag{60}$$

$$\begin{bmatrix} X_i & * \\ \overline{K}_i + \overline{L}_i & u_{0(k)}^2 \end{bmatrix} > 0, \quad k = 1,\ldots,m, \tag{61}$$

$$\sigma_1 R_i^{-1} < X_i < R_i^{-1}, \tag{62}$$

*with*

$$\begin{aligned} \overline{C}_{zi} &= \left[\left(C_{pz,i} + D_{p,zu,i} D_{ci} C_{py,i}\right) X_i \;\; D_{p,zu,i} X_i \;\; I_2 \overline{C}_{aw,i}\right], \\ \overline{D}_{zq,i}^T &= I_2 \overline{D}_{aw,i} + D_{p,zu,i}. \end{aligned} \tag{63}$$

*Proof.* Choose the similar Lyapunov function as Theorem 7 and denote

$$\begin{aligned} \Pi(x(k), w(k), r(k) = i) &= E\{V(k+1)\} - \mu V(k) + z(k)^T z(k) \\ &\quad - \gamma^2\mu^{-N} w(k)^T w(k). \end{aligned} \tag{64}$$

Thus, in light of Lemma 5, we have

$$\begin{aligned} &\Pi(x(k), w(k), r_k = i) \\ &\leq \xi(k)^T [L_{1i} \;\; L_{2i} \;\; L_{3i}]^T \overline{W} [L_{1i} \;\; L_{2i} \;\; L_{3i}]\, \xi(k) \\ &\quad + \xi(k)^T [C_{zi} \;\; D_{zw,i} \;\; D_{zq,i}]^T [C_{zi} \;\; D_{zw,i} \;\; D_{zq,i}]\, \xi(k) \\ &\quad + \xi^T(k) \begin{bmatrix} -\mu X_i & 0 & L_i^T T_i \\ * & -\gamma^2\mu^{-N} I & K_{wi}^T \\ * & * & -2T_i - He\{T_i K_{\phi,i}\} \end{bmatrix} \xi(k). \end{aligned} \tag{65}$$

Then pre- and postmultiply (59) by $\operatorname{diag}\{P_i, I, T_i, I\}$, and considering Schur complement lemma and (65), we derive that

$$\Pi(x(k), w(k), r(k) = i) < 0 \tag{66}$$

holds for all $r_k = i \in S$. According to (66), one can obtain that

$$\begin{aligned} E\{V(k+1)\} &< \mu E\{V(k)\} - E\{z(k)^T z(k)\} \\ &\quad + \gamma^2\mu^{-N} E\{w(k)^T w(k)\}. \end{aligned} \tag{67}$$

Then, we have

$$\begin{aligned} E\{V(k)\} &< \mu^k E\{V(0)\} - \sum_{j=0}^{k-1}\mu^{k-j-1} E\{z(j)^T z(j)\} \\ &\quad + \gamma^2\mu^{-N} E\left\{\sum_{j=0}^{k-1}\mu^{k-j-1} w(j)^T w(j)\right\}. \end{aligned} \tag{68}$$

Under the zero-value initial condition and noting that $V(k) \geq 0$, for all $K \in Z_{k\geq 0}$, it is shown that

$$\sum_{j=0}^{k-1}\mu^{k-j-1} E\{z(j)^T z(j)\} < \gamma^2\mu^{-N} E\left\{\sum_{j=0}^{k-1}\mu^{k-j-1} w(j)^T w(j)\right\}. \tag{69}$$

Since $\mu \geq 1$ and from (69), we have

$$\begin{aligned} E\left\{\sum_{j=0}^{N} z(j)^T z(j)\right\} &= \sum_{j=0}^{N} E\left\{z(j)^T z(j)\right\} \\ &\leq \sum_{j=0}^{N} E\left\{\mu^{N-j} z(j)^T z(j)\right\} \\ &\leq \gamma^2 \mu^{-N} E\left\{\sum_{j=0}^{N} \mu^{N-j} w(j)^T w(j)\right\} \\ &\leq \gamma^2 E\left\{\sum_{j=0}^{N} w(j)^T w(j)\right\}. \end{aligned} \quad (70)$$

The following proof is similar to the process of Zhang and Liu [17].

Since $\varepsilon(P_i, 1) \subset D(u_0)$, it follows that

$$\begin{bmatrix} P_i & * \\ K_i + L_i & u_{0(k)}^2 \end{bmatrix} > 0, \quad k = 1, \ldots, m, \quad (71)$$

and then pre- and post-multiply (71) by $\text{diag}(X_i, I)$ and its transpose, respectively; we derive condition (61). This completes the proof. □

## 4. Illustrative Examples

In this section, a numerical example is provided to demonstrate the effectiveness of the proposed method. Consider the following systems with four operation modes.

Mode 1 is

$$\begin{gathered} A_{p1} = \begin{bmatrix} 0.75 & -0.75 \\ 1.5 & -1.5 \end{bmatrix}, \quad B_{pu,1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad B_{pw,1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\ C_{py,1} = \begin{bmatrix} -0.1 & -0.2 \end{bmatrix}, \quad C_{pz,1} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \\ D_{pyw,1} = 1, \quad D_{pzu,1} = 1, \quad D_{pzw,1} = 0.8. \end{gathered} \quad (72)$$

Mode 2 is

$$\begin{gathered} A_{p2} = \begin{bmatrix} 0.15 & 4.5 \\ 2.10 & -0.4 \end{bmatrix}, \quad B_{pu,2} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad B_{pw,2} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\ C_{py,2} = \begin{bmatrix} -0.1 & -0.1 \end{bmatrix}, \quad C_{pz,2} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \\ D_{pyw,2} = 1, \quad D_{pzu,2} = 0.9, \quad D_{pzw,2} = 0.8. \end{gathered} \quad (73)$$

Mode 3 is

$$\begin{gathered} A_{p3} = \begin{bmatrix} 0.24 & 2.50 \\ 1.2 & -2.1 \end{bmatrix}, \quad B_{pu,3} = \begin{bmatrix} 0.9 \\ 0 \end{bmatrix}, \quad B_{pw,3} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\ C_{py,3} = \begin{bmatrix} -0.1 & 0 \end{bmatrix}, \quad C_{pz,3} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \\ D_{pyw,3} = 0.8, \quad D_{pzu,3} = 1, \quad D_{pzw,3} = 1.2. \end{gathered} \quad (74)$$

Mode 4 is

$$\begin{gathered} A_{p4} = \begin{bmatrix} 1 & -0.25 \\ 1.5 & -1.5 \end{bmatrix}, \quad B_{pu,4} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad B_{pw,4} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\ C_{py,4} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad C_{pz,4} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \\ D_{pyw,4} = 1, \quad D_{pzu,4} = 0.5, \quad D_{pzw,4} = 1. \end{gathered} \quad (75)$$

With the given designed controllers,

$$\begin{gathered} A_{c1} = -5.5, \quad B_{cy,1} = -1, \quad B_{cw,1} = 1, \\ C_{c1} = -1, \quad D_{cy,1} = -0.1, \quad D_{cw,1} = 0.5, \\ A_{c2} = -5, \quad B_{cy,2} = -0.9, \quad B_{cw,2} = 1, \\ C_{c2} = -1, \quad D_{cy,2} = 5.9, \quad D_{cw,2} = 1, \\ A_{c3} = -4.5, \quad B_{cy,3} = -1, \quad B_{cw,3} = 1, \\ C_{c3} = -1.5, \quad D_{cy,3} = 5.1, \quad D_{cw,3} = 1, \\ A_{c4} = -7, \quad B_{cy,4} = -1, \quad B_{cw,4} = 1, \\ C_{c4} = -1.5, \quad D_{cy,4} = -2, \quad D_{cw,4} = 1. \end{gathered} \quad (76)$$

The transition rate matrix is given by the following:

$$\begin{bmatrix} 0.3 & 0.3 & 0.2 & 0.2 \\ 0.4 & 0.3 & 0.2 & 0.1 \\ 0.2 & 0.1 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.1 & 0.4 \end{bmatrix}. \quad (77)$$

In this case, we choose the initial values for $R_i = I_2$, $i = 1, 2, 3, 4$, $\delta_x = 1$, $N = 5$, $\alpha = 10^{-10}$, $\mu = 2.5$, $d = 1$, and $w(k) = 0.5(1 + \cos x(k))$; Theorem 7 yields to $\epsilon = 36.2671$, $\sigma_1 = 0.4906$, $\sigma_2 = 13.7421$, and the bounds of the input saturation $u_0 = 0.08$.

Based on Theorem 9, we derive

$$\begin{gathered} A_{aw,1} = -2.67, \quad A_{aw,2} = -1.86, \\ A_{aw,3} = -1.88, \quad A_{aw,4} = -2.59, \\ B_{aw,1} = -0.02, \quad B_{aw,2} = -0.01, \\ B_{aw,3} = -0.01, \quad B_{aw,4} = -0.02, \\ C_{aw,1} = \begin{bmatrix} 17.27 \\ 0.68 \end{bmatrix}, \quad C_{aw,2} = \begin{bmatrix} 68.44 \\ -48.31 \end{bmatrix}, \\ C_{aw,3} = \begin{bmatrix} 68.74 \\ -49.29 \end{bmatrix}, \quad C_{aw,4} = \begin{bmatrix} 17.27 \\ 0.66 \end{bmatrix}, \end{gathered}$$

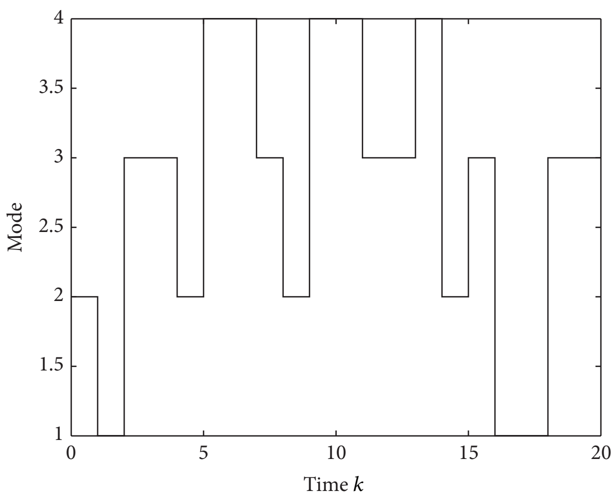


FIGURE 2: $r_k$ of jump rates.

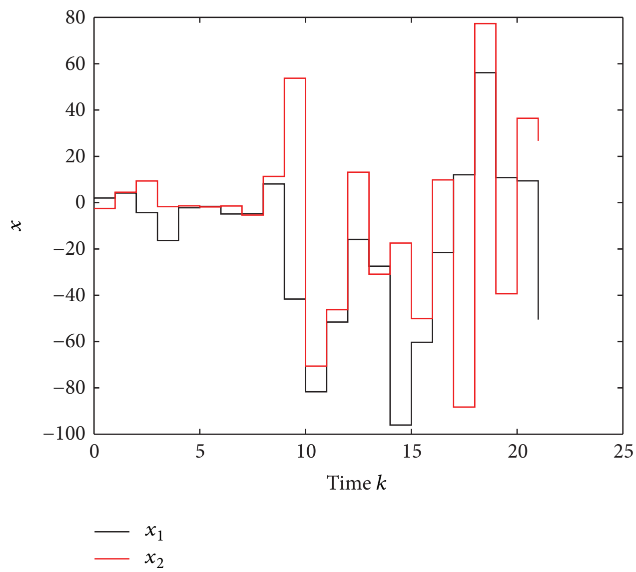


FIGURE 3: $x(k)$ of the system (1)–(5).

$$D_{aw,1} = \begin{bmatrix} 0.21 \\ 0.1 \end{bmatrix}, \qquad D_{aw,2} = \begin{bmatrix} 0.18 \\ -0.1 \end{bmatrix},$$
$$D_{aw,3} = \begin{bmatrix} 0.19 \\ 0 \end{bmatrix}, \qquad D_{aw,4} = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}. \tag{78}$$

*Remark 10.* Figures 2, 3, and 4 are given on the last page. Figure 1 is $r_k$ of the jump rates, Figure 2 and Figure 3 are state response of open and closed-loop system. Based on the figures provided, the controller and the compensator we designed guarantee that the resulting closed-loop systems are mean-square locally asymptotically finite-time stabilizable.

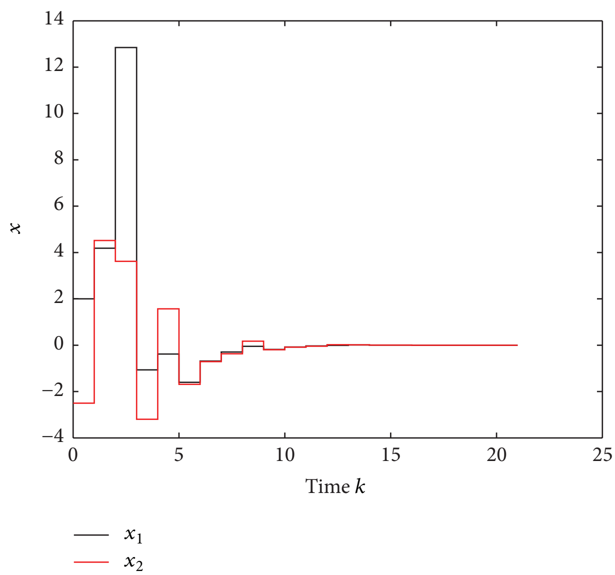


FIGURE 4: $x(k)$ of the closed-loop system (1)–(7).

## 5. Conclusions and Future Work

In this paper, the finite-time $H_\infty$ stabilization problem for a class of discrete-time Markov jump systems with input saturation has been investigated. Based on stochastic finite-time stability analysis, a controller designed for the unconstrained system with a dynamic antiwindup compensator subject to actuator saturation is given to guarantee the stochastic finite-time boundedness and stochastic finite-time stabilization of the considered closed-loop system for all admissible disturbances. Finally, the effectiveness of the proposed approach has been illustrated by simulation results. The finite-time stabilization problem of Markov jump systems with constrained input and time-delay will be considered in the future work.

## Conflict of Interests

The authors declare no conflict of interests.

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