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Research Article

Multigranulations Rough Set Method of Attribute Reduction in Information Systems Based on Evidence Theory

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Attribute reduction is one of the most important problems in rough set theory. However, from the granular computing point of view, the classical rough set theory is based on a single granulation. It is necessary to study the issue of attribute reduction based on multigranulations rough set. To acquire brief decision rules from information systems, this paper firstly investigates attribute reductions by combining the multigranulations rough set together with evidence theory. Concepts of belief and plausibility consistent set are proposed, and some important properties are addressed by the view of the optimistic and pessimistic multigranulations rough set. What is more, the multigranulations method of the belief and plausibility reductions is constructed in the paper. It is proved that a set is an optimistic (pessimistic) belief reduction if and only if it is an optimistic (pessimistic) lower approximation reduction, and a set is an optimistic (pessimistic) plausibility reduction if and only if it is an optimistic (pessimistic) upper approximation reduction.

1. Introduction

Rough set theory, originated by Pawlak in the early 1980s [1, 2], is an extension of the classical set theory and can be regarded as a soft computing tool to handle imprecision, vagueness, and uncertainty in date analysis. The theory has been found successful in applications, especially in the field of pattern recognition [3], medical diagnosis [4], data mining [5, 6], conflict analysis [7], algebra [8, 9], and other fields [10, 11]. Recently, the theory has generated a great deal of interest among more and more researchers.

Recently, several extensions of the rough set model have been proposed in terms of various requirements, such as the variable precision rough set (VPRS) model [12], the Bayesian rough set model [13], the fuzzy rough set model, and the rough fuzzy set model [14–16]. Equivalence relation is a basic notion in Pawlak's rough set model. The equivalence classes are employed to construct the lower and upper approximations of an arbitrary subset of the universe of discourse. However, the equivalence relation is a very restrictive condition that may limit applications of rough set. Hence, a variety of extensions of Pawlak's rough set were proposed by employing a more general mathematical concept, for example, arbitrary binary relations [17–19], neighborhood systems and Boolean algebras [20, 21], and partitions and coverings of the universe of discourse [22, 23]. In the view of granular computing (proposed by Zadeh [24]), a general concept described by a set is always characterized via the so-called lower and upper approximations under a single granulation; that is, the concept is depicted by known knowledge induced from a single relation (such as equivalence relation, tolerance relation, and reflexive relation) on the universe.

Since each set of the information granules can be considered as a granulation space, then one can call two or more than two information granules as multigranulations space. To make it more widely to apply the rough set theory in practical applications, Qian and Liang extended Pawlak's single-granulation rough set model to a multigranulations rough set model [25]. Multigranulations rough set was initially proposed by Qian and Liang [25], and later many researchers have extended the multigranulations rough set to the generalized multigranulations rough set. Xu et al. developed a variable multigranulations rough set model [26], a fuzzy multigranulations rough set model [27], a generalized multigranulations rough set approach [28], and a multigranulations rough set model in ordered information systems [29]. Yang et al. proposed the hierarchical structure properties of the multigranulations rough set [30–33] and multigranulations rough set in incomplete information system [34]. Lin et al. presented a neighborhood-based multigranulations rough set [35]. Qian et al. discussed the decision-theoretic rough set theory based on Bayesian decision procedure into the multigranulations [36]. She and He explored the topological structures and the properties of multigranulations rough set [37] and many others [38, 39].

Another important method used to deal with uncertainty in information systems is the Dempster-Shafer theory of evidence. It was originated by Dempster's concept of lower and upper probabilities [40] and extended by Shafer as a theory [41]. The basic representational structure in the theory is a belief structure, which can derive dual pairs of belief and plausibility functions. Subsequently, Zadeh generalized the Dempster-Shafer theory to the fuzzy environment based on his work on the concepts of information granularity [42] and the theory of possibility [43]. So as to evaluate the degrees of belief in fuzzy events, many authors have enriched the Dempster-Shafer theory in different ways. Interested readers can refer to [44] for a summary of some of these generalizations.

There are strong connections between rough set theory and Dempster-Shafer theory of evidence. It has been demonstrated that various belief structures are associated with various rough approximation spaces such that the different dual pairs of lower and upper approximation operators induced by rough approximation spaces may be used to interpret the corresponding dual pairs of belief and plausibility functions induced by belief structures [45–48].

It is well known that attribute reduction is one of the hot research topics of rough set theory. There are many attributes in information system in general. But some attributes are not always needed based on the various reasons. Several kinds of attribute reductions such as upper approximation reductions, lower approximation reductions, and positive region reductions were discussed in a decision system according to different requirements [49–51]. By now much study on this area had been reported and many useful results were obtained [50, 52–55]. While in our real life, we may face some problems in which the existing reductions cannot be disposed, on this situation, some new reductions are needed. In this paper, we attempt to investigate attribute reduction in multigranulations rough set based on evidence theory and its strong relations with existing reductions.

The organization of the rest of this paper is as follows. In Section 2, we give some basic concepts of information systems and Pawlak's rough set, multigranulations rough set, and evidence theory. In Section 3, evidence theory in optimistic multigranulations rough set and pessimistic multigranulations rough set has been constructed, and some important properties are discussed. In Section 4, we introduce the optimistic and pessimistic multigranulations rough set method of belief and plausibility reductions; then we also combine the belief structure with the lower and upper approximations. And in Section 5, we conclude the paper with a summary and outlook for further research.

2. Preliminaries

Throughout this paper, we assume that the universe is a nonempty finite set.

An information system is an order triple $S = (U, \mathrm{AT}, f)$, where $U = \{x_1, x_2, \ldots, x_n\}$ is a nonempty finite set of objects, $\mathrm{AT} = \{a_1, a_2, \ldots, a_m\}$ is a nonempty finite set of conditional attributes, and for any $a_i \in \mathrm{AT}$, $f_{a_i} : U \to V_{a_i}$ is a map, where V_{a_i} is the domain of the attribute a_i. In particular, a target information system is given by $S = (U, \mathrm{AT}, f, D, g)$, where $D = \{d_1, d_2, \ldots, d_p\}$ is a nonempty finite set of decision attributes, and for any $d_j \in D$, $g_{d_j} : U \to V_{d_j}$ is a map, where V_{d_j} is the domain of the attribute d_j. In general, a target information system is consistent, if the partitions induced from the set of condition attributes AT are finer than the partitions induced from the set of decision attributes D. Otherwise, it is inconsistent.

For an information system, any attribute domain V_a may contain special symbol "$*$" to represent that the value of an attribute is unknown. Here, we assume that an object $x \in U$ possesses only one value for an attribute a, $a \in \mathrm{AT}$. Thus, if the value of an attribute a is missing, then the attribute value is the symbol "$*$", and the real value of the attribute must be from the set $V_a \backslash \{*\}$. Any domain value different from "$*$" will be called regular. A system in which values of all attributes for all objects from U are regular (known) is called complete; otherwise it is called incomplete [56, 57].

Throughout this paper, we assume that the information system is the complete information system.

Suppose that $S = (U, \mathrm{AT}, f)$ is an information system, $R_A = \{(x, y) \mid f_{a_i}(x) = f_{a_i}(y), \forall a_i \in A\}$; let U/R_A be a partition of U induced by the attribute subset $A \subseteq \mathrm{AT}$. For any $x \in U$, $[x]_A = \{y \mid (x, y) \in R_A\}$; more information can be found in [58–60].

In the multigranulations rough set model, unlike Pawlak's rough set theory, the target concept is approximated via multiple partitions induced by multiple equivalence relations. Suppose that $S = (U, \mathrm{AT}, f)$ is an information system, $A_1, A_2, \ldots, A_s$ ($s \leq 2^{|\mathrm{AT}|}$) are attribute subsets, and $X \subseteq U$. Then the optimistic multigranulations lower approximation and upper approximation of X related to $A_1, A_2, \ldots, A_s$ in U are defined as follows:

$$\underline{\mathrm{OM}_{\sum_{i=1}^{s} A_i}}(X) = \left\{x \mid \vee\left([x]_{A_i} \subseteq X\right)\right\},$$
$$\overline{\mathrm{OM}_{\sum_{i=1}^{s} A_i}}(X) = \left\{x \mid \wedge\left([x]_{A_i} \cap X \neq \emptyset\right)\right\}. \quad (1)$$

And the pessimistic multigranulations lower approximation and upper approximation of X related to $A_1, A_2, \ldots, A_s$ in U are defined as follows:

$$\underline{\mathrm{PM}_{\sum_{i=1}^{s} A_i}}(X) = \left\{x \mid \wedge\left([x]_{A_i} \subseteq X\right)\right\},$$
$$\overline{\mathrm{PM}_{\sum_{i=1}^{s} A_i}}(X) = \left\{x \mid \vee\left([x]_{A_i} \cap X \neq \emptyset\right)\right\}. \quad (2)$$

More detailed introductions can be seen in [58, 59, 61, 62].

In evidence theory [40, 41], a mass function of a universe U can be defined by a map $m : P(U) \rightarrow [0, 1]$, and this mass function satisfies two axioms:

$$(M1) \quad m(\emptyset) = 0,$$
$$(M2) \quad \sum_{X \subseteq U} m(X) = 1. \tag{3}$$

The value $m(X)$ represents the degree of belief that a specific element of U belongs to set X but not to any particular subset of X.

If $m(X) > 0$, then X is called a focal element. The family of all focal elements of m are denoted by M. Then the pair (M, m) is called a belief structure.

In information systems, each belief structure can derive a pair of belief and plausibility functions based on classical equivalence relation.

Definition 1 (see [40, 41]). Let (M, m) be a belief structure. A set function $\mathrm{Bel} : P(U) \rightarrow [0, 1]$ is referred to as a belief function on U, if for any $X \in P(U)$,

$$\mathrm{Bel}(X) = \sum_{Y \subseteq X} m(Y). \tag{4}$$

A set function $\mathrm{Pl} : P(U) \rightarrow [0, 1]$ is referred to as a plausibility function on U, if for any $X \in P(U)$,

$$\mathrm{Pl}(X) = \sum_{Y \cap X \neq \emptyset} m(Y). \tag{5}$$

Remark 2. The above definition about belief and plausibility functions can also be defined as follows.

A set function $\mathrm{Bel} : P(U) \rightarrow [0, 1]$ is referred to as a belief function on U, if it satisfies the following three axioms [48]:

(1) $\mathrm{Bel}(\emptyset) = 0$,

(2) $\mathrm{Bel}(U) = 1$,

(3) for any positive integer n and every collection $X_1, X_2, \ldots, X_n \subseteq U$,

$$\mathrm{Bel}\left(\bigcup_{i=1}^{n} X_i\right) \geq \sum_{\emptyset \neq J \subseteq \{1,2,\ldots,n\}} (-1)^{|J|+1} \mathrm{Bel}\left(\bigcap_{i \in J} X_i\right). \tag{6}$$

A set function $\mathrm{Pl} : P(U) \rightarrow [0, 1]$ is referred to as a plausibility function on U, if it satisfies the following three axioms [48]:

(1) $\mathrm{Pl}(\emptyset) = 0$,

(2) $\mathrm{Pl}(U) = 1$,

(3) for any positive integer n and every collection $X_1, X_2, \ldots, X_n \subseteq U$,

$$\mathrm{Pl}\left(\bigcap_{i=1}^{n} X_i\right) \leq \sum_{\emptyset \neq J \subseteq \{1,2,\ldots,n\}} (-1)^{|J|+1} \mathrm{Pl}\left(\bigcup_{i \in J} X_i\right). \tag{7}$$

From the definition and remark above, one can test and verify belief and plausibility functions by validating the three axioms above, respectively.

3. Evidence Theory in Multigranulations Rough Set

In this section, we will introduce the evidence theory in the optimistic multigranulations rough set and pessimistic multigranulations rough set and discuss some important properties of evidence theory in information system.

Definition 3. Let $S = (U, \mathrm{AT}, f)$ be an information system, $A_1, A_2, \ldots, A_s \subseteq \mathrm{AT}$ $(s \leq 2^{|\mathrm{AT}|})$, and let R_i be the equivalence relation associated with A_i. For any $X \in U/R_i$, a mass function m_i of S can be defined as $m_i(X) = |X|/|U|$, where $|X|$ denotes the cardinality of a set X.

By the above definition, we can easily find that a mass function of information system satisfies two basic axioms. That is to say, for any $X \in U/R_i$ in information system, the following two axioms hold directly:

$$(M1) \quad m_i(\emptyset) = 0,$$
$$(M2) \quad \sum_{X \subseteq U/R_i} m_i(X) = 1. \tag{8}$$

Similarly, the family of all focal elements of m is denoted by M in optimistic multigranulations rough set. The pair (M, m) is called a belief structure of the optimistic multigranulations rough set in information system; a pair of belief and plausibility function in the optimistic multigranulations rough set can be derived immediately.

Definition 4. Let $S = (U, \mathrm{AT}, f)$ be an information system and (M, m) a belief structure.

(1) A set function $\mathrm{Bel} : P(U) \rightarrow [0, 1]$ is referred to as an optimistic belief function on U, if for any $X \in P(U)$,

$$\mathrm{Bel}^{o}_{\sum_{i=1}^{s} A_i}(X) = \sum_{Y \subseteq X, Y \in U/R_{A_i} (\exists A_i \subseteq \mathrm{AT})} m_i(Y). \tag{9}$$

A set function $\mathrm{Pl} : P(U) \rightarrow [0, 1]$ is referred to as an optimistic plausibility function on U, if for any $X \in P(U)$,

$$\mathrm{Pl}^{o}_{\sum_{i=1}^{s} A_i}(X) = \sum_{Y \cap X \neq \emptyset, Y \in U/R_{A_i} (\forall A_i \subseteq \mathrm{AT})} m_i(Y). \tag{10}$$

(2) A set function $\mathrm{Bel} : P(U) \rightarrow [0, 1]$ is referred to as a pessimistic belief function on U, if for any $X \in P(U)$,

$$\mathrm{Bel}^{P}_{\sum_{i=1}^{s} A_i}(X) = \sum_{Y \subseteq X, Y \in U/R_{A_i} (\forall A_i \subseteq \mathrm{AT})} m_i(Y). \tag{11}$$

A set function $\mathrm{Pl} : P(U) \rightarrow [0, 1]$ is referred to as a pessimistic plausibility function on U, if for any $X \in P(U)$,

$$\mathrm{Pl}^{P}_{\sum_{i=1}^{s} A_i}(X) = \sum_{Y \cap X \neq \emptyset, Y \in U/R_{A_i} (\exists A_i \subseteq \mathrm{AT})} m_i(Y). \tag{12}$$

Theorem 5. *Let $S = (U, AT, f)$ be an information system, for any $X \subseteq U$, $A_1, A_2, \ldots, A_s \subseteq AT$ ($s \leq 2^{|AT|}$), denoted by*

$$Bel^o_{\sum_{i=1}^s A_i}(X) = \frac{\left|\underline{OM_{\sum_{i=1}^s A_i}}(X)\right|}{|U|},$$
$$Pl^o_{\sum_{i=1}^s A_i}(X) = \frac{\left|\overline{OM_{\sum_{i=1}^s A_i}}(X)\right|}{|U|},$$
$$Bel^P_{\sum_{i=1}^s A_i}(X) = \frac{\left|\underline{PM_{\sum_{i=1}^s A_i}}(X)\right|}{|U|},$$
$$Pl^P_{\sum_{i=1}^s A_i}(X) = \frac{\left|\overline{PM_{\sum_{i=1}^s A_i}}(X)\right|}{|U|}. \tag{13}$$

Then

$Bel^o_{\sum_{i=1}^s A_i}(X)$ is the optimistic belief function and $Pl^o_{\sum_{i=1}^s A_i}(X)$ is the optimistic plausibility function of U;

$Bel^P_{\sum_{i=1}^s A_i}(X)$ is the pessimistic belief function and $Pl^P_{\sum_{i=1}^s A_i}(X)$ is the pessimistic plausibility function of U.

Proof. We only prove $\mathrm{Bel}^o_{\sum_{i=1}^s A_i}(X)$ is the optimistic belief function of U; then analogously we can prove $\mathrm{Pl}^o_{\sum_{i=1}^s A_i}(X)$ is the optimistic plausibility function of U. Consider

(1) $\mathrm{Bel}^o_{\sum_{i=1}^s A_i}(\emptyset) = |\underline{\mathrm{OM}_{\sum_{i=1}^s A_i}}(\emptyset)|/|U| = 0/|U| = 0$,

(2) $\mathrm{Bel}^o_{\sum_{i=1}^s A_i}(U) = |\underline{\mathrm{OM}_{\sum_{i=1}^s A_i}}(U)|/|U| = |U|/|U| = 1$,

(3) for any positive integer n and every collection $X_1, X_2, \ldots, X_n \subseteq U$, we have

$$\begin{aligned}
&\mathrm{Bel}^o_{\sum_{i=1}^s A_i}(X_1 \cup X_2 \cup \cdots \cup X_n) \\
&= \frac{\left|\underline{\mathrm{OM}_{\sum_{i=1}^s A_i}}(X_1 \cup X_2 \cup \cdots \cup X_n)\right|}{|U|} \\
&\geq \frac{\left|\underline{\mathrm{OM}_{\sum_{i=1}^s A_i}}(X_1) \cup \underline{\mathrm{OM}_{\sum_{i=1}^s A_i}}(X_2) \cup \cdots \cup \underline{\mathrm{OM}_{\sum_{i=1}^s A_i}}(X_n)\right|}{|U|} \\
&= \sum_{j=1}^n \frac{\left|\underline{\mathrm{OM}_{\sum_{i=1}^s A_i}}(X_j)\right|}{|U|} \\
&\quad - \sum_{k<j} \frac{\left|\underline{\mathrm{OM}_{\sum_{i=1}^s A_i}}(X_k) \cap \underline{\mathrm{OM}_{\sum_{i=1}^s A_i}}(X_j)\right|}{|U|} + \cdots + (-1)^{n+1} \\
&\quad \times \frac{\left|\underline{\mathrm{OM}_{\sum_{i=1}^s A_i}}(X_1) \cap \underline{\mathrm{OM}_{\sum_{i=1}^s A_i}}(X_2) \cap \cdots \cap \underline{\mathrm{OM}_{\sum_{i=1}^s A_i}}(X_n)\right|}{|U|} \\
&\geq \sum_{j=1}^n \frac{\left|\underline{\mathrm{OM}_{\sum_{i=1}^s A_i}}(X_j)\right|}{|U|} - \sum_{k<j} \frac{\left|\underline{\mathrm{OM}_{\sum_{i=1}^s A_i}}(X_k \cap X_j)\right|}{|U|} \\
&\quad + \cdots + (-1)^{n+1} \frac{\left|\underline{\mathrm{OM}_{\sum_{i=1}^s A_i}}(X_1 \cap X_2 \cdots \cap X_n)\right|}{|U|} \\
&= \sum_{j=1}^n \mathrm{Bel}^o_{\sum_{i=1}^s A_i}(X_j) - \sum_{k<j} \mathrm{Bel}^o_{\sum_{i=1}^s A_i}(X_k \cap X_j) \\
&\quad + \cdots + (-1)^{n+1} \mathrm{Bel}^o_{\sum_{i=1}^s A_i}(X_1 \cap X_2 \cap \cdots \cap X_n).
\end{aligned} \tag{14}$$

Similarly we can prove that $\mathrm{Bel}^P_{\sum_{i=1}^s A_i}(X)$ is the pessimistic belief function and $\mathrm{Pl}^P_{\sum_{i=1}^s A_i}(X)$ is the pessimistic plausibility function of U.

Thus, the theorem is proved. □

From the theorem above, one can get the following properties of the belief function and plausibility function in the optimistic and pessimistic multigranulations rough set.

Theorem 6. *Belief function and plausibility function based on the same belief structure are connected by the dual property in the optimistic multigranulations rough set and the pessimistic multigranulations rough set, respectively,*

(1) $Bel^o_{\sum_{i=1}^s A_i}(X) = 1 - Pl^o_{\sum_{i=1}^s A_i}(\sim X)$,

(2) $Bel^P_{\sum_{i=1}^s A_i}(X) = 1 - Pl^P_{\sum_{i=1}^s A_i}(\sim X)$.

Proof. (1) It is clear that $\underline{\mathrm{OM}_{\sum_{i=1}^s A_i}}(X) =\sim \overline{\mathrm{OM}_{\sum_{i=1}^s A_i}}(\sim X)$, so $|\underline{\mathrm{OM}_{\sum_{i=1}^s A_i}}(X)| = |U| - |\overline{\mathrm{OM}_{\sum_{i=1}^s A_i}}(\sim X)|$.

Then

$$\begin{aligned}
\frac{\left|\underline{\mathrm{OM}_{\sum_{i=1}^s A_i}}(X)\right|}{|U|} &= \frac{|U| - \left|\overline{\mathrm{OM}_{\sum_{i=1}^s A_i}}(\sim X)\right|}{|U|} \\
&= 1 - \frac{\left|\overline{\mathrm{OM}_{\sum_{i=1}^s A_i}}(\sim X)\right|}{|U|}.
\end{aligned} \tag{15}$$

That is to say

$$\mathrm{Bel}^o_{\sum_{i=1}^s A_i}(X) = 1 - \mathrm{Pl}^o_{\sum_{i=1}^s A_i}(\sim X). \tag{16}$$

(2) One can prove $\mathrm{Bel}^P_{\sum_{i=1}^s A_i}(X) = 1 - \mathrm{Pl}^P_{\sum_{i=1}^s A_i}(\sim X)$ to be similar. □

Corollary 7. *Let $S = (U, AT, f)$ be an information system, for any $X \subseteq U$, $A_1, A_2, \ldots, A_s \subseteq AT$ ($s \leq 2^{|AT|}$); then*

(1) $Bel^o_{A_i}(X) \leq Bel^o_{\sum_{1\leq j\leq s}^{i\neq j} A_i}(X) \leq Bel^o_{\sum_{i=1}^s A_i}(X) \leq |X|/|U| \leq Pl^o_{\sum_{i=1}^s A_i}(X) \leq Pl^o_{\sum_{1\leq j\leq s}^{i\neq j} A_i}(X) \leq Pl^o_{A_i}(X)$,

(2) $Bel^P_{\sum_{i=1}^s A_i}(X) \leq Bel^P_{\sum_{1\leq j\leq s}^{i\neq j} A_i}(X) \leq Bel^P_{A_i}(X) \leq |X|/|U| \leq Pl^P_{A_i}(X) \leq Pl^P_{\sum_{1\leq j\leq s}^{i\neq j} A_i}(X) \leq Pl^P_{\sum_{i=1}^s A_i}(X)$.

Proof. It can be obtained directly by combining the properties of the optimistic multigranulations rough set, pessimistic multigranulations rough set, and Theorem 5. □

The difference between (1) and (2) in Corollary 7 is mainly because of the difference between the definition of the optimistic and pessimistic multigranulations rough set lower and upper approximations.

Example 8. Table 1 depicts a target information system containing some information about an emporium investment project. *Locus*, *Investment*, and *Population density* are the conditional attributes of the system, and *Decision* is the decision attribute. (In the sequel, L, I, P, and D will stand for *Locus*, *Investment*, *Population density*, and *Decision*, resp.) The attribute domains are as follows: $V_L = \{\text{Good}, \text{Common}, \text{Bad}\}$, $V_I = \{\text{High}, \text{Low}\}$, $V_P = \{\text{Big}, \text{Small}, \text{Medium}\}$, and $V_D = \{\text{Yes}, \text{No}\}$.

Let $X = \{x_1, x_2, x_6, x_8\}$; we have gotten

$$\begin{gathered}
\underline{\mathrm{OM}_L}(X) = \underline{R_L}(X) = \{x_8\}, \\
\underline{\mathrm{PM}_L}(X) = \underline{R_L}(X) = \{x_8\}, \\
\overline{\mathrm{OM}_L}(X) = \overline{R_L}(X) = \{x_1, x_2, \ldots, x_8\}, \\
\overline{\mathrm{PM}_L}(X) = \overline{R_L}(X) = \{x_1, x_2, \ldots, x_8\}, \\
\underline{\mathrm{OM}_{L+P}}(X) = \{x_1, x_2, x_8\}, \qquad \underline{\mathrm{PM}_{L+P}}(X) = \emptyset, \\
\overline{\mathrm{OM}_{L+P}}(X) = \{x_1, x_2, x_6, x_7, x_8\}, \\
\overline{\mathrm{PM}_{L+P}}(X) = \{x_1, x_2, \ldots, x_8\}, \\
\underline{\mathrm{OM}_{L+I+P}}(X) = \{x_1, x_2, x_8\}, \qquad \underline{\mathrm{PM}_{L+I+P}}(X) = \emptyset, \\
\overline{\mathrm{OM}_{L+I+P}}(X) = \{x_1, x_2, x_6, x_7, x_8\}, \\
\overline{\mathrm{PM}_{L+I+P}}(X) = \{x_1, x_2, \ldots, x_8\}.
\end{gathered} \tag{17}$$

So, we can calculate

$$\begin{gathered}
\mathrm{Bel}^o_L(X) = \frac{\left|\underline{\mathrm{OM}_L}(X)\right|}{|U|} = \frac{1}{8}, \\
\mathrm{Bel}^P_L(X) = \frac{\left|\underline{\mathrm{PM}_L}(X)\right|}{|U|} = \frac{1}{8}, \\
\mathrm{Pl}^o_L(X) = \frac{\left|\overline{\mathrm{OM}_L}(X)\right|}{|U|} = 1, \\
\mathrm{Pl}^P_L(X) = \frac{\left|\overline{\mathrm{PM}_L}(X)\right|}{|U|} = 1, \\
\mathrm{Bel}^o_{L+P}(X) = \frac{\left|\underline{\mathrm{OM}_{L+P}}(X)\right|}{|U|} = \frac{3}{8}, \\
\mathrm{Bel}^P_{L+P}(X) = \frac{\left|\underline{\mathrm{PM}_{L+P}}(X)\right|}{|U|} = 0, \\
\mathrm{Pl}^o_{L+P}(X) = \frac{\left|\overline{\mathrm{OM}_{L+P}}(X)\right|}{|U|} = \frac{5}{8}, \\
\mathrm{Pl}^P_{L+P}(X) = \frac{\left|\overline{\mathrm{PM}_{L+P}}(X)\right|}{|U|} = 1, \\
\mathrm{Bel}^o_{L+I+P}(X) = \frac{\left|\underline{\mathrm{OM}_{L+I+P}}(X)\right|}{|U|} = \frac{3}{8}, \\
\mathrm{Bel}^P_{L+I+P}(X) = \frac{\left|\underline{\mathrm{PM}_{L+I+P}}(X)\right|}{|U|} = 0, \\
\mathrm{Pl}^o_{L+I+P}(X) = \frac{\left|\overline{OM_{L+I+P}}(X)\right|}{|U|} = \frac{5}{8}, \\
\mathrm{Pl}^P_{L+I+P}(X) = \frac{\left|\overline{\mathrm{PM}_{L+I+P}}(X)\right|}{|U|} = 1.
\end{gathered} \tag{18}$$

Hence, the following is obvious:

$$\begin{aligned}
&\mathrm{Bel}^o_L(X) \le \mathrm{Bel}^o_{L+P}(X) \le \mathrm{Bel}^o_{L+I+P}(X) \\
&\quad \le \frac{|X|}{|U|} \le \mathrm{Pl}^o_{L+I+P}(X) \le \mathrm{Pl}^o_{L+P}(X) \le \mathrm{Pl}^o_L(X), \\
&\mathrm{Bel}^P_{L+I+P}(X) \le \mathrm{Bel}^P_{L+P}(X) \le \mathrm{Bel}^P_L(X) \\
&\quad \le \frac{|X|}{|U|} \le \mathrm{Pl}^P_L(X) \le \mathrm{Pl}^P_{L+P}(X) \le \mathrm{Pl}^P_{L+I+P}(X).
\end{aligned} \tag{19}$$

4. Attribute Reduction Based on Evidence Theory

In this section, we consider the optimistic multiple granulation rough set and the pessimistic multigranulations rough set method of the attribute reductions by introducing the concepts of belief and plausibility reductions in information system and compare them with the existing reductions.

Let $S = (U, \mathrm{AT}, f, D, g)$ be a decision information system, $A \subseteq \mathrm{AT}$, and let R_A and R_D be the equivalence relations of U, which are induced by the conditional attribute set A and the decision attribute set $D = \{d\}$, respectively; we denote

$$\begin{gathered}
\frac{U}{R_A} = \{[x_i]_A \mid x_i \in U\}, \\
\frac{U}{R_D} = \{D_1, D_2, \ldots, D_r\}, \\
\overline{\eta}^o_A = \Big(\overline{\mathrm{OM}_{\sum_{A_i \in A} A_i}}(D_1), \overline{\mathrm{OM}_{\sum_{A_i \in A} A_i}}(D_2), \\
\ldots, \overline{\mathrm{OM}_{\sum_{A_i \in A} A_i}}(D_r)\Big),
\end{gathered}$$

Table 1: A target information system about emporium investment project.

Project	Locus	Investment	Population density	Decision
x_1	Common	High	Big	Yes
x_2	Bad	High	Big	Yes
x_3	Bad	Low	Small	No
x_4	Bad	Low	Small	No
x_5	Bad	Low	Small	No
x_6	Bad	High	Medium	Yes
x_7	Common	High	Medium	No
x_8	Good	High	Medium	Yes

$$\underline{\eta}_A^o = \left(\underline{\mathrm{OM}_{\sum_{A_i\in A}A_i}}(D_1), \underline{\mathrm{OM}_{\sum_{A_i\in A}A_i}}(D_2), \ldots, \underline{\mathrm{OM}_{\sum_{A_i\in A}A_i}}(D_r)\right),$$
$$\overline{\eta}_A^P = \left(\overline{\mathrm{PM}_{\sum_{A_i\in A}A_i}}(D_1), \overline{\mathrm{PM}_{\sum_{A_i\in A}A_i}}(D_2), \ldots, \overline{\mathrm{PM}_{\sum_{A_i\in A}A_i}}(D_r)\right),$$
$$\underline{\eta}_A^P = \left(\underline{\mathrm{PM}_{\sum_{A_i\in A}A_i}}(D_1), \underline{\mathrm{PM}_{\sum_{A_i\in A}A_i}}(D_2), \ldots, \underline{\mathrm{PM}_{\sum_{A_i\in A}A_i}}(D_r)\right), \tag{20}$$

where $[x_i]_A = \{y \in U \mid (x_i, y) \in R_A\}$.

Definition 9 (see [58]). Let $S = (U, \mathrm{AT}, f, D, g)$ be a decision information system in which $A \subseteq \mathrm{AT}$.

(1) If $\overline{\eta}_A^o = \overline{\eta}_{\mathrm{AT}}^o$, then A is referred to as an optimistic upper approximation consistent set of S with respect to the equivalence relation R_A; if A is an optimistic upper approximation consistent set of S and for any $A' \subseteq A$, A' is not the optimistic upper approximation consistent set of S, then A is referred to as an optimistic upper approximation reduction of S with respect to the equivalence relation R_A.

(2) If $\underline{\eta}_A^o = \underline{\eta}_{\mathrm{AT}}^o$, then A is referred to as an optimistic lower approximation consistent set of S with respect to the equivalence relation R_A; if A is an optimistic lower approximation consistent set of S and for any $A' \subseteq A$, A' is not the optimistic lower approximation consistent set of S, then A is referred to as an optimistic lower approximation reduction of S with respect to the equivalence relation R_A.

(3) If $\overline{\eta}_A^P = \overline{\eta}_{\mathrm{AT}}^P$, then A is referred to as a pessimistic upper approximation consistent set of S with respect to the equivalence relation R_A; if A is a pessimistic upper approximation consistent set of S and for any $A' \subseteq A$, A' is not the pessimistic upper approximation consistent set of S, then A is referred to as a pessimistic upper approximation reduction of S with respect to the equivalence relation R_A.

(4) If $\underline{\eta}_A^P = \underline{\eta}_{\mathrm{AT}}^P$, then A is referred to as a pessimistic lower approximation consistent set of S with respect to the equivalence relation R_A; if A is a pessimistic lower approximation consistent set of S and for any $A' \subseteq A$, A' is not the pessimistic lower approximation consistent set of S, then A is referred to as a pessimistic lower approximation reduction of S with respect to the equivalence relation R_A. From the definitions above, one can get the following theorem directly.

Theorem 10. *Let $S = (U, AT, f, D, g)$ be a decision information system in which $A \subseteq AT$; then*

(1) *A is an optimistic upper approximation consistent set of S if and only if, for any $D_j \in U/R_D$, $\overline{OM_{\sum_{A_i\in A}A_i}}(D_j) = \overline{R_{AT}}(D_j)$ holds;*

(2) *A is an optimistic lower approximation consistent set of S if and only if, for any $D_j \in U/R_D$, $\underline{OM_{\sum_{A_i\in A}A_i}}(D_j) = \underline{R_{AT}}(D_j)$ holds;*

(3) *A is a pessimistic upper approximation consistent set of S if and only if, for any $D_j \in U/R_D$, $\overline{PM_{\sum_{A_i\in A}A_i}}(D_j) = \overline{R_{AT}}(D_j)$ holds;*

(4) *A is a pessimistic lower approximation consistent set of S if and only if, for any $D_j \in U/R_D$, $\underline{PM_{\sum_{A_i\in A}A_i}}(D_j) = \underline{R_{AT}}(D_j)$ holds.*

Proof. It can be derived easily from the definition of the optimistic upper and lower approximation reduction and the pessimistic upper and lower approximation reduction. □

Definition 11. Let $S = (U, \mathrm{AT}, f, D, g)$ be a decision information system in which $A \subseteq \mathrm{AT}$.

(1) If for any $X \in U/R_{\mathrm{AT}}$, $\mathrm{Bel}^o_{\sum_{A_i\in A}A_i}(X) = \mathrm{Bel}^o_{\mathrm{AT}}(X)$, then A is referred to as an optimistic belief consistent set of S; if A is an optimistic belief consistent set of S and for any $A' \subseteq A$, A' is not the optimistic belief consistent set of S, then A is referred to as an optimistic belief reduction of S.

(2) If for any $X \in U/R_{\mathrm{AT}}$, $\mathrm{Pl}^o_{\sum_{A_i\in A}A_i}(X) = \mathrm{Pl}^o_{\mathrm{AT}}(X)$, then A is referred to as an optimistic plausibility consistent set of S; if A is an optimistic plausibility consistent

set of S and for any $A' \subseteq A$, A' is not the optimistic plausibility consistent set of S, then A is referred to as an optimistic plausibility reduction of S.

(3) If for any $X \in U/R_{\mathrm{AT}}$, $\mathrm{Bel}^{P}_{\sum_{A_i \in A} A_i}(X) = \mathrm{Bel}^{P}_{\mathrm{AT}}(X)$, then A is referred to as a pessimistic belief consistent set of S; if A is a pessimistic belief consistent set of S and for any $A' \subseteq A$, A' is not the pessimistic belief consistent set of S, then A is referred to as a pessimistic belief reduction of S.

(4) If for any $X \in U/R_{\mathrm{AT}}$, $\mathrm{Pl}^{P}_{\sum_{A_i \in A} A_i}(X) = \mathrm{Pl}^{P}_{\mathrm{AT}}(X)$, then A is referred to as a pessimistic plausibility consistent set of S; if A is a pessimistic plausibility consistent set of S and for any $A' \subseteq A$, A' is not the pessimistic plausibility consistent set of S, then A is referred to as a pessimistic plausibility reduction of S.

Theorem 12. *Let $S = (U, AT, f, D, g)$ be a decision information system in which $A \subseteq AT$; then*

(1) *A is an optimistic belief consistent set of S if and only if A is an optimistic lower approximation consistent set of S;*

(2) *A is an optimistic plausibility consistent set of S if and only if A is an optimistic upper approximation consistent set of S;*

(3) *A is a pessimistic belief consistent set of S if and only if A is a pessimistic lower approximation consistent set of S;*

(4) *A is a pessimistic plausibility consistent set of S if and only if A is a pessimistic upper approximation consistent set of S.*

Proof. (1) Assume that A is an optimistic belief consistent set of S; for any $X \in U/R_{\mathrm{AT}}$, we have $\mathrm{Bel}^{o}_{\sum_{A_i \in A} A_i}(X) = \mathrm{Bel}^{o}_{\mathrm{AT}}(X)$. That is to say, $|\underline{\mathrm{OM}_{\sum_{A_i \in A} A_i}}(X)| = |\underline{R_{\mathrm{AT}}}(X)|$. Then by the definition of the optimistic lower approximation we have $[x]_{\mathrm{AT}} \subseteq [x]_{A_i}$; then $\underline{\mathrm{OM}_{\sum_{A_i \in A} A_i}}(X) \subseteq \underline{R_{\mathrm{AT}}}(X)$. So $\underline{\mathrm{OM}_{\sum_{A_i \in A} A_i}}(X) = \underline{R_{\mathrm{AT}}}(X)$. By Theorem 10, we obtain A is an optimistic lower approximation consistent set of S.

Conversely, if A is an optimistic lower approximation consistent set of S, for any $D_j \in U/R_D$, we have $\underline{\mathrm{OM}_{\sum_{A_i \in A} A_i}}(D_j) = \underline{R_{\mathrm{AT}}}(D_j)$. Then $|\underline{\mathrm{OM}_{\sum_{A_i \in A} A_i}}(D_j)|/|U| = |\underline{R_{\mathrm{AT}}}(D_j)|/|U|$. That is to say, $\mathrm{Bel}^{o}_{\sum_{A_i \in A} A_i}(D_j) = \mathrm{Bel}^{o}_{\mathrm{AT}}(D_j)$. Thus A is an optimistic belief consistent set of S.

(2) Assume that A is an optimistic plausibility consistent set of S; for any $X \in U/R_{\mathrm{AT}}$, we have $\mathrm{Pl}^{o}_{\sum_{A_i \in A} A_i}(X) = \mathrm{Pl}^{o}_{\mathrm{AT}}(X)$. That is to say, $|\overline{\mathrm{OM}_{\sum_{A_i \in A} A_i}}(X)| = |\overline{R_{\mathrm{AT}}}(X)|$. By the definition of the optimistic upper approximation we have, $[x]_{\mathrm{AT}} \subseteq [x]_{A_i}$. Then $\overline{\mathrm{OM}_{\sum_{A_i \in A} A_i}}(X) \supseteq \overline{R_{\mathrm{AT}}}(X)$. So $\overline{\mathrm{OM}_{\sum_{A_i \in A} A_i}}(X) = \overline{R_{\mathrm{AT}}}(X)$. By Theorem 10, we obtain A is an optimistic upper approximation consistent set of S.

Conversely, if A is an optimistic upper approximation consistent set of S, for any $D_j \in U/R_D$, we have $\overline{\mathrm{OM}_{\sum_{A_i \in A} A_i}}(D_j) = \overline{R_{\mathrm{AT}}}(D_j)$. Then $|\overline{\mathrm{OM}_{\sum_{A_i \in A} A_i}}(D_j)|/|U| = |\overline{R_{\mathrm{AT}}}(D_j)|/|U|$. That is to say, $\mathrm{Pl}^{o}_{\sum_{A_i \in A} A_i}(D_j) = \mathrm{Pl}^{o}_{\mathrm{AT}}(D_j)$. Thus A is an optimistic plausibility consistent set of S.

(3) It is straightforward by (1).

(4) It is straightforward by (2).

Hence, the proof is completed. □

Table 2: A system about emporium investment project.

Project	Decision
x_1	Yes
x_2	Yes
x_3	No
x_4	No
x_5	No
x_6	Yes
x_7	No
x_8	Yes

Corollary 13. *Let $S = (U, AT, f, D, g)$ be a decision information system in which $A \subseteq AT$; then*

(1) *A is an optimistic belief reduction of S if and only if A is an optimistic lower approximation reduction of S;*

(2) *A is an optimistic plausibility reduction of S if and only if A is an optimistic upper approximation reduction of S;*

(3) *A is a pessimistic belief reduction of S if and only if A is a pessimistic lower approximation reduction of S;*

(4) *A is a pessimistic plausibility reduction of S if and only if A is a pessimistic upper approximation reduction of S.*

We have shown that the belief and plausibility reduction are the same with lower and upper approximation reduction. One may deal with some issue in which the lower and upper approximation reduction cannot be explained, while the belief and plausibility reduction can be well explained. Just like this, indefinite integral and definite integral are two basic problems in the integral calculus. Indefinite integral is the inverse operation of differentiation, while definite integral is the limit of a particular type of the sum. There are both difference and connection between them. The definite integral and the indefinite integral can be connected in theory using Newton-leibniz formula, while some functions still can be integrated, though they have no antiderivative on closed interval. Under this circumstance, indefinite integral cannot be evaluated by Newton-leibniz formula. Next we use an example to illustrate this problem.

Example 14. Table 2 depicts a system containing some information about an emporium investment project. All projects are $U = \{x_1, x_2, \ldots, x_8\}$, and *Decision* is the decision attribute. (In the sequel, D will stand for *Decision*.)

In this example, there are no conditional attributes. So we cannot use the lower and upper approximations to deal with this system. Based on evidence theory, each object can be given corresponding confidence degree according to the existing information. And this confidence degree after normalization can be used as a new mass function. Then we can apply the evidence theory to dispose this system. Until now, we have not found a good method to evaluate the confidence degree. This will be an important issue of our research in the future.

5. Conclusions

It is well known that rough set theory has been regarded as a generalization of the classical set theory in some cases. In this paper, we have combined the rough set theory and evidence theory, in order to study the problem of attribute reductions. By introducing mass function based on the multigranulations rough set, we have considered the notions of the multigranulations rough set method of belief and plausibility reductions. What is more, we have found that a set is an optimistic (pessimistic) belief reduction if and only if it is an optimistic (pessimistic) lower approximation reduction, and a set is an optimistic (pessimistic) plausibility reduction if and only if it is an optimistic (pessimistic) upper approximation reduction by optimistic and pessimistic frames, respectively. And we will investigate the specific application of theories obtained in our further study.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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