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Research Article

Existence and Uniqueness of Almost Periodic Solutions for Neural Networks with Neutral Delays

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A class of neural networks system with neutral delays is investigated. The existence and uniqueness of almost periodic solution for the system are obtained by using fixed point theorem; we extend some results in the references.

1. Introduction

In recent years, neural networks have been deeply investigated due to their applicability in solving some image processing, signal processing, and pattern recognition problems. And neural networks have been applied in artificial intelligence and automatic control engineering because of their good abilities of information memory and information association ([1, 2]).

Cellular neural networks (for short CNN) have been introduced by Chua and Yang [3] in 1988. Usually, in the electronic implementations of analog neural networks, time delays will inevitably occur in the communication and response of neurons because of the unavoidable finite switching speed of amplifiers ([4–9]). Due to the complicated dynamic properties of the neural cells in the real world, some complicated dynamic properties have been described by delayed cellular neural networks (DCNNs) ([10–13]).

Bai [10] proposed a neural networks model which takes the following form:

$$\begin{aligned} x_i'(t) &= -c_i(t)\,x_i(t) + \sum_{j=1}^{n} a_{ij}(t)\, f_j\left(x_j\left(t-\tau_{ij}(t)\right)\right) \\ &\quad + \sum_{j=1}^{n} b_{ij}(t)\, g_j\left(x_j'\left(t-\sigma_{ij}(t)\right)\right) + I_i(t), \end{aligned} \tag{1}$$

where $i = 1, 2, \dots, n$, with initial condition

$$\begin{gathered} x_i(s) = \varphi_i(s), \quad s \in [-\tau, 0], \\ \varphi = (\varphi_1, \varphi_2, \dots, \varphi_n)^T \in C^1([-\tau, 0], R^n). \end{gathered} \tag{2}$$

By using fixed point theorem, Bai studied the global stability of almost periodic solutions for the above neural networks.

Since neural networks with neutral delays contain some very important information about the derivative of the past state, it is very important for us to study such complicated system. Some authors studied some more complicated neutral neural networks and several important results have been obtained in ([14–20]). For example, Pinto and Robledo [14] studied an impulsive neural network of n-units and distributed delays as follows:

$$\begin{aligned} x_i'(t) &= -a_i(t)\,x_i(t) \\ &\quad + \sum_{j=1}^{n} \left\{ b_{ij}(t)\, f_j\left(x_j(t)\right) + c_{ij}(t)\, g_j\left(\left(k_{ij} * x_j\right)(t)\right) \right\} \\ &\quad + \gamma_i(t), \quad t \neq t_k, \\ \Delta x_i(\tau_k) &= x_i(\tau_k^+) - x_i(\tau_k) \\ &= \sum_{j=1}^{n} A_{ij}(k)\, x_j(\tau_k) + I_j\left(x(\tau_k)\right) + \mu_i(k), \quad t = t_k, \end{aligned} \tag{3}$$

where $(k_{ij} * x_j)(t) = \int_0^\infty k_{ij}(r)x_j(t-r)dr$, $x_i(\tau_k^+) = \lim_{\delta\to 0^+} x_i(\tau_k + \delta)$, $i = 1,2,\ldots,n$, $k = 1,2,\ldots$. By using spectral radius theorem they obtained a result on the existence and stability of an almost periodic solution for the system (3).

Feng et al. [17] considered delayed neural network as follows:

$$\begin{aligned} u_i'(t) &= -c_i u_i(t) + \sum_{j=1}^n w_{ij} g_j\left(u_j(t)\right) + \sum_{j=1}^n a_{ij} g_j\left(u_j(t-\tau)\right) \\ &\quad + \sum_{j=1}^n b_{ij} \int_{-\infty}^t k_j(t-s)\, g_j\left(u_j(s)\right) ds \\ &\quad + \sum_{j=1}^n d_{ij} u_j'(t-\tau) + \varrho_j, \\ u_i(t) &= \phi_i(t), \quad -\infty < t \le 0,\ i = 1,2,\ldots,n, \end{aligned} \tag{4}$$

and obtained the system (4) having a unique equilibrium point, which is globally asymptotically stable.

Wang and Zhu [19] were concerned with the following generalized neutral-type neural networks with delays:

$$\begin{aligned} \left(A_i x_i\right)'(t) &= -a_i(t)\, x_i(t) \\ &\quad + \sum_{j=1}^n \Big[b_{ij}(t)\, f_j\left(x_j(t)\right) \\ &\qquad + d_{ij}(t)\, g_j\left(x_j\left(t-\tau_{ij}(t)\right)\right)\Big] + I_i(t), \\ x_i(t) &= \phi_i(t), \quad t \in [-r, 0],\ i = 1,2,\ldots,n, \end{aligned} \tag{5}$$

where A_i is a difference defined by $(A_i x_i)(t) = x_i(t) - \sum_{j=1}^n c_{ij}(t) x_i(t - \delta_{ij}(t))$. By using fixed point theorem, Lyapunov function method, and comparison theorem, the authors studied the existence, global asymptotic stability, and exponential stability of almost periodic solution for the system (5).

Motivated by the above papers, in this paper, we consider the neural networks with neutral delays

$$\begin{aligned} x_i'(t) &= -a_i(t)\, x_i(t) + \sum_{j=1}^n b_{ij}(t)\, f_j\left(x_j(t)\right) \\ &\quad + \sum_{j=1}^n c_{ij}(t)\, f_j\left(x_j\left(t-\tau_{ij}(t)\right)\right) \\ &\quad + \sum_{j=1}^n d_{ij}(t) \int_{-\infty}^t k_j(t-s)\, f_j\left(x_j(s)\right) ds \\ &\quad + \sum_{j=1}^n e_{ij}(t)\, g_j\left(x_j'\left(t-\sigma_{ij}(t)\right)\right) + I_i(t), \\ &\qquad i = 1,2,\ldots,n, \end{aligned} \tag{6}$$

with initial condition

$$x_i(s) = \psi_i(s), \quad s \in [-\tau, 0],\ i = 1,2,\ldots,n, \tag{7}$$

where $\psi = (\psi_1, \psi_2, \ldots, \psi_n)^T \in C^1([-\tau,0], R^n)$, $\tau = \max_{1\le i,j\le n}\{\sup_{t\in R}\tau_{ij}(t), \sup_{t\in R}\sigma_{ij}(t)\} > 0$, $a_i \ge 0$, $\tau_{ij} \ge 0$, $\sigma_{ij} \ge 0$, b_{ij}, c_{ij}, d_{ij}, e_{ij}, and I_i are almost periodic functions, $i, j = 1,2,\ldots,n$, with ecological meaning are as follows:

$x_i(t)$: the potential (or voltage) of cell i at time t;

$a_i(t)$: represents the rate with which the ith unit will reset its potentialto the resting state in isolation when disconnected from the networkand external inputs at time t;

$b_{ij}(t)$, $c_{ij}(t)$, $d_{ij}(t)$, $e_{ij}(t)$: represent some strengths of connectivity and neutraldelayed strengths of connectivity between cell i and j at time t;

f_j, g_j, k_j: the activation functions and k_j is a scalar integrable function defined in $[0, +\infty)$;

$I_i(t)$: an external input on the ith unit at time t;

τ_{ij}, σ_{ij}: correspond to the transmission delays of the ith unit along the axon of the jth unit at time t.

The aim of this paper is to obtain sufficient conditions for the existence and uniqueness of almost periodic solutions to system (6), by using fixed point theorem and differential inequality theory and the analysis technique.

The remaining part of this paper is organized as follows. In Section 2, we will state several definitions and lemmas which will be useful in proving the main results. In Section 3, by using fixed point theorem and differential inequality techniques, the existence of almost periodic solution for system (6) is obtained. In Section 4, globally exponential stability of almost periodic solution for system (6) is obtained; thus the uniqueness of almost periodic solution for system (6) is obtained.

2. Preliminaries

For the sake of convenience, we introduce the following notations:

$$\begin{gathered} a_i^- = \inf_{t\in R}\left|a_i(t)\right|, \qquad a_i^+ = \sup_{t\in R}\left|a_i(t)\right|, \\ b_{ij}^+ = \sup_{t\in R}\left|b_{ij}(t)\right|, \\ c_{ij}^+ = \sup_{t\in R}\left|c_{ij}(t)\right|, \qquad d_{ij}^+ = \sup_{t\in R}\left|d_{ij}(t)\right|, \\ e_{ij}^+ = \sup_{t\in R}\left|e_{ij}(t)\right|, \\ I_i^+ = \sup_{t\in R}\left|I_i(t)\right|, \\ H = \max\left\{\max_{1\le i\le n}\left\{\frac{I_i^+}{a_i^-}\right\}, \max_{1\le i\le n}\left\{I_i^+\left(1 + \frac{a_i^+}{a_i^-}\right)\right\}\right\}, \end{gathered}$$

$$\phi_0(t) = \left(\int_{-\infty}^{t} e^{-\int_s^t a_1(u)du} I_1(s)\, ds, \ldots, \int_{-\infty}^{t} e^{-\int_s^t a_n(u)du} I_n(s)\, ds \right)^T. \quad (8)$$

For system (6), we introduce the following assumptions.

(H_1) $f_j(0) = 0$, $g_j(0) = 0$, $f_j, g_j : R \to R$ are Lipschitz continuous with Lipschitz constants L_j^f and L_j^g, respectively, $|f_j(u) - f_j(v)| \le L_j^f |u - v|$ and $|g_j(u) - g_j(v)| \le L_j^g |u - v|$, for all $u, v \in R$:

$$(H_2)\ \theta = \max \left\{ \max_{1\le i\le n} \left\{ \frac{1}{a_i^-}, 1 + \frac{a_i^+}{a_i^-} \right\} \times \left[\sum_{j=1}^{n} \left(b_{ij}^+ L_j^f + c_{ij}^+ L_j^f + d_{ij}^+ L_j^f + e_{ij}^+ L_j^g \right) \right] \right\} < 1. \quad (9)$$

(H_3) $\int_0^{+\infty} k_j(s) ds = 1$ and $k_j(t)$ is a decreasing function about t.

In this paper, we will denote $\|m\|_x = \max\{\|m\|_0, \|m'\|_0\}$, where $\|m\|_0 = \max_{1\le i\le n} \sup_{t\in R} |m_i(t)|$, $\|m\|_1 = \max\{\|m\|, \|m'\|\}$, where $\|m\| = \max_{1\le i\le n} |m_i(t)|$, $\|M\| = \sup_{x\in R^n, \|x\|=1} |Mx|$, where M is matrix. Define the space X as $X = \{\phi \mid \phi = (\phi_1(t), \phi_2(t), \ldots, \phi_n(t))^T$, where $\phi_i : R \to R$ is continuously differentiable almost periodic function$\}$; then X is a Banach space with the norm defined by

$$\|\phi\|_x = \max\left\{\|\phi\|_0, \|\phi'\|_0\right\} = \max\left\{ \max_{1\le i\le n,} \sup_{t\in R} |\phi_i(t)|, \max_{1\le i\le n,} \sup_{t\in R} |\phi_i'(t)| \right\}. \quad (10)$$

We introduce some useful definitions and lemmas, which are important to establish our results.

Definition 1 (see [21, 22]). Let $f : R \to R^n$ be continuous in t, $f(t)$ is said to be almost periodic on R if, for any $\varepsilon > 0$, the set $T(f, \varepsilon) = \{\tau : |f(t+\tau) - f(t)| < \varepsilon$, for all $t \in R\}$ is relatively dense; that is, for $\forall \varepsilon > 0$, it is possible to find a real number $l = l(\varepsilon) > 0$, for any interval with length $l(\varepsilon)$, there exists a number $\tau = \tau(\varepsilon)$ in this interval such that $\|f(t+\tau) - f(t)\| < \varepsilon$, for $\forall t \in R$.

Definition 2 (see [21, 22]). Let $x \in R^n$ and $Q(t)$ be a $n \times n$ continuous matrix defined on R. The linear system

$$x'(t) = Q(t) x(t) \quad (11)$$

is said to admit an exponential dichotomy on R^n, if there exist positive constants k, α, projection P and the fundamental solution matrix $X(t)$ of (11) satisfying

$$\left\| X(t) P X^{-1}(s) \right\| \le k e^{-\alpha(t-s)}, \quad \text{for } t \ge s,$$
$$\left\| X(t) (I - P) X^{-1}(s) \right\| \le k e^{-\alpha(s-t)}, \quad \text{for } t \le s. \quad (12)$$

Definition 3 (see [10]). Let $x^*(t) = (x_1^*(t), x_2^*(t), \ldots, x_n^*(t))^T$ be a continuously differentiable almost periodic solution of system (6) with initial value $\varphi^* = (\varphi_1^*(t), \varphi_2^*(t), \ldots, \varphi_n^*(t))^T \in C^1([-\tau, 0]; R^n)$. If there exist constants $\lambda > 0$ and $M \ge 1$ such that for every solution $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ of system (6) with any initial value $\varphi = (\varphi_1(t), \varphi_2(t), \ldots, \varphi_n(t))^T \in C^1([-\tau, 0]; R^n)$,

$$\|x(t) - x^*(t)\|_1 = \max\left\{ \|x(t) - x^*(t)\|, \|x'(t) - x^{*\prime}(t)\| \right\} \le M \|\varphi - \varphi^*\|_1 e^{-\lambda t}, \quad \forall t > 0, \quad (13)$$

where $\|x(t) - x^*(t)\| = \max_{1\le i\le n} |x_i(t) - x_i^*(t)|$. Then $x^*(t)$ is said to be globally exponential stable.

Lemma 4 (see [21, 22]). *If the linear system* (11) *admits an exponential dichotomy, then almost periodic system*

$$x'(t) = Q(t) x(t) + g(t), \quad (14)$$

has a unique almost periodic solution $x(t)$, and

$$x(t) = \int_{-\infty}^{t} X(t) P X^{-1}(s) g(s)\, ds - \int_{t}^{+\infty} X(t) (I - P) X^{-1}(s) g(s)\, ds. \quad (15)$$

Lemma 5 (see [21, 22]). *Let $c_i(t)$ be an almost periodic function on R^n and*

$$M[c_i] = \lim_{T\to+\infty} \frac{1}{T} \int_t^{t+T} c_i(s)\, ds > 0, \quad i = 1, 2, \ldots, n. \quad (16)$$

Then the linear system

$$x'(t) = \operatorname{diag}\left(-c_1(t), -c_2(t), \ldots, -c_n(t)\right) x(t) \quad (17)$$

admits an exponential dichotomy on R^n.

3. Existence of Almost Periodic Solution

Theorem 6. *Assume that (H_1)–(H_3) hold; then there exists a unique continuously differentiable almost periodic solution of system* (6) *in the region $X_0 = \{\phi \mid \phi \in X, \|\phi - \phi_0\|_x \le \theta H/(1 - \theta)\}$.*

Proof. For $\forall\phi \in X$, we consider the almost periodic solution $x^{\phi}(t)$ of nonlinear almost periodic differential equations

$$\begin{aligned} x_i'(t) = {} & -a_i x_i(t) + \sum_{j=1}^{n} b_{ij} f_j(\phi_j(t)) \\ & + \sum_{j=1}^{n} c_{ij} f_j(\phi_j(t-\tau_{ij}(t))) \\ & + \sum_{j=1}^{n} d_{ij} \int_{-\infty}^{t} k_j(t-s) f_j(\phi_j(s))\, ds \\ & + \sum_{j=1}^{n} e_{ij} g_j(\phi_j'(t-\sigma_{ij})) + I_i(t), \end{aligned} \tag{18}$$

where $i = 1, 2, \ldots, n$.

From $a_i(t) > 0$, we have

$$M[a_i] = \lim_{T\to+\infty} \frac{1}{T} \int_{t}^{t+T} a_i(s)\, ds > 0, \quad i = 1, 2, \ldots, n. \tag{19}$$

From Lemmas 4 and 5, system (6) has a unique almost periodic solution $x^{\phi}(t)$ which can be expressed as follows:

$$\begin{aligned} x^{\phi}(t) & = \left(x_1^{\phi}(t), x_2^{\phi}(t), \ldots, x_n^{\phi}(t)\right)^T \\ & = \Bigg(\int_{-\infty}^{t} e^{-\int_s^t a_1(u)du} \times \Bigg[\sum_{j=1}^{n} b_{1j} f_j(\phi_j(s)) + \sum_{j=1}^{n} c_{1j} f_j(\phi_j(s-\tau_{1j}(s))) \\ & \quad + \sum_{j=1}^{n} d_{1j} \int_{-\infty}^{s} k_j(s-u) f_j(\phi_j(u))\, du + \sum_{j=1}^{n} e_{1j} g_j(\phi_j'(s-\sigma_{1j})) + I_1(s) \Bigg] ds, \\ & \quad \int_{-\infty}^{t} e^{-\int_s^t a_2(u)du} \times \Bigg[\sum_{j=1}^{n} b_{2j} f_j(\phi_j(s)) + \sum_{j=1}^{n} c_{2j} f_j(\phi_j(s-\tau_{2j}(s))) \\ & \quad + \sum_{j=1}^{n} d_{2j} \int_{-\infty}^{s} k_j(s-u) f_j(\phi_j(u))\, du + \sum_{j=1}^{n} e_{2j} g_j(\phi_j'(s-\sigma_{2j})) + I_2(s) \Bigg] ds, \ldots, \\ & \quad \int_{-\infty}^{t} e^{-\int_s^t a_n(u)du} \times \Bigg[\sum_{j=1}^{n} b_{nj} f_j(\phi_j(s)) + \sum_{j=1}^{n} c_{nj} f_j(\phi_j(s-\tau_{nj}(s))) \\ & \quad + \sum_{j=1}^{n} d_{nj} \int_{-\infty}^{s} k_j(s-u) f_j(\phi_j(u))\, du + \sum_{j=1}^{n} e_{nj} g_j(\phi_j'(s-\sigma_{nj})) + I_n(s) \Bigg] ds \Bigg). \end{aligned} \tag{20}$$

Define an operator: $T : X \to X$ by setting

$$(T\phi)(t) = x^{\phi}(t), \quad \forall\phi \in X. \tag{21}$$

By the definition of $\|\cdot\|_x$, one has

$$\begin{aligned} \|\phi_0\|_x & = \max\left\{\|\phi_0\|_0, \|\phi_0'\|_0\right\} \\ & = \max\Bigg\{ \max_{1\le i\le n,}\sup_{t\in R} \left\{ \left| \int_{-\infty}^{t} I_i(s) e^{-\int_s^t a_i(u)du} ds \right| \right\}, \\ & \qquad \max_{1\le i\le n,}\sup_{t\in R} \left\{ \left| I_i(t) - \int_{-\infty}^{t} I_i(s) a_i(t) e^{-\int_s^t a_i(u)du} ds \right| \right\} \Bigg\} \\ & \le \max\left\{ \max_{1\le i\le n} \left\{ \frac{I_i^+}{a_i^-} \right\}, \max_{1\le i\le n} \left\{ I_i^+ + a_i^+ \frac{I_i^+}{a_i^-} \right\} \right\} = H. \end{aligned} \tag{22}$$

Hence, for $\forall\phi \in X_0 = \{\phi \mid \phi \in X, \|\phi - \phi_0\|_x \le \theta H/(1-\theta)\}$, one has

$$\|\phi\|_x \le \|\phi - \phi_0\|_x + \|\phi_0\|_x \le \frac{\theta H}{1-\theta} + H = \frac{H}{1-\theta}. \tag{23}$$

Now we prove that T maps the set X_0 into itself.

Obviously, for all $\phi \in X_0$, it follows from (H_1)–(H_3) that

$$\begin{aligned}
&\|T\phi - \phi_0\|_0 \\
&= \max_{1\le i\le n,}\sup_{t\in R}\left\{\left|\int_{-\infty}^{t} e^{-\int_s^t a_i(u)du} \times \left[\sum_{j=1}^{n} b_{ij} f_j(\phi_j(s)) + \sum_{j=1}^{n} c_{ij} f_j(\phi_j(s-\tau_{ij}(s))) + \sum_{j=1}^{n} d_{ij}\int_{-\infty}^{s} k_j(s-u) f_j(\phi_j(u))\,du + \sum_{j=1}^{n} e_{ij} g_j(\phi_j'(s-\sigma_{ij}))\right] ds\right|\right\} \\
&\le \max_{1\le i\le n,}\sup_{t\in R}\left\{\int_{-\infty}^{t} e^{-\int_s^t a_i(u)du} \times \left[\sum_{j=1}^{n} |b_{ij}||f_j(\phi_j(s))| + \sum_{j=1}^{n} |c_{ij}||f_j(\phi_j(s-\tau_{ij}(s)))| + \sum_{j=1}^{n} |d_{ij}||f_j(\phi_j(s))| + \sum_{j=1}^{n} |e_{ij}||g_j(\phi_j'(s-\sigma_{ij}))|\right] ds\right\} \\
&\le \max_{1\le i\le n,}\sup_{t\in R}\left\{\int_{-\infty}^{t} e^{-\int_s^t a_i(u)du} \times \left[\sum_{j=1}^{n} b_{ij}^{+} L_j^{f}\|\phi\|_0 + \sum_{j=1}^{n} c_{ij}^{+} L_j^{f}\|\phi\|_0 + \sum_{j=1}^{n} d_{ij}^{+} L_j^{f}\|\phi\|_0 + \sum_{j=1}^{n} e_{ij}^{+} L_j^{g}\|\phi'\|_0\right] ds\right\} \\
&\le \max_{1\le i\le n,}\sup_{t\in R}\left\{\int_{-\infty}^{t} e^{-\int_s^t a_i(u)du} \times \left[\sum_{j=1}^{n}\left(b_{ij}^{+} L_j^{f} + c_{ij}^{+} L_j^{f} + d_{ij}^{+} L_j^{f} + e_{ij}^{+} L_j^{g}\right) \times \|\phi\|_x\right] ds\right\} \\
&\le \max_{1\le i\le n,}\sup_{t\in R}\left\{\int_{-\infty}^{t} e^{-(t-s)a_i^{-}} ds \times \sum_{j=1}^{n}\left(b_{ij}^{+} L_j^{f} + c_{ij}^{+} L_j^{f} + d_{ij}^{+} L_j^{f} + e_{ij}^{+} L_j^{g}\right)\|\phi\|_x\right\} \\
&= \max_{1\le i\le n}\left\{\frac{1}{a_i^{-}}\sum_{j=1}^{n}\left(b_{ij}^{+} L_j^{f} + c_{ij}^{+} L_j^{f} + d_{ij}^{+} L_j^{f} + e_{ij}^{+} L_j^{g}\right)\right\}\|\phi\|_x.
\end{aligned} \tag{24}$$

Moreover, we get

$$\begin{aligned}
&\left\|(T\phi - \phi_0)'\right\|_0 \\
&= \max_{1\le i\le n,}\sup_{t\in R}\left\{\left|\sum_{j=1}^{n} b_{ij} f_j(\phi_j(t)) + \sum_{j=1}^{n} c_{ij} f_j(\phi_j(t-\tau_{ij}(t))) + \sum_{j=1}^{n} d_{ij}\int_{-\infty}^{t} k_j(t-s) f_j(\phi_j(s))\,ds + \sum_{j=1}^{n} e_{ij} g_j(\phi_j'(t-\sigma_{ij}))\right.\right. \\
&\qquad - \int_{-\infty}^{t} a_i(t) e^{-\int_s^t a_i(u)du} \times \left[\sum_{j=1}^{n} b_{ij} f_j(\phi_j(s)) + \sum_{j=1}^{n} c_{ij} f_j(\phi_j(s-\tau_{ij}(s))) + \sum_{j=1}^{n} d_{ij}\int_{-\infty}^{s} k_j(s-u) f_j(\phi_j(u))\,du \right. \\
&\qquad \left.\left.\left. + \sum_{j=1}^{n} e_{ij} g_j(\phi_j'(s-\sigma_{ij}))\right] ds\right|\right\} \\
&\le \max_{1\le i\le n}\left\{\sum_{j=1}^{n} b_{ij}^{+} L_j^{f}\|\phi\|_0 + \sum_{j=1}^{n} c_{ij}^{+} L_j^{f}\|\phi\|_0 + \sum_{j=1}^{n} d_{ij}^{+} L_j^{f}\|\phi\|_0 + \sum_{j=1}^{n} e_{ij}^{+} L_j^{g}\|\phi'\|_0\right.
\end{aligned}$$

$$+a_i^+\int_{-\infty}^{t} e^{-\int_s^t a_i(u)du} \times\left[\sum_{j=1}^{n} b_{ij}^+ L_j^f\|\phi\|_0+\sum_{j=1}^{n} e_{ij}^+ L_j^g\|\phi'\|_0 +\sum_{j=1}^{n} c_{ij}^+ L_j^f\|\phi\|_0+\sum_{j=1}^{n} d_{ij}^+ L_j^f\|\phi\|_0\right] ds\Bigg\}$$

$$\leq \max_{1\leq i\leq n}\left\{\left(1+\frac{a_i^+}{a_i^-}\right) \times\sum_{j=1}^{n}\left(b_{ij}^+ L_j^f+c_{ij}^+ L_j^f+d_{ij}^+ L_j^f+e_{ij}^+ L_j^g\right)\|\phi\|_x\right\}. \tag{25}$$

Thus, by (23), (24), (25), and (H_2), one has

$$\begin{aligned}
&\|T\phi-\phi_0\|_x \\
&\quad=\max\left\{\|T\phi-\phi_0\|_0,\|(T\phi-\phi_0)'\|_0\right\} \\
&\quad\leq \max_{1\leq i\leq n}\left\{\left(\max\left\{\frac{1}{a_i^-},1+\frac{a_i^+}{a_i^-}\right\}\right) \times\left[\sum_{j=1}^{n}\left(b_{ij}^+ L_j^f+c_{ij}^+ L_j^f+d_{ij}^+ L_j^f+e_{ij}^+ L_j^g\right)\right]\right\}\|\phi\|_x \\
&\quad=\theta\|\phi\|_x\leq\frac{\theta H}{1-\theta},
\end{aligned} \tag{26}$$

which implies that $T\phi\in X_0$. So, the operator T is a self-operator from X_0 to X_0.

Next, we prove that T is a contraction operator of the X_0.

In fact, in view of (H_1)–(H_3), for all $\phi,\varphi\in x_0$, we have

$$\begin{aligned}
&\|T\phi-T\varphi\|_0 \\
&=\max_{1\leq i\leq n,\, t\in R}\sup\Bigg\{\bigg|\int_{-\infty}^{t} e^{-\int_s^t a_i(u)du} \times\bigg[\sum_{j=1}^{n} b_{ij}\left(f_j(\phi_j(s))-f_j(\varphi_j(s))\right) \\
&\qquad+\sum_{j=1}^{n} c_{ij}\left(f_j\left(\phi_j\left(s-\tau_{ij}(s)\right)\right)-f_j\left(\varphi_j\left(s-\tau_{ij}(s)\right)\right)\right) \\
&\qquad+\sum_{j=1}^{n} d_{ij}\int_{-\infty}^{s} k_j(s-u)\times\left(f_j(\phi_j(u))-f_j(\varphi_j(u))\right)du \\
&\qquad+\sum_{j=1}^{n} e_{ij}\left(g_j\left(\phi_j'\left(s-\sigma_{ij}\right)\right)-g_j\left(\varphi_j'\left(s-\sigma_{ij}\right)\right)\right)\bigg] ds\bigg|\Bigg\} \\
&\leq\max_{1\leq i\leq n,\, t\in R}\sup\Bigg\{\bigg|\int_{-\infty}^{t} e^{-\int_s^t a_i(u)du} \times\bigg[\sum_{j=1}^{n}\left|b_{ij}\right|\left|f_j(\phi_j(s))-f_j(\varphi_j(s))\right| \\
&\qquad+\sum_{j=1}^{n}\left|c_{ij}\right|\left|f_j\left(\phi_j\left(s-\tau_{ij}(s)\right)\right)-f_j\left(\varphi_j\left(s-\tau_{ij}(s)\right)\right)\right| \\
&\qquad+\sum_{j=1}^{n}\left|d_{ij}\right|\left|f_j(\phi_j(s))-f_j(\varphi_j(s))\right| \\
&\qquad+\sum_{j=1}^{n}\left|e_{ij}\right|\left|g_j\left(\phi_j'\left(s-\sigma_{ij}\right)\right)-g_j\left(\varphi_j'\left(s-\sigma_{ij}\right)\right)\right|\bigg] ds\bigg|\Bigg\} \\
&\leq\max_{1\leq i\leq n,\, t\in R}\sup\Bigg\{\int_{-\infty}^{t} e^{-\int_s^t a_i(u)du} \times\bigg[\sum_{j=1}^{n} b_{ij}^+ L_j^f\|\phi-\varphi\|_0+\sum_{j=1}^{n} c_{ij}^+ L_j^f\|\phi-\varphi\|_0 \\
&\qquad+\sum_{j=1}^{n} d_{ij}^+ L_j^f\|\phi-\varphi\|_0+\sum_{j=1}^{n} e_{ij}^+ L_j^g\|(\phi-\varphi)'\|_0\bigg] ds\Bigg\} \\
&\leq\max_{1\leq i\leq n,\, t\in R}\sup\Bigg\{\int_{-\infty}^{t} e^{-\int_s^t a_i(u)du} \times\left[\sum_{j=1}^{n}\left(b_{ij}^+ L_j^f+c_{ij}^+ L_j^f+d_{ij}^+ L_j^f+e_{ij}^+ L_j^g\right)\right]
\end{aligned}$$

$$
\begin{aligned}
&\times \|\phi-\varphi\|_x ds\Big\}\\
&\le \max_{1\le i\le n,}\sup_{t\in R}\left\{\int_{-\infty}^{t} e^{-(t-s)a_i^-}ds\right.\\
&\qquad \times \sum_{j=1}^{n}\left(b_{ij}^+L_j^f + c_{ij}^+L_j^f + d_{ij}^+L_j^f + e_{ij}^+L_j^g\right)\\
&\qquad \left.\times \|\phi-\varphi\|_x\right\}\\
&\le \max_{1\le i\le n}\left\{\frac{1}{a_i^-}\sum_{j=1}^{n}\left(b_{ij}^+L_j^f + c_{ij}^+L_j^f + d_{ij}^+L_j^f + e_{ij}^+L_j^g\right)\right\}\\
&\quad \times \|\phi-\varphi\|_x\\
&\le \theta\|\phi-\varphi\|_x,\\
&\left\|(T\phi-T\varphi)'\right\|_0\\
&= \max_{1\le i\le n,}\sup_{t\in R}\left\{\left|\sum_{j=1}^{n} b_{ij}\left(f_j\left(\phi_j(t)\right)-f_j\left(\varphi_j(t)\right)\right)\right.\right.\\
&\qquad + \sum_{j=1}^{n} c_{ij}\left(f_j\left(\phi_j\left(t-\tau_{ij}(t)\right)\right)\right.\\
&\qquad\qquad \left.-f_j\left(\varphi_j\left(t-\tau_{ij}(t)\right)\right)\right)\\
&\qquad + \sum_{j=1}^{n} d_{ij}\int_{-\infty}^{t} k_j(t-s)\\
&\qquad\qquad \times\left(f_j\left(\phi_j(s)\right)-f_j\left(\varphi_j(s)\right)\right)ds\\
&\qquad + \sum_{j=1}^{n} e_{ij}\left(g_j\left(\phi_j'\left(t-\sigma_{ij}\right)\right)\right.\\
&\qquad\qquad \left.-g_j\left(\varphi_j'\left(t-\sigma_{ij}\right)\right)\right)\\
&\qquad - \int_{-\infty}^{t} a_i(t)\, e^{-\int_s^t a_i(u)du}\\
&\qquad \times\left[\sum_{j=1}^{n} b_{ij}\left(f_j\left(\phi_j(s)\right)\right.\right.\\
&\qquad\qquad \left.-f_j\left(\varphi_j(s)\right)\right)\\
&\qquad\qquad + \sum_{j=1}^{n} c_{ij}\left(f_j\left(\phi_j\left(s-\tau_{ij}(s)\right)\right)\right.\\
&\qquad\qquad \left.-f_j\left(\varphi_j\left(s-\tau_{ij}(s)\right)\right)\right)\\
&\qquad\qquad + \sum_{j=1}^{n} d_{ij}\int_{-\infty}^{s} k_j(s-u)\\
&\qquad\qquad \times\left(f_j\left(\phi_j(u)\right)\right.\\
&\qquad\qquad \left.-f_j\left(\varphi_j(u)\right)\right)du\\
&\qquad\qquad + \sum_{j=1}^{n} e_{ij}\left(g_j\left(\phi_j'\left(s-\sigma_{ij}\right)\right)\right.\\
&\qquad\qquad \left.\left.\left.-g_j\left(\varphi_j'\left(s-\sigma_{ij}\right)\right)\right)\right]ds\right|\right\}\\
&\le \max_{1\le i\le n}\sum_{j=1}^{n}\left(b_{ij}^+L_j^f + c_{ij}^+L_j^f + d_{ij}^+L_j^f + e_{ij}^+L_j^g\right)\\
&\qquad \times \|\phi-\varphi\|_x\\
&\quad + \max_{1\le i\le n,}\sup_{t\in R}\left\{\int_{-\infty}^{t} a_i^+ e^{-\int_s^t a_i(u)du}\right.\\
&\qquad \times\left[\sum_{j=1}^{n}\left(b_{ij}^+L_j^f + c_{ij}^+L_j^f\right.\right.\\
&\qquad\qquad \left.\left.+d_{ij}^+L_j^f + e_{ij}^+L_j^g\right)\right]ds\\
&\qquad \left.\times\|\phi-\varphi\|_x\right\}\\
&\le \max_{1\le i\le n}\left\{\left(1+\frac{a_i^+}{a_i^-}\right)\right.\\
&\qquad \times\sum_{j=1}^{n}\left(b_{ij}^+L_j^f + c_{ij}^+L_j^f\right.\\
&\qquad\qquad \left.\left.+d_{ij}^+L_j^f + e_{ij}^+L_j^g\right)\right\}\\
&\quad \times\|\phi-\varphi\|_x\\
&\le \theta\|\phi-\varphi\|_x.
\end{aligned}
\tag{27}
$$

Thus,

$$\|T\phi-T\varphi\|_x \le \theta\|\phi-\varphi\|_x. \tag{28}$$

In view of (H_2), we have $\theta < 1$; it means that the T is a contraction operator. By Banach fixed point theorem, there exists a fixed point $z \in X_0$ such that $Tz = z$, which implies system (6) has an almost periodic solution. □

4. Uniqueness of Almost Periodic Solution

Theorem 7. *Assume that (H_1)–(H_3) hold; then system (6) has a unique continuously differentiable almost periodic solution $z(t)$ which is globally exponentially stable.*

Proof. It follows from Theorem 6 that system (6) has at least one almost periodic solution $z(t) = (z_1(t), z_2(t), \ldots, z_n(t))^T \in X_0$ with initial value $\mu(t) = (\mu_1(t), \mu_2(t), \ldots, \mu_n(t))^T$. Let $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ be an arbitrary solution of system (6) with initial value $\psi^*(t) = (\psi_1^*(t), \psi_2^*(t), \ldots, \psi_n^*(t))^T$. Let $y_i(t) = x_i(t) - z_i(t)$, $\psi_i(t) = \psi_i^*(t) - \mu_i(t)$, $i = 1, 2, \ldots, n$. Then

$$\begin{aligned} y_i'(s) + a_i y_i(s) &= \sum_{j=1}^{n} b_{ij}\left[f_j\left(y_j(s) + z_j(s)\right) - f_j\left(z_j(s)\right)\right] \\ &\quad + \sum_{j=1}^{n} c_{ij}\left[f_j\left(y_j\left(s - \tau_{ij}(s)\right) + z_j\left(s - \tau_{ij}(s)\right)\right) - f_j\left(z_j\left(s - \tau_{ij}(s)\right)\right)\right] \\ &\quad + \sum_{j=1}^{n} d_{ij} \int_{-\infty}^{s} k_j(s-u) \times \left[f_j\left(y_j(u) + z_j(u)\right) - f_j\left(z_j(u)\right)\right] du \\ &\quad + \sum_{j=1}^{n} e_{ij}\left[g_j\left(y_j'\left(s - \sigma_{ij}\right) + z_j'\left(s - \sigma_{ij}\right)\right) - g_j\left(z_j'\left(s - \sigma_{ij}\right)\right)\right], \end{aligned} \tag{29}$$

where $i = 1, 2, \ldots, n$. Let $F_i(\cdot)$ and $G_i(\cdot)$ be defined by

$$\begin{aligned} F_i(\eta_i) &= a_i^- - \eta_i - \sum_{j=1}^{n}\left(b_{ij}^+ L_j^f + c_{ij}^+ L_j^f + d_{ij}^+ L_j^f + e_{ij}^+ L_j^g\right) e^{\eta_i \tau}, \\ G_i(\varepsilon_i) &= a_i^- - \varepsilon_i - \left(a_i^+ + a_i^-\right) \times \sum_{j=1}^{n}\left(b_{ij}^+ L_j^f + c_{ij}^+ L_j^f + d_{ij}^+ L_j^f + e_{ij}^+ L_j^g\right) e^{\varepsilon_i \tau}, \end{aligned} \tag{30}$$

where $\eta_i, \varepsilon_i \in [0, \infty)$, $i = 1, 2, \ldots, n$. From (H_2), we have

$$\begin{aligned} F_i(0) &= a_i^- - \sum_{j=1}^{n}\left(b_{ij}^+ L_j^f + c_{ij}^+ L_j^f + d_{ij}^+ L_j^f + e_{ij}^+ L_j^g\right) > 0, \\ G_i(0) &= a_i^- - \left(a_i^+ + a_i^-\right) \times \sum_{j=1}^{n}\left(b_{ij}^+ L_j^f + c_{ij}^+ L_j^f + d_{ij}^+ L_j^f + e_{ij}^+ L_j^g\right) > 0, \\ & \qquad i = 1, 2, \ldots, n. \end{aligned} \tag{31}$$

Since $F_i(\cdot)$ and $G_i(\cdot)$ are continuous on $[0, \infty)$ and $F_i(\eta_i)$, $G_i(\varepsilon_i) \to -\infty$ as $\eta_i, \varepsilon_i \to \infty$, there exist $\eta_i^*, \varepsilon_i^* > 0$ such that $F_i(\eta_i^*) = G_i(\varepsilon_i^*) = 0$ and $F_i(\eta_i) > 0$ for $\eta_i \in (0, \eta_i^*)$, $G_i(\varepsilon_i) > 0$ for $\varepsilon_i \in (0, \varepsilon_i^*)$. It is easy to check that $\xi = \min\{\eta_1^*, \eta_2^*, \ldots, \eta_n^*, \varepsilon_1^*, \varepsilon_2^*, \ldots, \varepsilon_n^*\}$. We obtain

$$F_i(\xi) \geq 0, \quad G_i(\xi) \geq 0, \quad i = 1, 2, \ldots, n. \tag{32}$$

So, we can choose a positive constant $0 < \lambda < \min\{\xi, a_1^-, a_2^-, \ldots, a_n^-\}$ such that $F_i(\lambda) > 0$, $G_i(\lambda) > 0$, which implies that

$$\begin{aligned} &\frac{\sum_{j=1}^{n}\left(b_{ij}^+ L_j^f + c_{ij}^+ L_j^f + d_{ij}^+ L_j^f + e_{ij}^+ L_j^g\right) e^{\lambda \tau}}{a_i - \lambda} < 1, \\ &\left(1 + \frac{a_i^+}{a_i^- - \lambda}\right) \sum_{j=1}^{n}\left(b_{ij}^+ L_j^f + c_{ij}^+ L_j^f + d_{ij}^+ L_j^f + e_{ij}^+ L_j^g\right) e^{\lambda \tau} < 1. \end{aligned} \tag{33}$$

By (29), we have

$$\begin{aligned} y_i(t) &= \psi_i(0) e^{-\int_0^t a_i(u) du} + \int_0^t e^{-\int_s^t a_i(u) du} \\ &\quad \times \left\{ \sum_{j=1}^{n} b_{ij}\left[f_j\left(y_j(s) + z_j(s)\right) - f_j\left(z_j(s)\right)\right] \right. \\ &\quad + \sum_{j=1}^{n} c_{ij}\left[f_j\left(y_j\left(s - \tau_{ij}(s)\right) + z_j\left(s - \tau_{ij}(s)\right)\right) - f_j\left(z_j\left(s - \tau_{ij}(s)\right)\right)\right] \\ &\quad + \sum_{j=1}^{n} d_{ij} \int_{-\infty}^{s} k_j(s-u) \times \left[f_j\left(y_j(u) + z_j(u)\right) - f_j\left(z_j(u)\right)\right] du \\ &\quad \left. + \sum_{j=1}^{n} e_{ij}\left[g_j\left(y_j'\left(s - \sigma_{ij}\right) + z_j'\left(s - \sigma_{ij}\right)\right) - g_j\left(z_j'\left(s - \sigma_{ij}\right)\right)\right] \right\} ds. \end{aligned} \tag{34}$$

Let $M = a_i^- / \sum_{j=1}^{n}(b_{ij}^+ L_j^f + c_{ij}^+ L_j^f + d_{ij}^+ L_j^f + e_{ij}^+ L_j^g)$. By (H_2) we have $M > 1$ and

$$\|y(t)\|_1 = \|\psi\|_1 \leq M\|\psi\|_1 e^{-\lambda t}, \quad \forall t \in [-\tau, 0], \ \lambda > 0. \tag{35}$$

We claim that

$$\|y(t)\|_1 \leq M\|\psi\|_1 e^{-\lambda t}, \quad t > 0. \tag{36}$$

To prove (36), we first prove for any $p > 1$, the following inequality holds:

$$\|y(t)\|_1 < pM\|\psi\|_1 e^{-\lambda t}, \quad t > 0. \tag{37}$$

Otherwise, there must be some $t_1 > 0$ and some $i \in \{1, 2, \dots, n\}$, such that

$$\begin{aligned}
\|y(t)\|_1 &= \max\left\{\|y(t)\|, \|y'(t)\|\right\} \\
&= \max\left\{\max_{1\le j\le n}|y_j(t)|, \max_{1\le j\le n}|y'_j(t)|\right\} \\
&= \max\left\{|y_i(t_1)|, |y'_i(t_1)|\right\} \\
&= pM\|\psi\|_1 e^{-\lambda t_1} \\
&\le pM\|\psi\|_1 e^{-\lambda t}, \quad \forall t \in [-\tau, t_1].
\end{aligned} \tag{38}$$

By (33), (34), (38), (H_1), and (H_3), we have

$$\begin{aligned}
&|y_i(t_1)| \\
&\le \|\psi\|_1 e^{-\int_0^{t_1} a_i(u)du} \\
&\quad + \int_0^{t_1} e^{-\int_s^{t_1} a_i(u)du} \\
&\qquad \times \left[\sum_{j=1}^n b_{ij}^+ L_j^f |y_j(s)| \right.\\
&\qquad\quad + \sum_{j=1}^n c_{ij}^+ L_j^f \left|y_j\left(s - \tau_{ij}(s)\right)\right| \\
&\qquad\quad + \sum_{j=1}^n d_{ij} \int_{-\infty}^s k_j(s-u) L_j^f |y_j(u)| du \\
&\qquad\quad \left.+ \sum_{j=1}^n e_{ij}^+ L_j^g \left|y'_j\left(s - \sigma_{ij}\right)\right|\right] ds \\
&\le \|\psi\|_1 e^{-a_i t_1} \\
&\quad + \int_0^{t_1} e^{-\int_s^{t_1} a_i(u)du} \\
&\qquad \times \left[\sum_{j=1}^n b_{ij}^+ L_j^f pM\|\psi\|_1 e^{-\lambda(s)} \right.\\
&\qquad\quad + \sum_{j=1}^n c_{ij}^+ L_j^f pM\|\psi\|_1 e^{-\lambda(s-\tau_{ij}(s))} \\
&\qquad\quad + \sum_{j=1}^n d_{ij} \int_{-\infty}^s k_j(s-u) L_j^f pM\|\psi\|_1 e^{-\lambda(u)} du \\
&\qquad\quad \left.+ \sum_{j=1}^n e_{ij}^+ L_j^g pM\ \|\psi\|_1 e^{-\lambda(s-\sigma_{ij})}\right] ds \\
&= pM\|\psi\|_1 e^{-\lambda t_1} \\
&\quad \times \left\{ \frac{1}{pM} e^{(\lambda - a_i)t_1} \right.\\
&\qquad + \int_0^{t_1} e^{-\int_s^{t_1} a_i(u)du} e^{\lambda(t_1-s)} \\
&\qquad\quad \times \left[\sum_{j=1}^n b_{ij}^+ L_j^f + \sum_{j=1}^n c_{ij}^+ L_j^f e^{\lambda\tau_{ij}} \right.\\
&\qquad\qquad + \sum_{j=1}^n d_{ij} \int_{-\infty}^s k_j(s-u) L_j^f du \\
&\qquad\qquad \left.\left.+ \sum_{j=1}^n e_{ij}^+ L_j^g e^{\lambda\sigma_{ij}}\right] ds \right\} \\
&\le pM\|\psi\|_1 e^{-\lambda t_1} \\
&\quad \times \left\{ \frac{1}{M} e^{(\lambda - a_i)t_1} \right.\\
&\qquad + \int_0^{t_1} e^{-(t_1-s)a_i} e^{\lambda(t_1-s)} \\
&\qquad\quad \times \left[\sum_{j=1}^n b_{ij}^+ L_j^f + \sum_{j=1}^n c_{ij}^+ L_j^f e^{\lambda\tau_{ij}} \right.\\
&\qquad\qquad + \sum_{j=1}^n d_{ij} \int_{-\infty}^s k_j(s-u) L_j^f du \\
&\qquad\qquad \left.\left.+ \sum_{j=1}^n e_{ij}^+ L_j^g e^{\lambda\sigma_{ij}}\right] ds \right\} \\
&\le pM\|\psi\|_1 e^{-\lambda t_1} \\
&\quad \times \left\{ \frac{1}{M} e^{(\lambda - a_i)t_1} \right.\\
&\qquad + \frac{1}{a_i - 1}\left(1 - e^{(\lambda - a_i)t_1}\right) \\
&\qquad \times \sum_{j=1}^n \left(b_{ij}^+ L_j^f + c_{ij}^+ L_j^f \right.\\
&\qquad\qquad + d_{ij} \int_{-\infty}^s k_j(t-s) L_j^f du \\
&\qquad\qquad \left.\left. + e_{ij}^+ L_j^g\right) e^{\lambda\tau} \right\} \\
&\le pM\|\psi\|_1 e^{-\lambda t_1} \\
&\quad \times \left\{ \frac{1}{M} e^{(\lambda - a_i)t_1} \right.
\end{aligned}$$

$$
\begin{aligned}
&\quad + \frac{1}{a_i - 1}\left(1 - e^{(\lambda - a_i)t_1}\right) \\
&\quad \times \sum_{j=1}^{n}\left(b_{ij}^{+}L_j^f + c_{ij}^{+}L_j^f\right. \\
&\qquad \left.+ d_{ij}L_j^f + e_{ij}^{+}L_j^g\right)e^{\lambda\tau}\Big\} \\
&\le pM\|\psi\|_1 e^{-\lambda t_1} \\
&\quad \times \left\{\left(\frac{1}{M} - \frac{\sum_{j=1}^{n}\left(b_{ij}^{+}L_j^f + c_{ij}^{+}L_j^f + d_{ij}L_j^f + e_{ij}^{+}L_j^g\right)e^{\lambda\tau}}{a_i - \lambda}\right)\right. \\
&\qquad \times e^{(\lambda - a_i)t_1} \\
&\qquad \left.+ \frac{\sum_{j=1}^{n}\left(b_{ij}^{+}L_j^f + c_{ij}^{+}L_j^f + d_{ij}L_j^f + e_{ij}^{+}L_j^g\right)e^{\lambda\tau}}{a_i - \lambda}\right\} \\
&\le pM\|\psi\|_1 e^{-\lambda t_1} \\
&\quad \times \frac{\sum_{j=1}^{n}\left(b_{ij}^{+}L_j^f + c_{ij}^{+}L_j^f + d_{ij}L_j^f + e_{ij}^{+}L_j^g\right)e^{\lambda\tau}}{a_i - \lambda} \\
&< pM\|\psi\|_1 e^{-\lambda t_1}.
\end{aligned} \tag{39}
$$

Direct differentiation of (34) gives

$$
\begin{aligned}
&y_i'(t) \\
&= -a_i(t)\psi_i(0)e^{-\int_0^t a_i(u)du} \\
&\quad - \int_0^t a_i(t)e^{-\int_s^t a_i(u)du} \\
&\quad \times \left\{\sum_{j=1}^{n} b_{ij}\left[f_j\left(y_j(s) + z_j(s)\right) - f_j\left(z_j(s)\right)\right]\right. \\
&\qquad + \sum_{j=1}^{n} c_{ij}\left[f_j\left(y_j\left(s - \tau_{ij}(s)\right)\right.\right. \\
&\qquad\qquad \left.+ z_j\left(s - \tau_{ij}(s)\right)\right) \\
&\qquad\qquad \left.- f_j\left(z_j\left(s - \tau_{ij}(s)\right)\right)\right] \\
&\qquad + \sum_{j=1}^{n} d_{ij}\int_{-\infty}^{s} k_j(s - u) \\
&\qquad\qquad \times \left[f_j\left(y_j(u) + z_j(u)\right) - f_j\left(z_j(u)\right)\right]du \\
&\qquad + \sum_{j=1}^{n} e_{ij}\left[g_j\left(y_j'\left(s - \sigma_{ij}\right) + z_j'\left(s - \sigma_{ij}\right)\right)\right. \\
&\qquad\qquad \left.\left. - g_j\left(z_j'\left(s - \sigma_{ij}\right)\right)\right]\right\} ds \\
&\quad + \sum_{j=1}^{n} b_{ij}\left[f_j\left(y_j(s) + z_j(s)\right) - f_j\left(z_j(s)\right)\right] \\
&\quad + \sum_{j=1}^{n} c_{ij}\left[f_j\left(y_j\left(s - \tau_{ij}(s)\right) + z_j\left(s - \tau_{ij}(s)\right)\right)\right. \\
&\qquad \left.- f_j\left(z_j\left(s - \tau_{ij}(s)\right)\right)\right] \\
&\quad + \sum_{j=1}^{n} d_{ij}\int_{-\infty}^{s} k_j(s - u) \\
&\qquad \times \left[f_j\left(y_j(u) + z_j(u)\right) - f_j\left(z_j(u)\right)\right]du \\
&\quad + \sum_{j=1}^{n} e_{ij}\left[g_j\left(y_j'\left(s - \sigma_{ij}\right)\right.\right. \\
&\qquad \left.\left.+ z_j'\left(s - \sigma_{ij}\right)\right) - g_j\left(z_j'\left(s - \sigma_{ij}\right)\right)\right].
\end{aligned} \tag{40}
$$

Thus, from (33), (40), and (H_1), we obtain

$$
\begin{aligned}
&\left|y_i'(t_1)\right| \\
&\le a_i^{+}\|\psi\|_1 e^{-\int_0^{t_1} a_i(u)du} \\
&\quad + \int_0^{t_1} a_i(t_1)e^{-\int_s^{t_1} a_i(u)du} \\
&\quad \times \left[\sum_{j=1}^{n} b_{ij}^{+}L_j^f\left|y_j(s)\right|\right. \\
&\qquad + \sum_{j=1}^{n} c_{ij}^{+}L_j^f\left|y_j\left(s - \tau_{ij}(s)\right)\right| \\
&\qquad + \sum_{j=1}^{n} d_{ij}\int_{-\infty}^{s} k_j(s - u)L_j^f\left|y_j(u)\right|du \\
&\qquad \left.+ \sum_{j=1}^{n} e_{ij}^{+}L_j^g\left|y_j'\left(s - \sigma_{ij}\right)\right|\right]ds \\
&\quad + \sum_{j=1}^{n} b_{ij}^{+}L_j^f\left|y_j(t_1)\right| \\
&\quad + \sum_{j=1}^{n} c_{ij}^{+}L_j^f\left|y_j\left(t_1 - \tau_{ij}(t_1)\right)\right| \\
&\quad + \sum_{j=1}^{n} d_{ij}\int_{-\infty}^{t_1} k_j(t_1 - u)L_j^f\left|y_j(u)\right|du \\
&\quad + \sum_{j=1}^{n} e_{ij}^{+}L_j^g\left|y_j'\left(t_1 - \sigma_{ij}\right)\right|
\end{aligned}
$$

$$\begin{aligned}
&\le a_i^+ \|\psi\|_1 e^{-a_i} t_1 \\
&\quad + \int_0^{t_1} a_i^+ e^{-\int_s^{t_1} a_i(u)du} \\
&\qquad \times \left[\sum_{j=1}^n b_{ij}^+ L_j^f pM \|\psi\|_1 e^{-\lambda(s)} \right. \\
&\qquad\quad + \sum_{j=1}^n c_{ij}^+ L_j^f pM \|\psi\|_1 e^{-\lambda(s-\tau_{ij}(s))} \\
&\qquad\quad + \sum_{j=1}^n d_{ij} \int_{-\infty}^s k_j(s-u) L_j^f pM \|\psi\|_1 e^{-\lambda(u)} du \\
&\qquad\quad \left. + \sum_{j=1}^n e_{ij}^+ L_j^g pM \|\psi\|_1 e^{-\lambda(s-\sigma_{ij})} \right] ds \\
&\quad + \sum_{j=1}^n b_{ij}^+ L_j^f pM \|\psi\|_1 e^{-\lambda(t_1)} \\
&\quad + \sum_{j=1}^n c_{ij}^+ L_j^f pM \|\psi\|_1 e^{-\lambda(t_1-\tau_{ij}(t_1))} \\
&\quad + \sum_{j=1}^n d_{ij} \int_{-\infty}^{t_1} k_j(t_1-u) L_j^f pM \|\psi\|_1 e^{-\lambda(u)} du \\
&\quad + \sum_{j=1}^n e_{ij}^+ L_j^g pM \|\psi\|_1 e^{-\lambda(t_1-\sigma_{ij})} \\
&= pM \|\psi\|_1 e^{-\lambda t_1} \\
&\quad \times \left\{ \frac{a_i^+}{pM} e^{(\lambda - a_i)t_1} \right. \\
&\qquad + \int_0^{t_1} a_i^+ e^{-\int_s^{t_1} a_i(u)du} e^{\lambda(t_1-s)} \\
&\qquad\quad \times \left[\sum_{j=1}^n b_{ij}^+ L_j^f + \sum_{j=1}^n c_{ij}^+ L_j^f e^{\lambda \tau_{ij}} \right. \\
&\qquad\qquad + \sum_{j=1}^n d_{ij} \int_{-\infty}^s k_j(s-u) L_j^f du \\
&\qquad\qquad \left. + \sum_{j=1}^n e_{ij}^+ L_j^g e^{\lambda \sigma_{ij}} \right] ds \\
&\qquad + \sum_{j=1}^n b_{ij}^+ L_j^f + \sum_{j=1}^n c_{ij}^+ L_j^f e^{\lambda \tau_{ij}} \\
&\qquad + \sum_{j=1}^n d_{ij} \int_{-\infty}^{t_1} k_j(t_1-u) L_j^f du \\
&\qquad \left. + \sum_{j=1}^n e_{ij}^+ L_j^g e^{\lambda \sigma_{ij}} \right\} \\
&\le pM \|\psi\|_1 e^{-\lambda t_1} \\
&\quad \times \left\{ \frac{a_i^+}{M} e^{(\lambda - a_i)t_1} \right. \\
&\qquad + \left(\sum_{j=1}^n b_{ij}^+ L_j^f + \sum_{j=1}^n c_{ij}^+ L_j^f \right. \\
&\qquad\qquad + \sum_{j=1}^n d_{ij} \int_{-\infty}^s k_j(s-u) L_j^f du \\
&\qquad\qquad \left. + \sum_{j=1}^n e_{ij}^+ L_j^g \right) e^{\lambda\tau} \\
&\qquad \times \int_0^{t_1} a_i^+ e^{-\int_s^{t_1} a_i(u)du} e^{\lambda(t_1-s)} ds \\
&\qquad + \sum_{j=1}^n \left(b_{ij}^+ L_j^f + c_{ij}^+ L_j^f \right. \\
&\qquad\qquad \left. \left. + d_{ij} \int_{-\infty}^{t_1} k_j(t_1-u) L_j^f du + e_{ij}^+ L_j^g \right) e^{\lambda\tau} \right\} \\
&= pM \|\psi\|_1 e^{-\lambda t_1} \\
&\quad \times \left\{ \left(\frac{1}{M} - \frac{\sum_{j=1}^n \left(b_{ij}^+ L_j^f + c_{ij}^+ L_j^f + d_{ij} L_j^f + e_{ij}^+ L_j^g \right) e^{\lambda\tau}}{a_i - \lambda} \right) \right. \\
&\qquad \times a_i^+ e^{(\lambda - a_i)t_1} \\
&\qquad + \left(1 + \frac{a_i^+}{a_i^- - \lambda} \right) \\
&\qquad \left. \times \sum_{j=1}^n \left(b_{ij}^+ L_j^f + c_{ij}^+ L_j^f + d_{ij} L_j^f + e_{ij}^+ L_j^g \right) e^{\lambda\tau} \right\} \\
&< pM \|\psi\|_1 e^{-\lambda t_1} \left(1 + \frac{a_i^+}{a_i^- - \lambda} \right) \\
&\quad \times \sum_{j=1}^n \left(b_{ij}^+ L_j^f + c_{ij}^+ L_j^f + d_{ij} L_j^f + e_{ij}^+ L_j^g \right) e^{\lambda\tau} \\
&< pM \|\psi\|_1 e^{-\lambda t_1}.
\end{aligned} \tag{41}$$

Therefore, in view of (39) and (41), we have

$$\|y(t_1)\|_1 = \max \left\{ |y_i(t_1)|, |y_i'(t_1)| \right\} < pM \parallel \psi\|_1 e^{-\lambda t_1}, \tag{42}$$

which contradicts (38); that is, the inequality (37) holds. Letting $p \to 1$, then the inequality (36) holds. Hence, the almost periodic solution of system (6) is globally exponentially stable; that is, the almost periodic solution is unique. □

Here we would like to give some remarks.

Remark 8. If $b_{ij} = 0$ and $d_{ij} = 0$, the system (6) reduces to the system (1) in [10], and we improve the corresponding results of [10].

Remark 9. When $g_j(x) = x$, the system (6) can be reduced to the system (3) in [17]. The methods and results in this paper are different from [17]. We use Banach fixed point theorem to study the existence and uniqueness of almost periodic solution for the system (6). Linear matrix inequalities and delay-dependent conditions are given to guarantee the considered delayed neural network to have a unique equilibrium point, which is globally asymptotically stable in [17].

5. An Example

In this section, we give an example to illustrate the effectiveness of our results.

Example 10. Consider the neural networks with neutral delays:

$$\begin{aligned} x_1'(t) = &-a_1(t)x_1(t) + \sum_{j=1}^{2} b_{1j}(t) f_j(x_j(t)) \\ &+ \sum_{j=1}^{2} c_{1j}(t) f_j(x_j(t-\tau_{1j}(t))) \\ &+ \sum_{j=1}^{2} d_{1j}\int_{-\infty}^{t} k_j(t-s) f_j(x_j(s))\,ds \\ &+ \sum_{j=1}^{2} e_{1j} g_j(x_j'(t-\sigma_{1j}(t))) + I_1(t), \\ x_2'(t) = &-a_2(t)x_2(t) + \sum_{j=1}^{2} b_{2j}(t) f_j(x_j(t)) \\ &+ \sum_{j=1}^{2} c_{2j}(t) f_j(x_j(t-\tau_{2j}(t))) \\ &+ \sum_{j=1}^{2} d_{2j}\int_{-\infty}^{t} k_j(t-s) f_j(x_j(s))\,ds \\ &+ \sum_{j=1}^{2} e_{2j} g_j(x_j'(t-\sigma_{2j}(t))) + I_2(t), \end{aligned} \tag{43}$$

where

$$\begin{gathered} a_1(t) = \frac{2}{3} + \frac{1}{6}\cos t, \\ a_2(t) = \frac{5}{3} + \frac{5}{6}\cos t, \\ b_{11} = b_{12} = \frac{1}{2} - \frac{1}{6}\cos 3t, \\ b_{21} = b_{22} = \frac{1}{2} - \frac{1}{5}\cos 3t, \\ c_{11} = c_{12} = \frac{1}{12}\sin^2 t, \\ c_{21} = c_{22} = \frac{1}{12}\cos^2 t, \\ d_{11} = d_{12} = \frac{1}{12}\cos^2(\sqrt{7}t), \\ d_{21} = d_{22} = \frac{1}{12}\sin^2(\sqrt{7}t), \\ e_{11} = e_{12} = \frac{1}{50}\sin|50t|, \\ e_{21} = e_{22} = \frac{1}{50}\cos|50t|, \\ I_1 = \cos(2t), \qquad I_1 = \sin(2t), \\ k_{11}(t) = k_{12}(t) = k_{21}(t) = k_{22}(t) = e^{-t}, \\ f_1(x) = f_2(x) = \frac{1}{25}|x|, \\ g_1(x) = g_2(x) = \frac{1}{25}\sin x, \\ \tau_{11} = \tau_{12} = 1 + \frac{1}{2}\sin(\sqrt{2}t), \\ \tau_{21} = \tau_{22} = 1 + \frac{1}{2}\cos(\sqrt{2}t), \\ \sigma_{11} = \sigma_{12} = 1 - \frac{1}{4}\cos\sqrt{8}t, \\ \sigma_{21} = \sigma_{22} = 1 - \frac{1}{5}\cos\sqrt{8}t. \end{gathered} \tag{44}$$

By simple calculation, we have

$$\begin{gathered} L_1 = L_2 = \frac{1}{25}, \qquad l_1 = l_2 = \frac{1}{25}, \\ a_1^- = \frac{1}{2}, \qquad a_2^- = \frac{5}{6}, \\ a_1^+ = \frac{5}{6}, \qquad a_2^+ = \frac{5}{2}, \\ b_{11}^+ = b_{12}^+ = \frac{2}{3}, \qquad b_{21}^+ = b_{22}^+ = \frac{7}{10}, \\ c_{11}^+ = c_{12}^+ = \frac{1}{12}, \qquad c_{21}^+ = c_{22}^+ = \frac{1}{12}, \\ d_{11}^+ = d_{12}^+ = \frac{1}{12}, \qquad d_{21}^+ = d_{22}^+ = \frac{1}{12}, \\ e_{11}^+ = e_{12}^+ = \frac{1}{50}, \qquad e_{21}^+ = e_{22}^+ = \frac{1}{50}, \\ I_1^+ = I_2^+ = 1, \qquad \theta = 0.284 < 1. \end{gathered} \tag{45}$$

Clearly, (H_1)–(H_3) hold. From Theorems 6 and 7, system (6) has a unique continuously differentiable almost periodic solution, which is globally exponentially stable.

6. Conclusion

In this work, we are concerned with a neural network model with neutral delays. The existence and uniqueness of almost periodic solution for the system are explored by means of Banach fixed point theorem. Our result is in good agreement with some related results in the literature.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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