

Research Article

Delay-Dependent Robust Exponential Stability and H_∞ Analysis for a Class of Uncertain Markovian Jumping System with Multiple Delays

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Received 26 November 2013; Accepted 23 January 2014; Published 17 March 2014

Academic Editor: Hao Shen

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This paper deals with the problem of robust exponential stability and H_∞ performance analysis for a class of uncertain Markovian jumping system with multiple delays. Based on the reciprocally convex approach, some novel delay-dependent stability criteria for the addressed system are derived. At last, numerical examples is given presented to show the effectiveness of the proposed results.

1. Introduction

It is well known that time delay is usually the main reason for instability and poor performance of many practical control systems [1–5]. The stability results for delayed systems can be generally classified into two categories: delay-independent stability criteria and delay-dependent criteria. And the delay-dependent results are often less conservative than the delay-independent ones, especially when the time delays are small. Therefore, much more attention has been focused on study of the delay-dependent stability conditions in recent years. For example, the system transformation method in [6], the descriptor system method in [7], parameter-dependent Lyapunov-Krasovskii functional method in [8], Jensen inequality method in [9], Free-weighting matrix method in [10, 11], integral inequality method in [12], augmented Lyapunov functional method in [13], convex domain method in [14], interval partition method in [15, 16], reciprocally convex method in [17], and so forth. And those approaches have been widely used in the stability analysis for lots of delayed systems in recent years [18–20].

On the other hand, since Markovian jumping systems can model many types of dynamic systems subject to abrupt changes in their structures, such as failure prone manufacturing systems, power systems, and economics systems

[21–27], a great deal of results related to stability analysis and synthesis for this class of systems with time delays has been reported in recent years. For example, for the delay-independent results, sufficient conditions for mean squares to stochastic stability were obtained in [28], while exponential stability conditions were proposed in [29]. The robust H_∞ filtering problem was dealt with in [30]. For the delay-dependent ones, the stability and H_∞ control results were presented by resorting to some bounding techniques for some cross terms and using model transformation to the original delay system in [31]. The H_∞ control and Filtering problem were taken into account in [32] using the Free-weighting matrix method. The stability and H_∞ analysis was proposed in [33] with the idea of delay partition. Filtering problem with a new index was considered in [34] using the reciprocally convex method. It is worth mentioned that inspite of the deep study for the delayed stochastic in recent years as mentioned above, there are few papers that consider the problem of stability analysis for uncertain stochastic systems with multiple delays, which motivates our study.

In this paper, the robust exponential stability and H_∞ performance analysis for a class of uncertain Markovian system with multiple time-varying delays is investigated. Some new delay-dependent stability conditions are derived.

And numerical simulation is given to demonstrate the effectiveness of the result.

Notation. Throughout this paper, for symmetric matrices X and Y , the notation $X \geq Y$ (resp., $X > Y$) means that the matrix $X - Y$ is positive semidefinite (resp., positive definite); I is the identity matrix with appropriate dimension; M^T represents the transpose of the matrix M ; $\mathcal{E}\{\cdot\}$ denotes the expectation operator with respect to some probability measure \mathcal{P} ; $L_2[0, \infty]$ is the space of square-integrable vector functions over $[0, \infty]$; $|\cdot|$ refers to the Euclidean vector norm; $\|\cdot\|_2$ stands for the usual $L_2[0, \infty]$ norm; $(\Omega, \mathcal{F}, \mathcal{P})$ is a probability space with Ω the sample space and \mathcal{F} is the σ -algebra of subsets of the sample space. Matrices, if not explicitly mentioned, are assumed to have compatible dimensions.

2. System Description and Preliminaries

Consider the following uncertain Markovian jumping system with multiple time-varying delays:

$$\begin{aligned} \dot{x}(t) &= A_0(r(t), t)x(t) \\ &+ \sum_{i=1}^m A_i(r(t), t)x(t - h_i(t)) \\ &+ D_1(r(t), t)w(t), \end{aligned} \quad (1)$$

$$z(t) = \sum_{i=0}^m C_i(r(t))x(t - h_i(t)) + D_2(r(t))w(t), \quad (2)$$

$$x(t) = \phi(t), \quad t \in [-h, 0], \quad (3)$$

where $x(t) \in R^n$ is the state; $w(t) \in R^p$ is the noise disturbance which is assumed to be an arbitrary signal in $L^2([0, \infty])$; $z(t) \in R^q$ is the signal to be estimated; $r(t)$ is a homogenous stationary Markov chain defined on a complete probability space $\{\Omega, \mathcal{F}, \mathcal{P}\}$ and taking values in a finite set $S = \{1, 2, \dots, N\}$ with generator $\Pi = (\lambda_{m,n})$ ($m, n \in S$) given by

$$\begin{aligned} P\{r(t + \Delta) = j \mid r(t) = k\} \\ = \begin{cases} \lambda_{k,j}\Delta + o(\Delta) & \text{if } k \neq j, \\ 1 + \lambda_{k,k}\Delta + o(\Delta) & \text{if } k = j, \end{cases} \end{aligned} \quad (4)$$

where $\Delta > 0$ and $\lim_{\Delta \rightarrow 0} o(\Delta)/\Delta = 0$, $\lambda_{k,j} \geq 0$ is the transition rate from k to j if $k \neq j$ and $\lambda_{k,k} = -\sum_{k \neq j} \lambda_{k,j}$. The scalar $h_i(t)$ is the time-varying delay with $0 \leq h_{1i} \leq h_i(t) \leq h_{2i}$, $\dot{h}_i(t) \leq \mu$, $i = 1, 2, \dots, m$, for any $t > 0$, where h_{1i}, h_{2i} , and μ are positive scalar constants; $\phi(t)$ is the initial function defined in $t \in [-h, 0]$ with $h = \max\{h_{21}, h_{22}, \dots, h_{2m}\}$; $A_i(r(t))$, $i = 0, 1, \dots, m$, and $D_1(r(t))$ are matrix functions with time-varying uncertainties described as $A_i(r(t), t) = A_i(r(t)) + \Delta A_i(r(t), t)$, $D_1(r(t), t) = D_1(r(t)) + \Delta D_1(r(t), t)$, where $A_i(r(t))$, $D_1(r(t))$ are known constant matrices, while

uncertainties $\Delta A_i(r(t), t)$, $\Delta D_1(r(t), t)$ are assumed to be norm bounded as

$$\begin{aligned} [\Delta A_i(r(t), t) \quad \Delta D_1(r(t), t)] \\ = E(r(t))F(r(t), t)[H_i(r(t)) \quad H_d(r(t))], \quad (5) \\ i = 0, 1, \dots, m, \end{aligned}$$

where $E(r(t))$, $H_i(r(t))$, $H_d(r(t))$, and $C_i(r(t))$, $D_2(r(t))$ in (2) are known constant matrices with appropriate dimensions. The unknown matrix functions $F(r(t))$ are having Lebesgue-measurable elements and satisfying

$$F(r(t)) \leq I, \quad \forall t > 0. \quad (6)$$

Remark 1. When $m = 1$, the system with multiple time-varying delays (1)–(3) is actually deduced to the uncertain Markovian jumping system with interval delay, which have been deeply studied in recent years. That is, the obtained results of multiple delayed systems can be directly deduced to the interval delayed systems.

Throughout this paper, we will use the following Definitions and Lemmas.

Definition 2. The uncertain Markovian jumping system with multiple time-varying delays (1)–(3) is said to be robustly exponentially stable in mean square for all admissible uncertainties, if there exist scalars $\alpha_1 > 0$ and $\alpha_2 > 0$ such that for all $t \geq 0$,

$$\|x(t, x_0, t_0)\|^2 \leq \alpha_1 e^{-\alpha_2 t} \sup_{-2h \leq s \leq 0} \{\|\phi(s)\|^2\}, \quad (7)$$

where $x(t, x_0, t_0)$ is the trivial solution of systems (1)–(3) with $w(t) = 0$.

Definition 3. Given a scalar $\gamma > 0$, uncertain Markovian jumping system with multiple time-varying delays (1)–(3) is said to be robustly exponentially stable with a prescribed H_∞ performance level γ if it is robustly exponentially stable, and under the zero initial condition, satisfies

$$\|z\|_{E_2} < \gamma \|w\|_2, \quad (8)$$

for all admissible uncertainties and nonzero $w(t) \in L_2[0, \infty)$, where

$$\|z\|_{E_2} = \left(\int_0^\infty |z(t)|^2 dt \right)^{1/2}. \quad (9)$$

Lemma 4 (see [9]). For any constant matrix $M \in R^{m \times m}$, $M = M^T > 0$, scalar $\gamma > 0$, vector function $\omega : [0, \gamma] \rightarrow R^m$ such that the integrations in the following are well defined; then

$$\gamma \int_0^\gamma \omega(\beta)^T M \omega(\beta) d\beta \geq \left(\int_0^\gamma \omega(\beta) d\beta \right)^T M \int_0^\gamma \omega(\beta) d\beta. \quad (10)$$

Lemma 5 (see [35]). Let A, D, E be real constant matrices with appropriate dimensions; matrix $F(t)$ satisfies $F^T(t)F(t) \leq I$. For any $\varepsilon > 0$, such that $P^{-1} - \varepsilon DD^T > 0$,

$$DF(t)E + E^T F^T(t)D^T \leq \varepsilon^{-1} DD^T + \varepsilon E^T E. \quad (11)$$

Lemma 6 (see [36]). Consider system (1) with $0 \leq h_{1i} \leq h_i(t) \leq h_{2i}$, $i = 1, 2, \dots, m$, for any matrices $Z_i \in \mathbb{R}^{n \times n}$ and $U_i \in \mathbb{R}^{n \times n}$ satisfying $\begin{bmatrix} Z_i & U_i \\ * & Z_i \end{bmatrix} \geq 0$; the following inequality holds

$$-\widehat{d}_i \int_{t-h_{2i}}^{t-h_{1i}} \dot{x}(s)^T Z_i \dot{x}(s) ds \leq \xi_i(t)^T \Omega_i \xi_i(t), \quad (12)$$

where $\widehat{d}_i = h_{2i} - h_{1i}$, and

$$\xi_i(t) = \begin{bmatrix} x(t-h_{1i})^T & x(t-h_i(t))^T & x(t-h_{2i})^T \end{bmatrix}^T, \quad (13)$$

$$\Omega_i = \begin{bmatrix} -Z_i & Z_i - U_i & U_i \\ * & -2Z_i + U_i + U_i^T & Z_i - U_i \\ * & * & -Z_i \end{bmatrix}.$$

3. Main Results

For simplicity, we define

$$\begin{aligned} \chi(t) &= \begin{bmatrix} x(t)^T & x(t-h_{11})^T & x(t-h_1(t))^T & x(t-h_{21})^T & \dots & x(t-h_{1m})^T & x(t-h_m(t))^T & x(t-h_{2m})^T & \dot{x}(t)^T \end{bmatrix}^T, \\ e_1 &= [1 \ 0 \ \dots \ 0]_{1 \times 3m+2}^T, \\ e_2 &= [0 \ 1 \ \dots \ 0]_{1 \times 3m+2}^T, \\ &\vdots \\ e_{3m+2} &= [0 \ 0 \ \dots \ 1]_{1 \times 3m+2}^T, \\ \beta_1 &= [1 \ 0 \ \dots \ 0]_{1 \times 3m+3}^T, \\ \beta_2 &= [0 \ 1 \ \dots \ 0]_{1 \times 3m+3}^T, \\ &\vdots \\ \beta_{3m+3} &= [0 \ 0 \ \dots \ 1]_{1 \times 3m+3}^T, \\ \chi_1(t) &= [\chi(t)^T \ w(t)^T]^T. \end{aligned} \quad (14)$$

3.1. Robust Exponential Stability Analysis. The criteria of the robust exponential stability for the systems (1)–(3) are proposed in the following Theorem.

Theorem 7. Systems (1)–(3) with $w(t) = 0$ is robustly exponentially stable if there exist positive matrices $P_k = P_k^T > 0$, $Q_i = Q_i^T > 0$, $R_{1i} = R_{1i}^T > 0$, $R_{2i} = R_{2i}^T > 0$, $S_i = S_i^T > 0$, $Z_i = Z_i^T > 0$, any matrices U_i , M_j with appropriate dimensions satisfying $\begin{bmatrix} Z_i & U_i \\ * & Z_i \end{bmatrix} \geq 0$, $i = 1, 2, \dots, m$; $j = 1, 2, k = 1, 2, \dots, N$, and positive scalars $\varepsilon > 0$, such that the following LMI holds

$$\begin{bmatrix} \Theta_{11k} & \Theta_{12k} \\ * & -\varepsilon I \end{bmatrix} < 0, \quad k = 1, 2, \dots, N, \quad (15)$$

where

$$\begin{aligned} \Theta_{11k} &= e_1 P_k e_{3m+2}^T + e_{3m+2} P_k e_1^T \\ &+ \left[\sum_{j=1}^N e_1 \lambda_{k,j} P_j e_1^T \right] + \left[e_1 \sum_{i=1}^m Q_i e_1^T \right] \end{aligned}$$

$$\begin{aligned} &- \left[\sum_{i=1}^m (1 - \mu) e_{3i} Q_i e_{3i}^T \right] \\ &+ \left[e_1 \sum_{i=1}^m (R_{1i} + R_{2i}) e_1^T \right] - \left[\sum_{i=1}^m e_{3i-1} R_{1i} e_{3i-1}^T \right] \\ &- \left[\sum_{i=1}^m e_{3i+1} R_{2i} e_{3i+1}^T \right] \\ &+ \left[\sum_{i=1}^m h_{1i}^2 e_{3m+2} S_i e_{3m+2}^T \right] + \left[\sum_{i=1}^m \widehat{d}_i^2 e_{3m+2} Z_i e_{3m+2}^T \right] \\ &- \left[\sum_{i=1}^m (e_1 - e_{3i-1}) S_i (e_1 - e_{3i-1})^T \right] \\ &+ \left(\sum_{i=1}^m \text{diag} (0, \Omega_i, 0) \right) \\ &+ [e_1 M_1 + e_{3m+2} M_2] \end{aligned}$$

$$\begin{aligned}
& \cdot [A_0(k) e_1^T + A_1(k) e_3^T + \cdots + A_m(k) e_{3m}^T - e_{3m+2}^T] \\
& + [e_1 A_0^T(k) + e_3 A_1^T(k) + \cdots + e_{3m} A_m^T(k) - e_{3m+2}^T] \\
& \times [M_1^T e_1^T + M_2^T e_{3m+2}^T] \\
& + \varepsilon [H_0(k) e_1^T + H_1(k) e_3^T + \cdots + H_m(k) e_{3m}^T]^T \\
& \times [H_0(k) e_1^T + H_1(k) e_3^T + \cdots + H_m(k) e_{3m}^T], \\
\Theta_{12k} & = [e_1 M_1 + e_{3m+2} M_2] E(k).
\end{aligned} \tag{16}$$

And Ω_i, \hat{d}_i are defined in (3).

Proof. On one hand, using Lemma 5 and Schur complement lemma to (15), we have

$$\begin{aligned}
\Pi & = e_1 P_k e_{3m+2}^T + e_{3m+2} P_k e_1^T \\
& + \left[\sum_{j=1}^N e_1 \lambda_{k,j} P_j e_1^T \right] + \left[e_1 \sum_{i=1}^m Q_i e_1^T \right] \\
& - \left[\sum_{i=1}^m (1 - \mu) e_{3i} Q_i e_{3i}^T \right] \\
& + \left[e_1 \sum_{i=1}^m (R_{1i} + R_{2i}) e_1^T \right] - \left[\sum_{i=1}^m e_{3i-1} R_{1i} e_{3i-1}^T \right] \\
& - \left[\sum_{i=1}^m e_{3i+1} R_{2i} e_{3i+1}^T \right] \\
& + \left[\sum_{i=1}^m h_{1i}^2 e_{3m+2} S_i e_{3m+2}^T \right] + \left[\sum_{i=1}^m \hat{d}_i^2 e_{3m+2} Z_i e_{3m+2}^T \right] \\
& - \left[\sum_{i=1}^m (e_1 - e_{3i-1}) S_i (e_1 - e_{3i-1})^T \right] \\
& + \left(\sum_{i=1}^m \text{diag}(0, \Omega_i, 0) \right) + [e_1 M_1 + e_{3m+2} M_2] \\
& \times [A_0(k, t) e_1^T + A_1(k, t) e_3^T + \cdots \\
& \quad + A_m(k, t) e_{3m}^T - e_{3m+2}^T] \\
& + [A_0(k) e_1^T + A_1(k) e_3^T + \cdots + A_m(k) e_{3m}^T - e_{3m+2}^T]^T \\
& \times [e_1 M_1 + e_{3m+2} M_2]^T \\
& \leq e_1 P e_{3m+2}^T + e_{3m+2} P e_1^T \\
& + \left[\sum_{j=1}^N e_1 P_k e_1^T \right] + \left[e_1 \sum_{i=1}^m Q_i e_1^T \right] \\
& - \left[\sum_{i=1}^m (1 - \mu) e_{3i} Q_i e_{3i}^T \right]
\end{aligned}$$

$$\begin{aligned}
& + \left[e_1 \sum_{i=1}^m (R_{1i} + R_{2i}) e_1^T \right] - \left[\sum_{i=1}^m e_{3i-1} R_{1i} e_{3i-1}^T \right] \\
& - \left[\sum_{i=1}^m e_{3i+1} R_{2i} e_{3i+1}^T \right] \\
& + \left[\sum_{i=1}^m h_{1i}^2 e_{3m+2} S_i e_{3m+2}^T \right] + \left[\sum_{i=1}^m \hat{d}_i^2 e_{3m+2} Z_i e_{3m+2}^T \right] \\
& - \left[\sum_{i=1}^m (e_1 - e_{3i-1}) S_i (e_1 - e_{3i-1})^T \right] \\
& + \left(\sum_{i=1}^m \text{diag}(0, \Omega_i, 0) \right) + [e_1 M_1 + e_{3m+2} M_2] \\
& \cdot [A_0(k) e_1^T + A_1(k) e_3^T + \cdots + A_m(k) e_{3m}^T - e_{3m+2}^T] \\
& + [e_1 A_0^T(k) + e_3 A_1^T(k) + \cdots + e_{3m} A_m^T(k) - e_{3m+2}^T] \\
& \times [M_1^T e_1^T + M_2^T e_{3m+2}^T] \\
& + \varepsilon [H_0(k) e_1^T + H_1(k) e_3^T + \cdots + H_m(k) e_{3m}^T]^T \\
& \times [H_0(k) e_1^T + H_1(k) e_3^T + \cdots + H_m(k) e_{3m}^T] \\
& + \varepsilon^{-1} [e_1 M_1 + e_{3m+2} M_2] E(k) E(k)^T \\
& \times [e_1 M_1 + e_{3m+2} M_2]^T \\
& = \Theta_{11} + \varepsilon^{-1} \Theta_{12}^T \Theta_{12} < 0.
\end{aligned} \tag{17}$$

On the other hand, define a new process $x_t(s) = x(t+s)$, $s \in [-2h, 0]$. Choose a Lyapunov-Krasovskii functional

$$V(x_t, t, r(t)) = \sum_{i=1}^5 V_i(x_t, t, r(t)), \tag{18}$$

where

$$\begin{aligned}
V_1(x_t, t, r(t)) & = x(t)^T P_{r(t)} x(t), \\
V_2(x_t, t, r(t)) & = \sum_{i=1}^m \int_{t-h_i(t)}^t x(s)^T Q_i x(s) ds, \\
V_3(x_t, t, r(t)) & = \sum_{i=1}^m \int_{t-h_{1i}}^t x(s)^T R_{1i} x(s) ds \\
& \quad + \sum_{i=1}^m \int_{t-h_{2i}}^t x(s)^T R_{2i} x(s) ds, \\
V_4(x_t, t, r(t)) & = \sum_{i=1}^m \int_{-h_{1i}}^0 \int_{t+s}^t h_{1i} \dot{x}(\alpha)^T S_i \dot{x}(\alpha) d\alpha ds, \\
V_5(x_t, t, r(t)) & = \sum_{i=1}^m \int_{-h_{2i}}^{-h_{1i}} \int_{t+s}^t \hat{d}_i \dot{x}(\alpha)^T Z_i \dot{x}(\alpha) d\alpha ds.
\end{aligned} \tag{19}$$

Let \mathcal{L} be the weak infinitesimal generator of the random process $\{x_t, t \geq 0\}$. Then for each $r(t) = k, k \in S$, we have

$$\begin{aligned} \mathcal{L}V_1(x_t, t, k) &= 2x(t)P_k\dot{x}(t) + \sum_{j=1}^N x(t)^T \lambda_{k,j} P_j x(t) \\ &= 2\chi(t)^T [e_1 P_k e_{3m+2}^T] \chi(t) \\ &\quad + \chi(t)^T \left[\sum_{j=1}^N e_1 \lambda_{k,j} P_j e_1^T \right] \chi(t), \end{aligned}$$

$$\begin{aligned} \mathcal{L}V_2(x_t, t, k) &\leq x(t)^T \sum_{i=1}^m Q_i x(t) \\ &\quad - (1-\mu) \sum_{i=1}^m x(t-h_i(t))^T Q_i x(t-h_i(t)) \\ &= \chi(t)^T \left[e_1 \sum_{i=1}^m Q_i e_1^T \right] \chi(t) \\ &\quad - \chi(t)^T \left[\sum_{i=1}^m (1-\mu) e_{3i} Q_i e_{3i}^T \right] \chi(t), \end{aligned}$$

$$\begin{aligned} \mathcal{L}V_3(x_t, t, k) &\leq x(t)^T \left[\sum_{i=1}^m (R_{1i} + R_{2i}) \right] x(t) \\ &\quad - \sum_{i=1}^m [x(t-h_{1i})^T R_{1i} x(t-h_{1i}) \\ &\quad \quad - x(t-h_{2i})^T R_{2i} x(t-h_{2i})] \\ &= \chi(t)^T \left[e_1 \sum_{i=1}^m (R_{1i} + R_{2i}) e_1^T \right] \chi(t) \\ &\quad - \chi(t)^T \left[\sum_{i=1}^m e_{3i-1} R_{1i} e_{3i-1}^T \right] \chi(t) \\ &\quad - \chi(t)^T \left[\sum_{i=1}^m e_{3i+1} R_{2i} e_{3i+1}^T \right] \chi(t), \end{aligned}$$

$$\begin{aligned} \mathcal{L}V_4(x_t, t, k) &= \sum_{i=1}^m h_{1i}^2 \dot{x}(t)^T S_i \dot{x}(t) \\ &\quad - \sum_{i=1}^m \int_{t-h_{1i}}^t h_{1i} \dot{x}(s)^T S_i \dot{x}(s) ds \\ &= \chi(t)^T \left[\sum_{i=1}^m h_{1i}^2 e_{3m+2} S_i e_{3m+2}^T \right] \chi(t) \end{aligned}$$

$$- \sum_{i=1}^m \int_{t-h_{1i}}^t h_{1i} \dot{x}(s)^T S_i \dot{x}(s) ds,$$

$$\begin{aligned} \mathcal{L}V_5(x_t, t, k) &= \sum_{i=1}^m \tilde{d}_i^2 \dot{x}(t)^T Z_i \dot{x}(t) \\ &\quad - \sum_{i=1}^m \int_{t-h_{2i}}^{t-h_{1i}} \hat{d}_i \dot{x}(s)^T Z_i \dot{x}(s) ds \\ &= \chi(t)^T \left[\sum_{i=1}^m \tilde{d}_i^2 e_{3m+2} Z_i e_{3m+2}^T \right] \chi(t) \\ &\quad - \sum_{i=1}^m \int_{t-h_{2i}}^{t-h_{1i}} \hat{d}_i \dot{x}(s)^T Z_i \dot{x}(s) ds. \end{aligned} \tag{20}$$

Applying Lemma 4 to $\mathcal{L}V_4(x_t)$ results in

$$\begin{aligned} &- \sum_{i=1}^m \int_{t-h_{1i}}^t h_{1i} \dot{x}(s)^T S_i \dot{x}(s) ds \\ &\leq - \sum_{i=1}^m \left(\int_{t-h_{1i}}^t \dot{x}(s) ds \right)^T S_i \left(\int_{t-h_{1i}}^t \dot{x}(s) ds \right) \\ &= - \sum_{i=1}^m [x(t) - x(t-h_{1i})]^T S_i [x(t) - x(t-h_{1i})] \\ &\leq -\chi(t)^T \sum_{i=1}^m [(e_1 - e_{3i-1}) S_i (e_1 - e_{3i-1})^T] \chi(t), \end{aligned} \tag{21}$$

and applying Lemma 6 to $\mathcal{L}V_5(x_t, t, k)$, we have that there exists U_i with $\begin{bmatrix} Z_i & U_i \\ * & Z_i \end{bmatrix} \geq 0, i = 1, 2, \dots, m$, such that

$$\begin{aligned} &- \sum_{i=1}^m \int_{t-h_{2i}}^{t-h_{1i}} \hat{d}_i f(s)^T Z_i f(s) ds \\ &\leq \sum_{i=1}^m \xi_i(t)^T \Omega_i \xi_i(t) \\ &= \chi(t)^T \left(\sum_{i=1}^m \text{diag}(0, \Omega_i, 0) \right) \chi(t), \end{aligned} \tag{22}$$

where $\xi_i(t)$ and Ω_i are defined in (13). Meanwhile, we note that

$$\begin{aligned} &2[x(t)^T M_1 + \dot{x}(t)^T M_2] \\ &\quad \times \left[A_0(k, t) x(t) + \sum_{i=1}^m A_i(k, t) x(t-h_i(t)) - \dot{x}(t) \right] = 0, \end{aligned} \tag{23}$$

that is,

$$\begin{aligned} & 2\chi(t)^T [e_1 M_1 + e_{3m+2} M_2] \\ & \times [A_0(k, t) e_1^T + A_1(k, t) e_3^T + \cdots + A_m(k, t) e_{3m}^T - e_{3m+2}^T] \\ & \times \chi(t) = 0. \end{aligned} \quad (24)$$

Then, we can deduce from (19)–(24) that

$$\mathcal{L}V(x_t, t, k) \leq \chi(t)^T \Pi \chi(t) < 0, \quad (25)$$

where Π is defined in (17). Therefore, by Definition 2 and the results in [37], we have that the system (1) is robustly stable. Now, we will prove the robust stochastic exponential stability in mean square for system (1). Setting $\lambda_0 = \lambda_{\min}\{-\Pi\} > 0$, we have

$$\mathcal{L}V(x_t, t, k) \leq \chi(t)^T \Pi \chi(t) \leq -\lambda_0 \|x(t)\|^2. \quad (26)$$

Choose $\bar{V}(x_t, t, k) = e^{2\alpha t} V(x_t, t, k)$, where $\alpha > 0$; then

$$\begin{aligned} \mathcal{L}\bar{V}(x_t, t, k) &= 2ke^{2\alpha t} V(x_t, t, k) + e^{2\alpha t} \mathcal{L}V(x_t, t, k) \\ &\leq 2ke^{2\alpha t} V(x_t, t, k) - \lambda_0 e^{2\alpha t} \|x(t)\|^2. \end{aligned} \quad (27)$$

Integrating the above inequality (27), we get

$$\begin{aligned} & \bar{V}(x_t, t, k) \\ & \leq V(x_0, 0, k) \\ & + \int_0^t \{2ke^{2\alpha s} V(x_s, s, k) - \lambda_0 e^{2\alpha s} \|x(s)\|^2\} ds. \end{aligned} \quad (28)$$

From (19), it can be inferred that

$$\begin{aligned} V(x_s, s, k) &\leq \lambda_{\max}(P_k) \|x(s)\|^2 \\ &+ \left[\sum_{i=1}^m (\lambda_{\max}(Q_i) + \lambda_{\max}(R_{1i}) + \lambda_{\max}(R_{2i})) \right] \\ &\times \int_{s-h}^s \|x(v)\|^2 dv \\ &+ \left[\sum_{i=1}^m h_{1i} \lambda_{\max}(S_i) + d_i \lambda_{\max}(Z_i) \right] \int_{s-h}^s \dot{x}(v)^T \dot{x}(v) dv. \end{aligned} \quad (29)$$

Note that

$$\begin{aligned} & \dot{x}(v)^T \dot{x}(v) \\ & \leq m [\lambda_{\max}(A_0^T(k, v) A_0(k, v)) \|x(v)\|^2 \\ & + \lambda_{\max}(A_1^T(k, v) A_1(v)) \|x(v - h_1(v))\|^2 \\ & + \lambda_{\max}(A_m^T(k, v) A_m(v)) \|x(v - h_m(v))\|^2]. \end{aligned} \quad (30)$$

We denote $\varrho = m[\lambda_{\max}(A_0^T(k, v) A_0(k, v)) + \lambda_{\max}(A_1^T(k, v) A_1(k, v)) + \lambda_{\max}(A_m^T(k, v) A_m(k, v))]$; then

$$\int_{s-h}^s \dot{x}(v)^T \dot{x}(v) dv \leq \varrho \int_{s-2h}^s \|x(v)\|^2 dv. \quad (31)$$

From (29) to (31), we obtain

$$V(x_s, s, k) \leq \Xi_0 \|x(s)\|^2 + \Xi_1 \int_{s-2h}^s \|x(v)\|^2 dv, \quad (32)$$

where

$$\begin{aligned} \Xi_0 &= \lambda_{\max}(P_k), \\ \Xi_1 &= \left[\sum_{i=1}^m (\lambda_{\max}(Q_i) + \lambda_{\max}(R_{1i}) + \lambda_{\max}(R_{2i})) \right] \\ &+ \varrho \left[\sum_{i=1}^m h_{1i} \lambda_{\max}(S_i) + d_i \lambda_{\max}(Z_i) \right]. \end{aligned} \quad (33)$$

By the similar method, we have

$$V(x_0, 0, k) \leq \theta \sup_{-2h \leq s \leq 0} \{\|\phi(s)\|^2\}, \quad (34)$$

where $\theta = 2h\Xi_1$. Therefore, by (28)–(34), we get

$$\begin{aligned} & \bar{V}(x_t, t, k) \\ & \leq \theta \sup_{-2h \leq s \leq 0} \{\|\phi(s)\|^2\} \\ & + \int_0^t \left\{ 2\alpha e^{2\alpha s} \left[\Xi_0 \|x(s)\|^2 + \Xi_1 \int_{s-2h}^s \|x(v)\|^2 dv \right] \right. \\ & \quad \left. - \lambda_0 e^{2\alpha s} \|x(s)\|^2 \right\} ds \\ & \leq \theta \sup_{-2h \leq s \leq 0} \{\|\phi(s)\|^2\} \\ & + (2\alpha\Xi_0 - \lambda_0) \int_0^t e^{2\alpha s} \|x(s)\|^2 ds \\ & + e^{2\alpha t} \Xi_1 \int_{-2h}^t \|x(v)\|^2 dv \\ & \leq (\theta + 2he^{2\alpha t} \Xi_1) \sup_{-2h \leq s \leq 0} \{\|\phi(s)\|^2\} \\ & + (e^{2\alpha t} \Xi_1 + 2\alpha\Xi_0 - \lambda_0) \int_0^t \|x(v)\|^2 dv. \end{aligned} \quad (35)$$

Choose $\alpha_0 > 0$ such that

$$e^{2\alpha_0 t} \Xi_1 + 2\alpha_0 \Xi_0 - \lambda_0 \leq 0; \quad (36)$$

then

$$\bar{V}(x_t, t, k) \leq (\theta + 2he^{2\alpha_0 t} \Xi_1) \sup_{-2h \leq s \leq 0} \{\|\phi(s)\|^2\}. \quad (37)$$

Since $\bar{V}(x_t, t, k) \geq e^{2\alpha_0 t} \lambda_{\min}(P_k) \|x(t)\|^2$, it can be shown from (37) that

$$\|x(t)\|^2 \leq \delta e^{-2\alpha_0 t} \sup_{-2h \leq s \leq 0} \{\|\phi(s)\|^2\}, \quad (38)$$

where

$$\delta = \frac{\theta + 2he^{2\alpha_0 t} \Xi_1}{\lambda_{\min}(P_k)}, \quad (39)$$

which implies that system (10) is robustly exponentially stable by Definition 2. This completes the proof. \square

3.2. Robust H_∞ Exponential Stability Analysis. The criteria of the robust exponential stability with H_∞ performance for the systems (1)–(3) are proposed in the following Theorem.

Theorem 8. *Given a scalar $\gamma > 0$, the systems (1)–(3) are robustly exponentially stable with a prescribed H_∞ performance level γ if there exist matrices $P_k = P_k^T > 0$, $Q_i = Q_i^T > 0$, $R_{1i} = R_{1i}^T > 0$, $R_{2i} = R_{2i}^T > 0$, $S_i = S_i^T > 0$, $Z_i = Z_i^T > 0$, any matrices U_i, M_j with appropriate dimensions satisfying $\begin{bmatrix} Z_i & U_i \\ * & Z_i \end{bmatrix} \geq 0$, $i = 1, 2, \dots, m$; $j = 1, 2$, $k = 1, 2, \dots, N$, and positive scalars $\varepsilon > 0$, such that the following LMI holds*

$$\begin{bmatrix} \Omega_{11k} & \Omega_{12k} & \Omega_{13k} \\ * & -\varepsilon I & 0 \\ * & * & -I \end{bmatrix} < 0, \quad (40)$$

where

$$\begin{aligned} \Omega_{11k} &= \beta_1 P_k \beta_{3m+2}^T + \beta_{3m+2} P_k \beta_1^T \\ &+ \left[\sum_{j=1}^N \beta_1 \lambda_{k,j} P_j \beta_1^T \right] + \left[\beta_1 \sum_{i=1}^m Q_i \beta_1^T \right] \\ &- \left[\sum_{i=1}^m (1 - \mu) \beta_{3i} Q_i \beta_{3i}^T \right] \\ &+ \left[\beta_1 \sum_{i=1}^m (R_{1i} + R_{2i}) \beta_1^T \right] - \left[\sum_{i=1}^m \beta_{3i-1} R_{1i} \beta_{3i-1}^T \right] \\ &- \left[\sum_{i=1}^m \beta_{3i+1} R_{2i} \beta_{3i+1}^T \right] \\ &+ \left[\sum_{i=1}^m h_{1i}^2 \beta_{3m+2} S_i \beta_{3m+2}^T \right] + \left[\sum_{i=1}^m \tilde{a}_i^2 \beta_{3m+2} Z_i \beta_{3m+2}^T \right] \\ &- \left[\sum_{i=1}^m (\beta_1 - \beta_{3i-1}) S_i (\beta_1 - \beta_{3i-1})^T \right] \\ &+ \left(\sum_{i=1}^m \text{diag}(0, \Omega_i, 0) \right) + [\beta_1 M_1 + \beta_{3m+2} M_2] \end{aligned}$$

$$\begin{aligned} &\cdot [A_0(k) \beta_1^T + A_1(k) \beta_3^T + \dots + A_m(k) \beta_{3m}^T \\ &\quad - \beta_{3m+2}^T + D_1(k) \beta_{3m+3}^T] \\ &+ [\beta_1 A_0^T(k) + \beta_3 A_1^T(k) + \dots + \beta_{3m} A_m^T(k) \\ &\quad - \beta_{3m+2} + \beta_{3m+3} D_1(k)^T] \\ &\cdot [M_1^T \beta_1^T + M_2^T \beta_{3m+2}^T] \\ &+ \varepsilon [H_0(k) \beta_1^T + H_1(k) \beta_3^T + \dots + H_m(k) \beta_{3m}^T]^T \\ &\times [H_0(k) \beta_1^T + H_1(k) \beta_3^T + \dots + H_m(k) \beta_{3m}^T] \\ &- \gamma^2 \beta_{3m+3} \beta_{3m+3}^T. \end{aligned}$$

$$\Omega_{12k} = [\beta_1 M_1 + \beta_{3m+2} M_2] E(k),$$

$$\Omega_{13k}$$

$$= [C_0(k) \beta_1^T + C_1(k) \beta_3^T + \dots + C_m(k) \beta_{3m}^T + D_2(k) \beta_{3m+3}^T]^T. \quad (41)$$

Proof. Implying Lemma 5 and Schur complement lemma to (40), we obtain

$$\begin{aligned} \Pi' &= \beta_1 P_k \beta_{3m+2}^T + \beta_{3m+2} P_k \beta_1^T \\ &+ \left[\sum_{j=1}^N \beta_1 \lambda_{k,j} P_j \beta_1^T \right] + \left[\beta_1 \sum_{i=1}^m Q_i \beta_1^T \right] \\ &- \left[\sum_{i=1}^m (1 - \mu) \beta_{3i} Q_i \beta_{3i}^T \right] \\ &+ \left[\beta_1 \sum_{i=1}^m (R_{1i} + R_{2i}) \beta_1^T \right] - \left[\sum_{i=1}^m \beta_{3i-1} R_{1i} \beta_{3i-1}^T \right] \\ &- \left[\sum_{i=1}^m \beta_{3i+1} R_{2i} \beta_{3i+1}^T \right] + \left[\sum_{i=1}^m h_{1i}^2 \beta_{3m+2} S_i \beta_{3m+2}^T \right] \\ &+ \left[\sum_{i=1}^m \tilde{a}_i^2 \beta_{3m+2} Z_i \beta_{3m+2}^T \right] \\ &- \left[\sum_{i=1}^m (\beta_1 - \beta_{3i-1}) S_i (\beta_1 - \beta_{3i-1})^T \right] \\ &+ \left(\sum_{i=1}^m \text{diag}(0, \Omega_i, 0) \right) \\ &+ [\beta_1 M_1 + \beta_{3m+2} M_2] \\ &\times [A_0(k, t) \beta_1^T + A_1(k, t) \beta_3^T + \dots + A_m(k, t) \beta_{3m}^T \\ &\quad - \beta_{3m+2}^T + D_1(k, t) \beta_{3m+3}^T] \end{aligned}$$

$$\begin{aligned}
 &+ [A_0(k, t) \beta_1^T + A_1(k, t) \beta_3^T + \dots + A_m(k, t) \beta_{3m}^T \\
 &\quad - \beta_{3m+2}^T + D_1(k, t) \beta_{3m+3}^T]^T \\
 &\times [\beta_1 M_1 + \beta_{3m+2} M_2]^T \\
 &- \gamma^2 \beta_{3m+3} \beta_{3m+3}^T.
 \end{aligned} \tag{42}$$

Set

$$J(t) = \left\{ \int_0^t [z(s)^T z(s) - \gamma^2 w(s)^T w(s)] \right\} ds. \tag{43}$$

Then, it is easy to have

$$\begin{aligned}
 J(t) &= \left\{ \int_0^t [z(s)^T z(s) - \gamma^2 w(s)^T w(s)] + \ell V(x(s), s, k) \right\} ds \\
 &\quad - \{V(x(t), t)\} \\
 &\leq \left\{ \int_0^t [z(s)^T z(s) - \gamma^2 w(s)^T w(s)] + \ell V(x(s), s, k) \right\} ds,
 \end{aligned} \tag{44}$$

where $V(x(t), t, k)$ is defined in (18). Similar to the proof of Theorem 7, we can obtain

$$z(t)^T z(t) - \gamma^2 w(t)^T w(t) + \ell V(x(t), t) \leq \chi_1(t)^T \Pi' \chi_1(t), \tag{45}$$

where Π' is given in (42) and $\chi_1(t)$ is defined in (14). Then, it follows from (40) and (45) that

$$J(t) < 0. \tag{46}$$

This implies that for any nonzero $v(t) \in L_2[0, \infty]$,

$$\|z\|_{E_2} < \gamma \|w\|_2. \tag{47}$$

Therefore, by Definition 3, the system is robustly exponentially stable with a prescribed H_∞ performance level γ . This completes the proof. \square

4. Numerical Example

In this section, we provide an example to demonstrate the effectiveness of the proposed method.

Let $m = 2$ and $N = 2$; consider the systems (1)–(3) with parameters as follows.

Mode 1

$$\begin{aligned}
 A_0(1) &= \begin{bmatrix} -5 & 0 \\ 0.5 & -6 \end{bmatrix}, & A_1(1) &= \begin{bmatrix} -2 & 0 \\ 1 & -3 \end{bmatrix}, \\
 A_2(1) &= \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0.5 \end{bmatrix}, & D_1(1) &= \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0.1 \end{bmatrix}, \\
 C_0(1) &= \begin{bmatrix} 0.1 & 0.1 \\ 0.2 & 0.1 \end{bmatrix}, & C_1(1) &= \begin{bmatrix} -0.1 & 0 \\ 0.1 & 0.2 \end{bmatrix}, \\
 C_2(1) &= \begin{bmatrix} -0.1 & 0.1 \\ 0 & 0.3 \end{bmatrix}, & D_2(1) &= \begin{bmatrix} -0.1 & -0.3 \\ 0.1 & -0.2 \end{bmatrix}, \\
 E(1) &= \begin{bmatrix} 0.1 & 0.1 \\ 0.2 & 0.3 \end{bmatrix}, & H_0(1) &= \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0.1 \end{bmatrix}, \\
 H_1(1) &= \begin{bmatrix} -0.3 & 0.4 \\ 0.5 & -0.1 \end{bmatrix}, & H_2(1) &= \begin{bmatrix} 0.2 & 0.2 \\ 0.3 & 0.1 \end{bmatrix}, \\
 H_d(1) &= \begin{bmatrix} -0.1 & 0.4 \\ 0.3 & -0.1 \end{bmatrix}, & \lambda_{11} &= -0.5, \quad \lambda_{12} = 0.5.
 \end{aligned} \tag{48}$$

Mode 2

$$\begin{aligned}
 A_0(2) &= \begin{bmatrix} -2 & 0 \\ 1 & -4 \end{bmatrix}, & A_1(2) &= \begin{bmatrix} -1 & 1 \\ 1 & -4 \end{bmatrix}, \\
 A_2(2) &= \begin{bmatrix} 0.2 & -0.2 \\ 0 & -0.3 \end{bmatrix}, & D_1(2) &= \begin{bmatrix} -0.1 & 0.2 \\ -0.1 & 0.3 \end{bmatrix}, \\
 C_0(2) &= \begin{bmatrix} 0.2 & -0.1 \\ 0.1 & -0.1 \end{bmatrix}, & C_1(2) &= \begin{bmatrix} -0.2 & 0 \\ -0.1 & 0.1 \end{bmatrix}, \\
 C_2(2) &= \begin{bmatrix} 0.1 & 0.2 \\ 0 & -0.1 \end{bmatrix}, & D_2(2) &= \begin{bmatrix} 0.1 & 0.2 \\ -0.1 & -0.3 \end{bmatrix}, \\
 E(2) &= \begin{bmatrix} 0.1 & -0.1 \\ 0.1 & -0.3 \end{bmatrix}, & H_0(2) &= \begin{bmatrix} 0.2 & 0.1 \\ 0.2 & 0.3 \end{bmatrix}, \\
 H_1(2) &= \begin{bmatrix} -0.1 & 0.5 \\ 0.3 & -0.3 \end{bmatrix}, & H_2(2) &= \begin{bmatrix} 0.1 & 0.1 \\ 0.6 & 0.2 \end{bmatrix}, \\
 H_d(2) &= \begin{bmatrix} 0.2 & -0.3 \\ 0.2 & -0.1 \end{bmatrix}, & \lambda_{21} &= 0.3, \quad \lambda_{22} = -0.3.
 \end{aligned} \tag{49}$$

And $\gamma = 2$, $\mu = 0.5$, $h_{11} = 0.1$, $h_{21} = 0.4$, $h_{12} = 0.4$, $h_{22} = 0.5$. Then, by solving the LMI (15) with the constraints in Theorem 7, we obtain

$$\begin{aligned}
 P_1 &= \begin{bmatrix} 59.1376 & 8.2356 \\ 8.2356 & 26.8113 \end{bmatrix}, \\
 P_2 &= \begin{bmatrix} 229.5038 & 50.5016 \\ 50.5016 & 124.2305 \end{bmatrix}, \\
 Q_1 &= \begin{bmatrix} 37.1817 & -0.5668 \\ -0.5668 & 41.5837 \end{bmatrix}, \\
 Q_2 &= \begin{bmatrix} 28.1504 & 6.9399 \\ 6.9399 & 7.1618 \end{bmatrix},
 \end{aligned}$$

$$\begin{aligned}
 R_{11} &= \begin{bmatrix} 18.1195 & 0.9123 \\ 0.9123 & 14.7239 \end{bmatrix}, \\
 R_{21} &= \begin{bmatrix} 31.6879 & -9.1485 \\ -9.1485 & 54.6448 \end{bmatrix}, \\
 R_{12} &= \begin{bmatrix} 31.9976 & 6.7097 \\ 6.7097 & 10.5708 \end{bmatrix}, \\
 R_{22} &= \begin{bmatrix} 32.5781 & 6.8947 \\ 6.8947 & 10.6649 \end{bmatrix}, \\
 S_1 &= \begin{bmatrix} 35.1285 & -0.1159 \\ -0.1159 & 25.3955 \end{bmatrix}, \\
 S_2 &= \begin{bmatrix} 0.9787 & 0.2268 \\ 0.2268 & 0.0975 \end{bmatrix}, \\
 Z_1 &= \begin{bmatrix} 52.7205 & -8.8649 \\ -8.8649 & 70.5952 \end{bmatrix}, \\
 Z_2 &= \begin{bmatrix} 1.7431 & 0.4057 \\ 0.4057 & 0.1638 \end{bmatrix}, \\
 U_1 &= \begin{bmatrix} 1.7807 & 18.4064 \\ 18.8018 & -53.4269 \end{bmatrix}, \\
 U_2 &= \begin{bmatrix} -10.2878 & -1.9164 \\ -1.8946 & -5.3788 \end{bmatrix}, \\
 M_1 &= \begin{bmatrix} 42.4713 & 5.0586 \\ 8.4912 & 25.0911 \end{bmatrix}, \\
 M_2 &= \begin{bmatrix} 11.7147 & 1.1250 \\ 1.9353 & 3.9491 \end{bmatrix}, \\
 \varepsilon &= 12.8944.
 \end{aligned}
 \tag{50}$$

If we fix the lower bound of $h_1(t)$ and $h_2(t)$, that is, $h_{12} = 0.3$ and $h_{22} = 0.5$, for the different h_{11} , we can get the upper bounds of h_{21} as in Table 1.

If we fix the lower bound of $h_1(t)$ and $h_2(t)$, that is, $h_{11} = 0.2$ and $h_{21} = 0.6$, for the different h_{12} , we can get the upper bounds of h_{22} as in Table 2.

5. Conclusion

The robust exponential stability and H_∞ performance analysis for uncertain Markovian jumping system with multiple time-varying delays has been investigated based on the reciprocally convex approach. Some new delay-dependent stability conditions are obtained in term of LMIs. Numerical example has been proposed to illustrate the effectiveness of result.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

TABLE 1: The upper bound of h_{21} for different h_{11} .

h_{11}	0.2	0.5	0.7	0.9	1	1.2
The upper bound of h_{21}	0.505	0.617	0.797	0.993	1.093	1.292

TABLE 2: The upper bound of h_{22} for different h_{12} .

h_{12}	0.3	0.5	0.7	0.9	1	1.2
The upper bound of h_{22}	0.397	0.597	0.797	0.997	1.097	1.297

Acknowledgment

This work was supported National Natural Science Foundation of China under Grant nos. 61004046, 61104117, and 61304064.

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