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Research Article

Existence and Stability of Almost Periodic Solution for a Stochastic Cellular Neural Network with External Perturbation

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A class of stochastic cellular neural networks with external perturbation is investigated. By employing fixed points principle and some stochastic analysis techniques, we establish some sufficient conditions for existence and exponential stability of a quadratic mean almost periodic solution of the model. The new criteria not only improve some classical results but also are applied in real problems due to the changes of external input.

1. Introduction

Cellular neural networks (CNNs) and the various generalizations have attracted many scientists' attention due to their important applications, such as associative memory, optimization problems, parallel computation, and so on [1–7]. Huang et al. [7] studied almost periodic solutions of a delayed cellular neural networks as follows:

$$\frac{dx_i(t)}{dt} = -c_i(t)\,x_i(t) + \sum_{j=1}^{N} a_{ij}(t)\,g_j\left(x_j(t-\tau(t))\right) + I_i(t), \quad i = 1, 2, \ldots, N. \tag{1}$$

The authors obtained some good criteria ensuring exponential global attractivity of almost periodic solution to (1).

The concept of almost periodic stochastic process is of great importance in probability for investigating stochastic process [8]. Recently, the existence and stability of almost periodic solution to stochastic cellular neural networks were considered [9]. To the best of our knowledge, there are few works about the quadratic mean almost periodic solution for stochastic cellular neural networks. Motivated by [7–13], in this paper, we will consider the existence and exponential stability of quadratic mean almost periodic for stochastic cellular neural networks with distributed delay as follows:

$$\begin{aligned} dx_i(t) = {} & \left[-c_i(t)\,x_i(t) + \sum_{j=1}^{n} a_{ij}(t)\,f_j\left(x_i\left(t-\tau_{ij}(t)\right)\right)\right. \\ & \left. + \sum_{j=1}^{n} b_{ij}(t)\,g_j\left(\int_{-\infty}^{0} k_j(s)\,x_i(s+t)\,ds\right)\right] dt \\ & + I_i(t)\,dt + \sigma_i\left(t, x_i(t)\right) d\omega_i(t), \quad i = 1, 2, \ldots, n, \end{aligned} \tag{2}$$

where $t \in [0, +\infty)$ and $x(t) = (x_1(t), x_2(t), \ldots, x_n(t)) \in R^n$, n is the number of neurons in the network; x_i denotes the state variable of the ith neuron; f_j and g_j denote the activation function of the jth neuron; the feedback function a_{ij} and b_{ij} indicate the strength of the neuron interconnections within the network; c_i represents an amplification function; I_i represents external input; σ_i can be viewed as a stochastic perturbation on the neuron states and ω_i is a Brownian motion; τ_{ij} is a variable delay function of the neuron x_i and the kernel function k_j satisfies $\int_{-\infty}^{0} k_j(t) = 1$. Some sufficient conditions ensuring the existence and stability of square mean almost periodic solutions are shown. The results in this paper improve some previous results and are applied in real problems such as signal processing and the design of networks and secure communication.

The rest of the paper is organized as follows. In Section 2, we introduce some definitions, lemmas, and some notations which would be useful to get the main results. In Section 3, the main results of existence and stability to (2) are obtained.

2. Preliminaries

Now let us state the following definitions and lemmas, which will be used to prove our main results.

Let $(\mathbb{B}, \|\cdot\|)$ be a Banach space and let $(\Omega, \mathbb{F}, \mathbb{P})$ be a probability space. Define $L^p(\mathbb{P}; \mathbb{B})$ for $p \geq 1$ to be the space of all $\mathbb{B}$-value random variable V such that

$$E\|V\|^p = \int_\Omega \|V\|^p d\mathbb{P} < \infty. \tag{3}$$

It is easy to find that $L^p(\mathbb{P}, \mathbb{B})$ is a Banach space when it is equipped with its natural norm $\|\cdot\|_p$ defined by

$$\|V\|_p := \left(\int_\Omega \|V\|^p d\mathbb{P}\right)^{1/p}. \tag{4}$$

Definition 1. A stochastic process $X : R \to L^p(\mathbb{P}; \mathbb{B})$ is said to be continuous whenever

$$\lim_{t \to s} E\|X(t) - X(s)\|^p = 0. \tag{5}$$

Definition 2. A stochastic process $X : R \to L^p(\mathbb{P}; \mathbb{B})$ is said to be stochastically bounded whenever

$$\lim_{N \to \infty} \sup_{t \in R} \mathbb{P}\{\|X(t) - X(s)\| > N\} = 0. \tag{6}$$

Definition 3. A stochastic process $X : R \to L^p(\mathbb{P}; \mathbb{B})$ is said to be p-mean almost periodic if for each $\varepsilon > 0$ there exists $l(\varepsilon) > 0$ such that any interval of length $l(\varepsilon)$ contains at least a number τ for which

$$\sup_{t \in R} E\|X(t+\tau) - X(t)\|^p < \varepsilon. \tag{7}$$

The number τ will be called an ε-translation of X and the set of all ε-translation of X is denoted by $\aleph(\varepsilon, X)$.

The collection of all stochastic processes $X : R \to L^p(\mathbb{P}; \mathbb{B})$ which are p-mean almost periodic is denoted by $\mathrm{AP}(R; L^p(\mathbb{P}; \mathbb{B}))$. Let $\mathrm{CUB}(R; L^p(\mathbb{P}; \mathbb{B}))$ denote the collection of all stochastic processes $X : R \to L^p(\mathbb{P}; \mathbb{B})$, which are continuous and uniformly bounded. Obviously, $\mathrm{CUB}(R; L^p(\mathbb{P}, \mathbb{B}))$ is a Banach space when it is equipped with the norm

$$\|X\|_\infty = \sup_{t \in R} \left(E\|X(t)\|^p\right)^{1/p}. \tag{8}$$

Lemma 4 (see [8]). *$AP(R; L^p(\mathbb{P}; \mathbb{B})) \subset CUB(R; L^p(\mathbb{P}; \mathbb{B}))$ is a closed subspace.*

Lemma 5 (see [8]). *If X belongs to $AP(R; L^p(\mathbb{P}; \mathbb{B}))$, then*

(i) *the mapping $t \to E\|X(t)\|^p$ is uniformly continuous;*

(ii) *there exists a constant $M > 0$ such that $E\|X(t)\|^p \leq M$, for each $t \in R$;*

(iii) *X is stochastically bounded.*

Let $(\mathbb{B}_1, \|\cdot\|_1)$ and $(\mathbb{B}_2, \|\cdot\|_2)$ be Banach spaces and let $L^p(\mathbb{P}; \mathbb{B}_1)$ and $L^p(\mathbb{P}; \mathbb{B}_2)$ be their corresponding L^p-spaces, respectively.

Definition 6. A function $f : R \times L^p(\mathbb{P}; \mathbb{B}_1) \to L^p(\mathbb{P}; \mathbb{B}_2), (t, y) \to f(t, y)$, which is jointly continuous, is said to be p-mean almost periodic in $t \in R$ uniformly in $y \in \mathbb{K}$ where $\mathbb{K} \subset L^p(\mathbb{P}; \mathbb{B}_1)$ is compact if, for any $\varepsilon > 0$, there exists $l(\varepsilon, \mathbb{K}) > 0$ such that any interval of length $l(\varepsilon, \mathbb{K})$ constants at least a number τ for which

$$\sup_{t \in R} \left(E\|f(t+\tau, y) - f(t, y)\|^p\right)^{1/p} < \varepsilon, \tag{9}$$

for each stochastic process $y : R \to \mathbb{K}$. The number τ will be called an ε-translation of f and the set of all ε-translation of f is denoted by $\aleph(\varepsilon, f, \mathbb{K})$.

Lemma 7 (see [8, 9]). *Let $f : R \times L^p(\mathbb{P}; \mathbb{B}_1) \to L^p(\mathbb{P}; \mathbb{B}_2)$, $(t, x) \to f(t, x)$ be a p-mean almost periodic process in $t \in R$ uniformly in $x \in \mathbb{K}$, where $\mathbb{K} \subset L^p(\mathbb{P}; \mathbb{B}_1)$ is compact. Suppose that f is Lipschitzian in the following sense:*

$$E\|f(t, x) - f(t, y)\|_2^p \leq ME\|x - y\|_1^p \tag{10}$$

for all $x, y \in L^p(\mathbb{P}; \mathbb{B}_1)$ and for each $t \in R$, where $M > 0$. Then for any p-mean almost periodic process $\phi : R \to L^p(\mathbb{P}; \mathbb{B}_1)$); then stochastic process $t \to f(t, \phi(t))$ is p-mean almost periodic.

We need to introduce the following notations. For every real sequence $\alpha = (\alpha_n)$ and a continuous stochastic process $f : R \to L^p(\mathbb{P}; \mathbb{B})$, if $\lim_{n \to \infty} E\|f(t + \alpha_n)\|^p$ exists, we define $T_\alpha E\|f\|^p = \lim_{n \to \infty} E\|f(t + \alpha_n)\|^p$. Like the proof of Fink [14], we have the following lemma.

Lemma 8. *$f : R \to L^p(\mathbb{P}; \mathbb{B})$ is p-mean almost periodic if and only if f is continuous and, for each $\alpha = (\alpha_n)$, there exists a subsequence α' of (α_n) such that $T_{\alpha'} E\|f\|^p = E\|g\|^p$ uniformly on R.*

Lemma 9. *If $u(t), g(t) : R \to L^2(\mathbb{P}; \mathbb{B})$ are square almost periodic stochastic process, then $u(t - g(t))$ is square mean almost periodic.*

Proof. It is obvious that $u(t - g(t))$ is continuous for $t \in R$; that is, $\lim_{t \to s} E\|u(t - g(t)) - u(s - g(s))\|^2 = 0$. For any sequence $\alpha' = (\alpha'_n)$, since $u(t), g(t) : R \to L^2(\mathbb{P}; \mathbb{B})$ are square almost periodic, we have

$$\begin{aligned} T_\alpha E\|u(t)\|^2 &= E\|\overline{u}(t)\|^2, \\ T_\alpha E\|g(t)\|^2 &= E\|\overline{g}(t)\|^2 \end{aligned} \tag{11}$$

uniformly for $t \in R$. On the other hand, since $u(t)$ is almost periodic, it is uniformly continuous on R. For any $\varepsilon > 0$, there exists a positive number $\delta(\varepsilon)$, such that $|t_1 - t_2| < \delta$ implies that $u(t_1) - u(t_2) < \varepsilon$. From (11), there exists a positive integer N, when $n > N$, we have

$$E\left\|u(t+\alpha_n) - \overline{u}(t)\right\|^2 < \frac{\varepsilon}{4}, \qquad E\left\|g(t+\alpha_n) - \overline{g}(t)\right\|^2 < \min\left\{\frac{\varepsilon}{4}, \delta\right\}, \quad t \in R. \tag{12}$$

Since $(a+b)^2 \leq 2a^2 + 2b^2$, we have

$$\begin{aligned} E&\left\|u\left(t+\alpha_n - g\left(t+\alpha_n\right)\right) - \overline{u}\left(t - \overline{g}(t)\right)\right\|^2 \\ &= E\left\|u\left(t+\alpha_n - g\left(t+\alpha_n\right)\right) - u\left(t+\alpha_n - \overline{g}(t)\right)\right. \\ &\quad \left. + u\left(t+\alpha_n - \overline{g}(t)\right) - \overline{u}\left(t - \overline{g}(t)\right)\right\|^2 \\ &\leq 2E\left\|u\left(t+\alpha_n - g\left(t+\alpha_n\right)\right) - u\left(t+\alpha_n - \overline{g}(t)\right)\right\|^2 \\ &\quad + 2E\left\|u\left(t+\alpha_n - \overline{g}(t)\right) - \overline{u}\left(t - \overline{g}(t)\right)\right\|^2 \\ &\leq 2\cdot\frac{\varepsilon}{4} + 2\cdot\frac{\varepsilon}{4} = \varepsilon, \end{aligned} \tag{13}$$

when $n > N$. Thus, $u(t - g(t))$ is square mean almost periodic. □

3. Main Results

In this section, we state and prove our main results concerning the existence and stability of square mean almost solutions of (2). Throughout the rest of the paper, the following assumptions are satisfied.

(H1) The functions $a_{ij}, b_{ij}, c_i, I_i : R \to L^2(\mathbb{P};\mathbb{B})$ $(i, j = 1, 2, \ldots, n)$ are square mean almost periodic functions, where $\inf_{t\in R} c_i(t) > 0$ $(i = 1, 2, \ldots, n)$.

(H2) The activation functions $f_j, g_j : R \to L^2(\mathbb{P};\mathbb{B})$ are square mean almost periodic, and f_j and g_j are Lipschitz in the following sense: there exist $L_{f_j} > 0$ and $L_{g_j} > 0$ for which

$$\begin{aligned} E\left\|f_j(u) - f_j(v)\right\|^2 &\leq L_{f_j}\|u - v\|^2, \\ & u, v \in L^2(\mathbb{P};\mathbb{B}), \\ & j = 1, 2, \ldots, n, \\ E\left\|g_j(u) - g_j(v)\right\|^2 &\leq L_{g_j}\|u - v\|^2, \\ & u, v \in L^2(\mathbb{P};\mathbb{B}), \\ & j = 1, 2, \ldots, n. \end{aligned} \tag{14}$$

(H3) The functions $\sigma_i(t, x_i(t)) : R \times L^2(\mathbb{P};\mathbb{B}) \to L^2(\mathbb{P};\mathbb{B})$ are square mean almost periodic in $t \in R$ uniformly in $x \in S$ ($S \subset L^2(\mathbb{P};\mathbb{B})$ being a compact subspace). Moreover, σ_i is Lipschitz in the following sense: there exists $L_{\sigma_i} > 0$ for which

$$\begin{aligned} E\left\|\sigma_i(u) - \sigma_i(v)\right\|^2 &\leq L_{\sigma_i}\|u - v\|^2, \\ & u, v \in L^2(\mathbb{P}, \mathbb{B}), \\ & t \in R,\ i = 1, 2, \ldots, n. \end{aligned} \tag{15}$$

Let denote the signs $h^* := \sup_{t\in R} h(t)$, $h_* := \inf_{t\in R} h(t)$ and $z^* := \sup_{(t,x)\in R\times L^2(\mathbb{P};\mathbb{B})} z(t, x)$.

Theorem 10. *Assume that conditions (H1)–(H3) are satisfied and $3({a^*}^2 n^2 L_f^* + {b^*}^2 n^2 L_g^* + L_\sigma^*) < c_*^2$; then (2) has a unique square mean almost periodic solution.*

Proof. By (2), we can obtain that

$$\begin{aligned} x_i(t) = \int_{-\infty}^{t} & e^{-c_i(s)(t-s)} \\ & \times \sum_{j=1}^{n}\left[a_{ij}(s) f_j\left(x_i\left(s - \tau_{ij}(s)\right)\right)\right. \\ & \left. + b_{ij}(s) g_j\left(\int_{-\infty}^{0} k_j(\theta) x_i(s+\theta)\, d\theta\right)\right] ds \\ & + \int_{-\infty}^{t} e^{-c_i(s)(t-s)} I_i(s)\, ds \\ & + \int_{-\infty}^{t} e^{-c_i(s)(t-s)} \sigma_i\left(s, x_i(s)\right) d\omega_i(s), \\ & i = 1, 2, \ldots, n, \end{aligned} \tag{16}$$

for all $t \in R$, $x_i(t)$ $(i = 1, 2, \ldots, n)$ given by (16) is the solution to (2).

Define $(\mathbb{L}x_i)(t) = \sum_{m=1}^{4}(\phi_m)x_i(t)$, where

$$\begin{aligned} \left(\phi_1 x_i\right)(t) &= \int_{-\infty}^{t} e^{-c_i(s)(t-s)} \times \sum_{j=1}^{n} a_{ij}(s) f_j\left(x_i\left(s - \tau_{ij}(s)\right)\right) ds, \\ \left(\phi_2 x_i\right)(t) &= \int_{-\infty}^{t} e^{-c_i(s)(t-s)} \times \sum_{j=1}^{n} b_{ij}(s) g_j\left(\int_{-\infty}^{0} k_j(\theta) x_i(s+\theta)\, d\theta\right) ds, \\ \left(\phi_3 x_i\right)(t) &= \int_{-\infty}^{t} e^{-c_i(s)(t-s)} I_i(s)\, ds, \\ \left(\phi_4 x_i\right)(t) &= \int_{-\infty}^{t} e^{-c_i(s)(t-s)} \sigma_i\left(s, x_i(s)\right) d\omega_i(s). \end{aligned} \tag{17}$$

For $i = 1, 2, \ldots, n$, we show that $(\mathbb{L}x_i)(\cdot)$ is square mean almost periodic if x_i is. Assuming that x_i $(i = 1, 2, \ldots, n)$ is square mean almost periodic and applying conditions (H1) and (H2) and Lemmas 7 and 9, one can easily obtain that $a_{ij}(s)f_j(x_i(s-\tau_{ij}(s)))$, $(i, j = 1, 2, \dots, n; s \in R)$ is square mean almost periodic. Therefore, for each $\varepsilon > 0$, there exists $l(\varepsilon) > 0$ such that any interval of length $l(\varepsilon)$ contains at least τ for which

$$E\left\|a_{ij}(s+\tau) f_j x_i\left(s+\tau-\tau_{ij}(s+\tau)\right) - a_{ij}(s) f_j x_i\left(s-\tau_{ij}(s)\right)\right\|^2 \le \left(\frac{c_*}{n}\right)^2 \varepsilon, \tag{18}$$

for each $s \in R$.

Now, by using Cauchy-Schwarz inequality, we can write

$$\begin{aligned}
&E\left\|(\phi_1 x_i)(t+\tau) - (\phi_1 x_i)(t)\right\|^2 \\
&= E\left\|\int_{-\infty}^{t+\tau} e^{-c_i(s)(t+\tau-s)} \sum_{j=1}^{n} a_{ij}(s) f_j\left(x_i\left(s-\tau_{ij}(s)\right)\right) ds - \int_{-\infty}^{t} e^{-c_i(s)(t-s)} \sum_{j=1}^{n} a_{ij}(s) f_j\left(x_i\left(s-\tau_{ij}(s)\right)\right) ds\right\|^2 \\
&\le E\left\|\int_{-\infty}^{t} e^{-c_*(t-s)} \sum_{j=1}^{n}\left[a_{ij}(s+\tau) f_j\left(x_i\left(s+\tau-\tau_{ij}(s+\tau)\right)\right) - a_{ij}(s) f_j\left(x_i\left(s-\tau_{ij}(s)\right)\right)\right] ds\right\|^2 \\
&\le \left(\int_{-\infty}^{t} e^{-c_*(t-s)} ds\right) \\
&\quad \times \left(\int_{-\infty}^{t} e^{-c_*(t-s)} E\left\|\sum_{j=1}^{n}\left[a_{ij}(s+\tau) f_j\left(x_i\left(s+\tau-\tau_{ij}(s+\tau)\right)\right) - a_{ij}(s) f_j\left(x_i\left(s-\tau_{ij}(s)\right)\right)\right]\right\| ds\right)^2 \\
&\le \left(\int_{-\infty}^{t} e^{-c_*(t-s)}\right)^2 \\
&\quad \times \sup_{s\in R}\left\{n\sum_{j=1}^{n} E\left\|a_{ij}(s+\tau) f_j\left(x_i\left(s+\tau-\tau_{ij}(s+\tau)\right)\right) - a_{ij}(s) f_j\left(x_i\left(s-\tau_{ij}(s)\right)\right)\right\|^2\right\} \\
&\le \frac{n}{c_*^2} \cdot nE\left\|a_{ij}(s+\tau) f_j x_i\left(s+\tau-\tau_{ij}(s+\tau)\right) - a_{ij}(s) f_j x_i\left(s-\tau_{ij}(s)\right)\right\|^2 \\
&\le \frac{n}{c_*^2} \cdot n\left(\frac{c_*}{n}\right)^2 \varepsilon = \varepsilon.
\end{aligned} \tag{19}$$

According to the above, for $i = 1, 2, \dots, n$, $E\|(\phi_1 x_i)(t+\tau) - (\phi_1 x_i)(t)\|^2 < \varepsilon$ for each $t \in R$; that is, $(\phi_1 x_i)(\cdot)$ is square mean almost periodic.

Next, for $i = 1, 2, \dots, n$, we show that $(\phi_2 x_i)(\cdot)$ is square mean almost periodic if x_i is. Since x_i $(i = 1, 2, \dots, n)$ is square mean almost periodic, for each $\varepsilon > 0$ there exists $l(\varepsilon) > 0$ such that any interval of length $l(\varepsilon)$ contains at least τ for which

$$E\left\|x_i(s+\tau+\theta) - x_i(s+\tau)\right\|^2 \le \frac{c_*^2}{4n^2 {b^*}^2 L_g^*}\varepsilon, \tag{20}$$

$$E\left\|b_{ij}(s+\tau) - b_{ij}(s)\right\|^2 \le \frac{c_*^2}{4n^2 {g^*}^2}\varepsilon, \tag{21}$$

for each $s \in R$.

So, we have

$$\begin{aligned}
&\left\|(\phi_2 x_i)(t+\tau) - (\phi_2 x_i)(t)\right\| \\
&= \left\|\int_{-\infty}^{t+\tau} e^{-c_i(s)(t+\tau-s)} \times \sum_{j=1}^{n} b_{ij}(s) g_j\left(\int_{-\infty}^{0} k_j(\theta) x_i(s+\theta) d\theta\right) ds \right. \\
&\quad \left. - \int_{-\infty}^{t} e^{-c_i(s)(t-s)} \sum_{j=1}^{n} b_{ij}(s) g_j \times \left(\int_{-\infty}^{0} k_j(\theta) x_i(s+\theta) d\theta\right) ds\right\| \\
&= \left\|\int_{-\infty}^{t} e^{-c_i(s)(t-s)} \times \sum_{j=1}^{n}\left[b_{ij}(s+\tau) g_j\left(\int_{-\infty}^{0} k_j(\theta) x_i(s+\tau+\theta) d\theta\right) - b_{ij}(s) g_j\left(\int_{-\infty}^{0} k_j(\theta) x_i(s+\theta) d\theta\right)\right] ds\right\|
\end{aligned}$$

$$
\begin{aligned}
&= \Bigg\| \int_{-\infty}^{t} e^{-c_i(s)(t-s)} \\
&\quad \times \Bigg\{ \sum_{j=1}^{n} \Bigg[b_{ij}(s+\tau) g_j \left(\int_{-\infty}^{0} k_j(\theta) x_i(s+\tau+\theta) d\theta \right) \\
&\qquad - b_{ij}(s) g_j \left(\int_{-\infty}^{0} k_j(\theta) x_i(s+\tau+\theta) d\theta \right) \Bigg] \\
&\quad + \sum_{j=1}^{n} \Bigg[b_{ij}(s) g_j \left(\int_{-\infty}^{0} k_j(\theta) x_i(s+\tau+\theta) d\theta \right) \\
&\qquad - b_{ij}(s) g_j \left(\int_{-\infty}^{0} k_j(\theta) x_i(s+\theta) d\theta \right) \Bigg] \Bigg\} ds \Bigg\| \\
&= \Bigg\| \int_{-\infty}^{t} e^{-c_i(s)(t-s)} \\
&\quad \times \Bigg\{ \sum_{j=1}^{n} \left[b_{ij}(s+\tau) - b_{ij}(s) \right] g_j \\
&\qquad \times \left(\int_{-\infty}^{0} k_j(\theta) x_i(s+\tau+\theta) d\theta \right) \\
&\quad + \sum_{j=1}^{n} b_{ij}(s) \Bigg[g_j \left(\int_{-\infty}^{0} k_j(\theta) x_i(s+\tau+\theta) d\theta \right) \\
&\qquad - g_j \left(\int_{-\infty}^{0} k_j(\theta) x_i(s+\theta) d\theta \right) \Bigg] \Bigg\} ds \Bigg\| .
\end{aligned}
\tag{22}
$$

Similarly, from (20)–(22), by using Cauchy-Schwarz inequality, we obtain that

$$
\begin{aligned}
&E \left\| (\phi_2 x_i)(t+\tau) - (\phi_2 x_i)(t) \right\|^2 \\
&\quad \le 2E \Bigg\| \int_{-\infty}^{t} e^{-c_i(s)(t-s)} \\
&\qquad \times \sum_{j=1}^{n} \left[b_{ij}(s+\tau) - b_{ij}(s) \right] g_j \\
&\qquad \times \left(\int_{-\infty}^{0} k_j(\theta) x_i(s+\tau+\theta) d\theta \right) ds \Bigg\|^2 \\
&\quad + 2E \Bigg\| \int_{-\infty}^{t} e^{-c_i(s)(t-s)} \\
&\qquad \times \sum_{j=1}^{n} b_{ij}(s) \\
&\qquad \times \Bigg[g_j \left(\int_{-\infty}^{0} k_j(\theta) x_i(s+\tau+\theta) d\theta \right) \\
&\qquad - g_j \left(\int_{-\infty}^{0} k_j(\theta) x_i(s+\theta) d\theta \right) \Bigg] ds \Bigg\|^2 \\
&\quad \le 2 \left(\int_{-\infty}^{t} e^{-c_i(s)(t-s)} ds \right) \\
&\qquad \times \int_{-\infty}^{t} e^{-c_i(s)(t-s)} \\
&\qquad \times E \Bigg\| \sum_{j=1}^{n} \left[b_{ij}(s+\tau) - b_{ij}(s) \right] g_j \\
&\qquad \times \left(\int_{-\infty}^{0} k_j(\theta) x_i(s+\tau+\theta) d\theta \right) \Bigg\|^2 ds \\
&\quad + 2 \left(\int_{-\infty}^{t} e^{-c_i(s)(t-s)} ds \right) \\
&\qquad \times \int_{-\infty}^{t} e^{-c_i(s)(t-s)} \\
&\qquad \times E \Bigg\| \sum_{j=1}^{n} b_{ij}(s) \\
&\qquad \times \Bigg[g_j \left(\int_{-\infty}^{0} k_j(\theta) x_i(s+\tau+\theta) d\theta \right) \\
&\qquad - g_j \left(\int_{-\infty}^{0} k_j(\theta) x_i(s+\theta) d\theta \right) \Bigg] \Bigg\|^2 ds \\
&\quad \le \frac{2g^{*2}}{c_*} \int_{-\infty}^{t} e^{-c_i(s)(t-s)} n \sum_{j=1}^{n} E \left\| b_{ij}(s+\tau) - b_{ij}(s) \right\|^2 ds + \frac{2b^{*2}}{c_*} \\
&\qquad \times \int_{-\infty}^{t} e^{-c_i(s)(t-s)} \\
&\qquad \times n \sum_{j=1}^{n} E \Bigg\| g_j \left(\int_{-\infty}^{0} k_j(\theta) x_i(s+\tau+\theta) d\theta \right) \\
&\qquad - g_j \left(\int_{-\infty}^{0} k_j(\theta) x_i(s+\theta) d\theta \right) \Bigg\|^2 ds \\
&\quad \le n^2 \frac{2g^{*2}}{c_*^2} \cdot \frac{c_*^2}{4n^2 g^{*2}} \varepsilon + n^2 \frac{2b^{*2}}{c_*^2} \\
&\qquad \cdot L_g^* \int_{-\infty}^{0} k_j(\theta) E \left\| x_i(s+\tau+\theta) - x_i(s+\theta) \right\|^2 d\theta \\
&\quad \le \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon.
\end{aligned}
\tag{23}
$$

So, for $i = 1, 2, \ldots, n$, $E\|(\phi_2 x_i)(t+\tau) - (\phi_2 x_i)(t)\|^2 < \varepsilon$ for each $t \in R$; that is, $(\phi_2 x_i)(\cdot)$ is square mean almost periodic. From (H1), it is easy to see that $(\phi_3 x_i)(\cdot) \in \mathrm{AP}(R; L^2(\mathbb{P}; \mathbb{B}))$.

Now, we prove that $(\phi_4 x_i)(\cdot) \in AP(R; L^2(\mathbb{P};\mathbb{B}))$ provided $x_i \in AP(R; L^2(\mathbb{P};\mathbb{B}))$ $(i = 1, 2, \ldots, n)$. From (H3) and Lemma 7, one can see that $\sigma_i(s, x_i(s))$ $(i = 1, 2, \ldots, n)$ is square mean almost periodic for $s \in R$; for each $\varepsilon > 0$ there exists $l(\varepsilon) > 0$ such that any interval of length $l(\varepsilon)$ contains at least τ for which

$$E\left\|\sigma_i(s+\tau, x_i(s+\tau)) - \sigma_i(s, x_i(s))\right\|^2 \le c_*^2 \varepsilon, \tag{24}$$

for each $s \in R$. We make extensive use of the Ito's isometry identity and the properties of $\overline{W}$ defined by $\overline{\omega_i} := \omega_i(s+\tau) - \omega_i(\tau)$ for each $s \in R$. And $\overline{\omega_i}$ is also a Brownian motion and has the same distribution as ω_i. Now,

$$\begin{aligned}&(\phi_4 x_i)(t+\tau) - (\phi_4 x_i)(t)\\&\quad= \int_{-\infty}^{t} e^{-c_i(s)(t-s)} \left[\sigma_i(s+\tau, x_i(s+\tau)) - \sigma_i(s, x_i(s))\right] d\omega_i(s).\end{aligned} \tag{25}$$

By making a change of variables $s = \nu - \tau$, we get

$$\begin{aligned}&E\left\|(\phi_4 x_i)(t+\tau) - (\phi_4 x_i)(t)\right\|^2\\&\quad= E\Bigg\|\int_{-\infty}^{t} e^{-c_i(s)(t-s)}\\&\qquad\times\left[\sigma_i(s+\tau, x_i(s+\tau)) - \sigma_i(s, x_i(s))\right] d\overline{\omega_i}(s)\Bigg\|^2.\end{aligned} \tag{26}$$

Thus, applying Ito's isometry identity, we have

$$\begin{aligned}&E\left\|(\phi_4 x_i)(t+\tau) - (\phi_4 x_i)(t)\right\|^2\\&\quad= \int_{-\infty}^{t} E\left\|e^{-c_i(s)(t-s)}\left[\sigma_i(s+\tau, x_i(s+\tau)) - \sigma_i(s, x_i(s))\right]\right\|^2 ds\\&\quad\le \left(\int_{-\infty}^{t} e^{-c_i(s)(t-s)} ds\right)\\&\qquad\times E\left(\int_{-\infty}^{t} e^{-c_i(s)(t-s)} \| \sigma_i(s+\tau, x_i(s+\tau))\right.\\&\qquad\qquad\left. - \sigma_i(s, x_i(s))\|^2\right) ds\\&\quad\le \frac{1}{c_*^2} E\left\|\sigma_i(s+\tau, x_i(s+\tau)) - \sigma_i(s, x_i(s))\right\|^2\\&\quad\le \frac{1}{c_*^2}\cdot c_*^2 \varepsilon = \varepsilon,\end{aligned} \tag{27}$$

which implies that $(\phi_4 x_i)(\cdot) \in AP(R; L^2(\mathbb{P};\mathbb{B}))$ $(i = 1, 2, \ldots, n)$.

From the above, for $i = 1, 2, \ldots, n$, it is clear that $\mathbb{L}$ maps $AP(R; L^p(\mathbb{P}, \mathbb{B}))$ into itself. In the next section, we show that $\mathbb{L}$ is a contraction mapping. Actually, for $i = 1, 2, \ldots, n$, we have

$$\begin{aligned}&\left\|(\mathbb{L}x_i)(t) - (\mathbb{L}y_i)(t)\right\|\\&\quad= \Bigg\|\int_{-\infty}^{t} e^{-c_i(s)(t-s)}\\&\qquad\times\sum_{j=1}^{n} a_{ij}(s)\left[f_j\left(x_i\left(s-\tau_{ij}(s)\right)\right)\right.\\&\qquad\qquad\left.-f_j\left(y_i\left(s-\tau_{ij}(s)\right)\right)\right] ds\\&\qquad+\int_{-\infty}^{t} e^{-c_i(s)(t-s)}\\&\qquad\times\sum_{j=1}^{n} b_{ij}(s)\left[g_j\left(\int_{-\infty}^{0} k_j(\theta)\, x_i(s+\theta)\, d\theta\right)\right.\\&\qquad\qquad\left.-g_j\left(\int_{-\infty}^{0} k_j(\theta)\, y_i(s+\theta)\, d\theta\right)\right] ds\\&\qquad+\int_{-\infty}^{t} e^{-c_i(s)(t-s)}\\&\qquad\times\left[\sigma_i(s, x_i(s)) - \sigma_i(s, y_i(s))\right] d\omega_i(s)\Bigg\|\\&\quad\le a^*\int_{-\infty}^{t} e^{-c_i(s)(t-s)}\\&\qquad\times\sum_{j=1}^{n}\left\|f_j\left(x_i\left(s-\tau_{ij}(s)\right)\right)-f_j\left(y_i\left(s-\tau_{ij}(s)\right)\right)\right\| ds\\&\qquad+b^*\int_{-\infty}^{t} e^{-c_i(s)(t-s)}\\&\qquad\times\sum_{j=1}^{n}\left\|g_j\left(\int_{-\infty}^{0} k_j(\theta)\, x_i(s+\theta)\, d\theta\right)\right.\\&\qquad\qquad\left.-g_j\left(\int_{-\infty}^{0} k_j(\theta)\, y_i(s+\theta)\, d\theta\right)\right\| ds\\&\qquad+\int_{-\infty}^{t} e^{-c_i(s)(t-s)}\left\|\sigma_i(s, x_i(s)) - \sigma_i(s, y_i(s))\right\| d\omega_i(s).\end{aligned} \tag{28}$$

Since $(a+b+c)^2 \le 3a^2 + 3b^2 + 3c^2$, we can get

$$\begin{aligned}&E\left\|(\mathbb{L}x_i)(t) - (\mathbb{L}y_i)(t)\right\|^2\\&\quad\le 3a^{*2}E\left(\int_{-\infty}^{t} e^{-c_i(s)(t-s)}\right.\\&\qquad\times\sum_{j=1}^{n}\left\|f_j\left(x_i\left(s-\tau_{ij}(s)\right)\right)\right.\end{aligned}$$

$$
\begin{aligned}
&\qquad\qquad -f_j\left(y_i\left(s-\tau_{ij}(s)\right)\right)\Big\| \,ds\Bigg)^2 + 3b^{*2}E \\
&\times\Bigg(\int_{-\infty}^{t} e^{-c_i(s)(t-s)} \\
&\qquad \times \sum_{j=1}^{n}\left\| g_j\left(\int_{-\infty}^{0} k_j(\theta)\, x_i(s+\theta)\, d\theta\right)\right. \\
&\qquad\qquad \left. -g_j\left(\int_{-\infty}^{0} k_j(\theta)\, y_i(s+\theta)\, d\theta\right)\right\| ds\Bigg)^2 \\
&+3E\Bigg(\int_{-\infty}^{t} e^{-c_i(s)(t-s)} \left\|\sigma_i\left(s, x_i(s)\right)\right. \\
&\qquad\qquad \left. -\sigma_i\left(s, y_i(s)\right)\right\| d\omega_i(s)\Bigg)^2 \\
&:= F(t)+G(t)+\Phi(t).
\end{aligned}
\tag{29}
$$

Firstly, we evaluate the first term of the right hand side as follows:

$$
\begin{aligned}
F(t) &\leq 3a^{*2}E\left(\int_{-\infty}^{t} e^{-c_i(s)(t-s)}ds\right) \\
&\quad\times\Bigg(\int_{-\infty}^{t} e^{-c_i(s)(t-s)} \\
&\qquad\times\Bigg(\sum_{j=1}^{n}\left\| f_j\left(x_i\left(s-\tau_{ij}(s)\right)\right)\right. \\
&\qquad\qquad \left. -f_j\left(y_i\left(s-\tau_{ij}(s)\right)\right)\right\| \Bigg)^2 ds\Bigg) \\
&\leq 3a^{*2}\left(\int_{-\infty}^{t} e^{-c_i(s)(t-s)}ds\right) \\
&\quad\times n\Bigg(\int_{-\infty}^{t} e^{-c_i(s)(t-s)} \\
&\qquad\times \sum_{j=1}^{n}E\left\| f_j\left(x_i\left(s-\tau_{ij}(s)\right)\right)\right. \\
&\qquad\qquad \left. -f_j\left(y_i\left(s-\tau_{ij}(s)\right)\right)\right\|^2 ds\Bigg) \\
&\leq 3a^{*2}n^2\left(\int_{-\infty}^{t} e^{-c_i(s)(t-s)}ds\right)^2 \\
&\quad\times L_f^{*}\sup_{t\in R} E\left\|x_i(t)-y_i(t)\right\|^2 \\
&\leq \frac{3a^{*2}n^2}{c_*^2} L_f^{*}\|x-y\|_{\infty}.
\end{aligned}
\tag{30}
$$

Secondly, we evaluate the second term of the right hand side as follows:

$$
\begin{aligned}
G(t) &\leq 3b^{*2}E\left(\int_{-\infty}^{t} e^{-c_i(s)(t-s)}ds\right) \\
&\quad\times\Bigg(\int_{-\infty}^{t} e^{-c_i(s)(t-s)} \\
&\qquad\times\sum_{j=1}^{n}\left\| g_j\left(\int_{-\infty}^{0} k_j(\theta)\, x_i(s+\theta)\, d\theta\right)\right. \\
&\qquad\qquad \left. -g_j\left(\int_{-\infty}^{0} k_j(\theta)\, y_i(s+\theta)\, d\theta\right)\right\| ds\Bigg) \\
&\leq \frac{3b^{*2}}{c_*}n \\
&\quad\times\int_{-\infty}^{t} e^{-c_i(s)(t-s)} \\
&\qquad\times\Bigg(\sum_{j=1}^{n}E\left\| g_j\left(\int_{-\infty}^{0} k_j(\theta)\, x_i(s+\theta)\, d\theta\right)\right. \\
&\qquad\qquad \left. -g_j\left(\int_{-\infty}^{0} k_j(\theta)\, y_i(s+\theta)\, d\theta\right)\right\|^2\Bigg) ds \\
&\leq \frac{3b^{*2}}{c_*}n^2\left(\int_{-\infty}^{t} e^{-c_i(s)(t-s)}ds\right) \\
&\quad\times L_g^{*}\sup_{t\in R} E\left\|\int_{-\infty}^{0} k_j(\theta)\left[x_i(t)-y_i(t)\right]d\theta\right\|^2 \\
&\leq \frac{3b^{*2}n^2}{c_*^2} L_g^{*}\sup_{t\in R} E\left\|x_i(t)-y_i(t)\right\|^2 \\
&\leq \frac{3b^{*2}n^2}{c_*^2} L_g^{*}\|x-y\|_{\infty}.
\end{aligned}
\tag{31}
$$

As to the third term, we apply isometry identity and have

$$
\begin{aligned}
\Phi(t) &\leq 3\left(\int_{-\infty}^{t} e^{-c_i(s)(t-s)}ds\right) \\
&\quad\times\int_{-\infty}^{t} e^{-c_i(s)(t-s)} \\
&\qquad\times E\left\|\sigma_i\left(s, x_i(s)\right)-\sigma_i\left(s, y_i(s)\right)\right\|^2 d\omega_i(s)
\end{aligned}
$$

$$\leq \frac{3}{c_*} L_\sigma^* \int_{-\infty}^{t} e^{-c_i(s)(t-s)} E\|x_i(s) - y_i(s)\|^2 ds$$
$$\leq \frac{3}{c_*^2} L_\sigma^* \sup_{t\in R} E\|x_i(t) - y_i(t)\|^2 \leq \frac{3}{c_*^2} L_\sigma^* \|x - y\|_\infty. \tag{32}$$

Thus, for $i = 1, 2, \ldots, n$, by combining (30), (31), and (32), we can obtain that

$$E\|(\mathbb{L}x_i)(t) - (\mathbb{L}y_i)(t)\|^2$$
$$\leq \left(\frac{3a^{*2}n^2}{c_*^2} L_f^* + \frac{3b^{*2}n^2}{c_*^2} L_g^* + \frac{3}{c_*^2} L_\sigma^*\right) \|x - y\|_\infty \tag{33}$$
$$\leq \frac{3}{c_*^2}\left(a^{*2}n^2 L_f^* + b^{*2}n^2 L_g^* + L_\sigma^*\right) \|x - y\|_\infty,$$

and it follows that

$$\|(\mathbb{L}x_i)(t) - (\mathbb{L}y_i)(t)\|_\infty$$
$$\leq \frac{3}{c_*^2}\left(a^{*2}n^2 L_f^* + b^{*2}n^2 L_g^* + L_\sigma^*\right) \|x - y\|_\infty. \tag{34}$$

From $(3/c_*^2)(a^{*2}n^2 L_f^* + b^{*2}n^2 L_g^* + L_\sigma^*) < 1$, it suffices to show that $\mathbb{L}$ has a unique fixed point, which is clearly the unique square mean almost periodic solution to (2). The proof is completed. □

Remark 11. According to the conditions of Theorem 10, we can find that the activation function of the neuron, the feedback function, the neuron interconnections, amplification function, external input, and stochastic perturbation have key effect on the unique of square mean almost periodic solution.

Theorem 12. *Assume that the conditions of Theorem 10 are held; then the square mean almost periodic solution of system* (2) *is global exponential stability.*

Proof. Take a positive constant α such that $0 < \alpha < c_*$. For $i = 1, 2, \ldots, n$, it follows from (16) that

$$e^{\alpha t} E\|x_i(t)\|^2$$
$$\leq 6e^{\alpha t} E\left\|\int_{-\infty}^{t} e^{-c_i(s)(t-s)} \times \sum_{j=1}^{n} a_{ij}(s) f_j\left(x_i\left(s - \tau_{ij}(s)\right)\right) ds\right\|^2$$
$$+ 6e^{\alpha t} E\left\|\int_{-\infty}^{t} e^{-c_i(s)(t-s)} \times \sum_{j=1}^{n} b_{ij}(s) g_j \times \left(\int_{-\infty}^{0} k_j(\theta) x_i(s+\theta) d\theta\right) ds\right\|^2$$
$$+ 3e^{\alpha t} E\left\|\int_{-\infty}^{t} e^{-c_i(s)(t-s)} I_i(s) ds\right\|^2$$
$$+ 3e^{\alpha t} E\left\|\int_{-\infty}^{t} e^{-c_i(s)(t-s)} \sigma_i(s, x_i(s)) d\omega_i(s)\right\|^2. \tag{35}$$

Now, we evaluate the first term on the right hand side of (35); we obtain

$$6e^{\alpha t} E\left\|\int_{-\infty}^{t} e^{-c_i(s)(t-s)} \times \sum_{j=1}^{n} a_{ij}(s) f_j\left(x_i\left(s - \tau_{ij}(s)\right)\right) ds\right\|^2$$
$$= 6e^{\alpha t}\left(\int_{-\infty}^{t} e^{-c_i(s)(t-s)} ds\right) \times \int_{-\infty}^{t} e^{-c_i(s)(t-s)} E\left\|\sum_{j=1}^{n} a_{ij}(s) f_j(x_i(s - \tau_{ij}(s)))\right\|^2 ds$$
$$\leq \frac{6}{c_*} e^{\alpha t} \times \int_{-\infty}^{t} e^{-c_*(t-s)} E\left\|\sum_{j=1}^{n} a_{ij}(s) f_j(x_i(s - \tau_{ij}(s)))\right\|^2 ds \tag{36}$$
$$\leq \frac{6n^2}{c_*} e^{-(c_*-\alpha)t} a^{*2} L_f^* \times \int_{-\infty}^{t} e^{(c_*-\alpha)s} e^{\alpha s} E\|x_i(s)\|^2 ds.$$

For any $x_i(t) \in R$ $(i = 1, 2, \ldots, n)$ and any $\varepsilon > 0$, there exists $t_1 \in R$ such that $e^{\alpha s} E\|x_i(s)\|^2 < \varepsilon$ for $t \geq t_1$. Therefore

$$6e^{\alpha t} E\left\|\int_{-\infty}^{t} e^{-c_i(s)(t-s)} \times \sum_{j=1}^{n} a_{ij}(s) f_j\left(x_i\left(s - \tau_{ij}(s)\right)\right) ds\right\|^2$$

$$\begin{aligned}
&\le \frac{6n^2}{c_*}e^{-(c_*-\alpha)t}a^{*2}L_f^* \\
&\quad\times \int_{-\infty}^{t} e^{(c_*-\alpha)s}e^{\alpha s}E\|x_i(s)\|^2 ds \\
&= \frac{6n^2}{c_*}e^{-(c_*-\alpha)t}a^{*2}L_f^* \\
&\quad\times \int_{-\infty}^{t_1} e^{(c_*-\alpha)s}e^{\alpha s}E\|x_i(s)\|^2 ds \\
&\quad+ \frac{6n^2}{c_*}e^{-(c_*-\alpha)t}a^{*2}L_f^* \\
&\quad\times \int_{t_1}^{t} e^{(c_*-\alpha)s}e^{\alpha s}E\|x_i(s)\|^2 ds \\
&\le \frac{6n^2}{c_*}e^{-(c_*-\alpha)t}a^{*2}L_f^* \\
&\quad\times \int_{-\infty}^{t_1} e^{(c_*-\alpha)s}e^{\alpha s}E\|x_i(s)\|^2 ds \\
&+ \frac{6n^2}{c_*}a^{*2}L_f^*\frac{1}{c_*(c_*-\alpha)}\varepsilon.
\end{aligned} \tag{37}$$

When $t \to +\infty$, $e^{-(c_*-\alpha)t} \to 0$, that is, there exists $t_2 \ge t_1$ such that for any $t \ge t_2$, one can get

$$\begin{aligned}
&\frac{6n^2}{c_*}e^{-(c_*-\alpha)t}a^{*2}L_f^*\int_{-\infty}^{t_1} e^{(c_*-\alpha)s}e^{\alpha s}E\|x_i(s)\|^2 ds \\
&\quad\le \varepsilon - \frac{6n^2}{c_*}a^{*2}L_f^*\frac{1}{c_*(c_*-\alpha)}\varepsilon.
\end{aligned} \tag{38}$$

Thus, from (37) and (38), we have for any $t_2 \le t$, when $t \to +\infty$,

$$6e^{\alpha t}E\left\|\int_{-\infty}^{t} e^{-c_i(s)(t-s)}\sum_{j=1}^{n}a_{ij}(s)f_j\left(x_i\left(s-\tau_{ij}(s)\right)\right)ds\right\|^2 \longrightarrow 0. \tag{39}$$

Like the discussion of other terms on the right hand side of (35), as $t \to +\infty$, we have

$$\begin{aligned}
&6e^{\alpha t}E\Bigg\|\int_{-\infty}^{t} e^{-c_i(s)(t-s)} \\
&\qquad\times\sum_{j=1}^{n}b_{ij}(s)g_j \\
&\qquad\quad\times\left(\int_{-\infty}^{0}k_j(\theta)x_i(s+\theta)d\theta\right)ds\Bigg\|^2 \longrightarrow 0, \\
&3e^{\alpha t}E\left\|\int_{-\infty}^{t} e^{-c_i(s)(t-s)}I_i(s)ds\right\|^2 \longrightarrow 0, \\
&3e^{\alpha t}E\left\|\int_{-\infty}^{t} e^{-c_i(s)(t-s)}\sigma_i(s,x_i(s))d\omega_i(s)\right\|^2 \longrightarrow 0.
\end{aligned} \tag{40}$$

Therefore, from the above, we get that $e^{\alpha t}E\|x_i(t)\|^2 \to 0$ ($i = 1, 2, \ldots, n$) as $t \to +\infty$. Thus we know that (2) has a unique square mean almost periodic solution, which is an exponentially stable. □

Remark 13. From the assumptions of Theorem 10, we can see that stochastic perturbations on the neuron states also have key effect on the exponentially stability of square mean almost periodic.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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