

## Editorial

# Nonlinear Dynamics in Applied Sciences Systems: Advances and Perspectives

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This editorial provides a brief review of some concepts related to the subject of the papers published in this special issue devoted to the onset of nonlinear dynamics in systems of the applied sciences. Nonlinear dynamics is currently an active and fashionable discipline that is having a profound effect on a wide variety of fields, including populations dynamics, physics, biology, economics, and sociology.

The origin of *nonlinear dynamics* is related to the gravitational three-body problem [1], which attempts to calculate the orbit of a planet around the sun in presence of two celestial bodies (planets or moon). In particular, the presence of a third celestial body can influence the dynamics of the plane and produce highly irregular dynamics (chaotic); see [2–4].

However, the development of a mathematical apparatus for irregular (hyperbolic) dynamics comes from mathematicians and theoretical physicists of the Russian school; see the review paper [5] and the reference cited therein. Moreover the development of high speed computers has also allowed for displaying the complex behavior of the solutions visually.

Nowadays, nonlinear dynamics can be found in almost every branch of the applied science. It includes systems in which feedback, iterations, nonlinear interactions, and the general dependency of each part of the system upon the behavior of all other parts demand the use of nonlinear differential equations rather than the well-known linear differential equations, for example, Bellman equation [6] (with applications in economics [7]), Boltzmann equation [8]

(with applications to gas dynamics), Colebrook equation [9] (with applications to turbulence), Ginzburg-Landau equation [10] (with applications to superconductors), Navier-Stokes equation [11] (with applications to fluid dynamics), Korteweg-De Vries equation [12] (for models of waves on shallow water surfaces), Sine-Gordon equation [13] (with applications to the study of crystal dislocations), Landau-Lifshitz-Gilbert equation [14] (with application to the precessional motion of magnetization in a solid), Ishimori equation [15], Vlasov equation [16] (with applications in plasma), nonlinear Schrodinger equation [17] (with applications to optics and water waves), Lienard equation [18] (with applications to oscillating circuits), Solow equation [19] (with applications to the economy), and Cournot-Bertrand equation [20] (with applications to the economy), Matsumoto-Nonaka equation [21] (with applications to the economy), Kaleckian equation [22] (with applications to the economy), Dullin-Gottwald-Holm equation [23] (with applications to the propagation of surface waves in a shallow water regime), Lotka-Volterra equation [24] (with applications in biology and economics), and thermostatted kinetic equations (with applications to physics, biology, vehicular traffic, crowds and swarms dynamics, and social and economic systems); see papers [25–34] and the review [35].

The motions involved in nonlinear equations are not simply combinations of a bunch of simpler motions. Moreover the dynamics involving nonlinear (ordinary, partial, or

integro) differential equations are extremely different, and the related mathematical methods and analysis are problem dependent. Numerical simulations are also carried out for supporting the results.

The qualitative analysis of nonlinear ordinary differential equations is usually performed by searching conserved quantities (an approach that is typically used in Hamiltonian systems) and/or dissipative quantities. Linearization of the equations by Taylor expansion, change of variables, bifurcation theory, and perturbation methods is the most used approaches.

Nonlinear partial differential equations are qualitatively analyzed by using change the variables, separation of variables, and integral transforms. Other methods include the examination of the characteristics curves, and scale analysis (typically in fluid and heat mechanics) that allows, in some cases, for simplifying the nonlinear Navier-Stokes equations.

The nonlinear dynamics that appear in the above mentioned equations, and in the papers of this special issues, include highly sensitive to initial conditions (chaos dynamics, see; among others, papers [36–38]); anomalous transport (see paper [39] and references cited therein); alternating between two or more exclusive states (multistability; see [40, 41]); aperiodic oscillations (known as chaotic oscillations; see papers [42, 43] and the references section); amplitude death (complete cessation of oscillations; see [44, 45]); solitons (self-reinforcing solitary wave; see [46]); bifurcations (changes in the qualitative or topological property).

In this special issue, the tools of nonlinear dynamics have been used in attempts to better understand irregularity in diverse mathematical models of population dynamics, physics, biology, and economy. The interested reader is addressed to explore these interesting and fascinating results further. Moreover applications can refer to more research fields.

The guest editors of this special issue hope that problems discussed and investigated in the papers by the authors of this issue can inspire and motivate researchers in these fields to discover new, innovative, and novel applications in all areas of pure and applied mathematics.

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