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Research Article

Local Fractional Z-Transforms with Applications to Signals on Cantor Sets

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The Z-transform has played an important role in signal processing. In this paper the Z-transform has been generalized by the coupling of both the Z-transform and the local fractional complex calculus. In the literature the local fractional Z-transform is applied to analyze signals, in the following it will be used to analyze signals on Cantor sets. Some examples are also given to show the efficiency and accuracy for handling the signals on Cantor sets.

1. Introduction

Integral transforms [1, 2], such as Fourier, Laplace, Mellin, Hilbert, and Hankel transforms, play important roles in solving the mathematical problems arising in applied mathematics, mathematical physics, and engineering science. In recent years, fractional calculus [3–11] was developed and used to model also some anomalous behaviors of diffusion [12–21] and transport [22–27]. Fractional integral transforms are suitable generalizations of the classical ones and were recently proposed by some researchers. For example, the fractional Fourier transforms were considered in [28, 29]. In [30], the fractional Hilbert transform was presented. The fractional Mellin transform [31, 32] was proposed to be used in image encryption. The fractional wavelet transform was presented and some applications were investigated in [33–35]. In [36], the fractional Hankel transform was reported in order to research the charge-amplitude state representations.

The Z-transform method [1, 2, 37] was applied to handle the linear time-invariant discrete-time systems (LTI discrete-time systems) and difference equations in Z-domain. However, the fractional derivative and integrals (the fractional PDIs) were used to transfer the fractional LTI discrete-time systems to Z-domain [38]. There appear signals defined on Cantor sets, which are the most striking properties of nondifferentiable functions. The classical Z-transform method and PDIs did not deal with them. In order to overcome them, local fractional calculus [39–43] may be applied to handle the function defined on Cantor sets shown in Figure 1. The local fractional integral transforms via local fractional calculus theory were proposed in [44–51]. For example, local fractional Fourier transforms reported in [40, 44] were used to find nondifferentiable solutions for local fractional ODEs and PDEs [45–47]. Laplace transforms via local fractional calculus [40] were generalized and reported in order to solve the local fractional ODEs and PDEs [48–50].

Fractal signal processing [51–59] is a hot topic for scientists and engineers. Very recently, the concept of the Z-transform method via local fractional calculus was considered only in [60]. However, there is no report on signal processing by using the local fractional Z-transforms. The main aim of this paper is to investigate the properties of local fractional Z-transforms and to present some examples for processing signals defined on Cantor sets.

The paper is organized as follows. In Section 2, the concepts of local fractional complex derivatives and integrals are given. In Section 3, the notions and properties of local

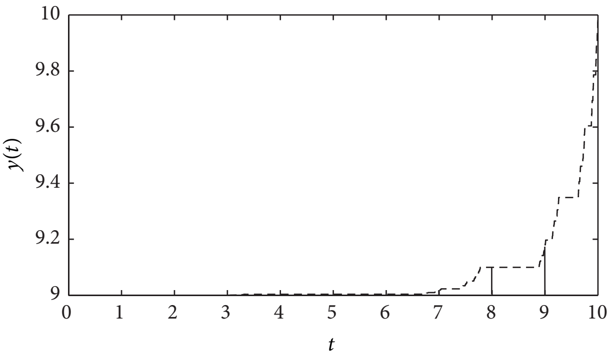

FIGURE 1: The chart of the signal $y(t)$ defined on Cantor sets.

fractional Z-transform method are presented. In Section 4, some examples and applications of this method are shown. Finally, Section 5 is the conclusions.

2. Local Fractional Derivatives and Integrals of Complex Functions and Recent Results

In this section, we introduce the concepts of local fraction derivative and integrals of complex functions. Let us first give the local fractional continuity of complex functions.

Definition 1 (see [40, 60]). The function $f(z)$ is said to be local fractional continuous at z_0 if there exists

$$\lim_{z\to z_0} f(z) = f(z_0). \tag{1}$$

There is the local fractional continuous relation in the following form:

$$f(z) \in C_\alpha(\mathfrak{R}), \tag{2}$$

where

$$\lim_{z\to z_0} f(z) = f(z_0), \quad z, z_0 \in \mathfrak{R}. \tag{3}$$

Definition 2 (see [40, 60]). The local fractional derivative of complex function $f(z)$ of order α is defined as

$$f^{(\alpha)}(z) = \frac{d^\alpha f(z)}{d^\alpha z} = \lim_{z\to z_0} \frac{\Delta^\alpha f(z)}{(z-z_0)^\alpha}, \quad \alpha \in (0,1], \tag{4}$$

where

$$\Delta^\alpha f(z) \cong \Gamma(1+\alpha)(f(z)-f(z_0)). \tag{5}$$

If the limit of (4) exists for all z_0 in a region $\mathfrak{R}$, then the complex function $f(z)$ is said to be local fractional analytic in a region $\mathfrak{R}$.

The properties of the local fractional derivatives of some complex functions are presented as follows [40]:

$$\begin{aligned} \frac{d^\alpha z^{k\alpha}}{dz^\alpha} &= \frac{\Gamma(1+k\alpha)}{\Gamma(1+(k-1)\alpha)} z^{(k-1)\alpha}, \\ \frac{d^\alpha E_\alpha(z^\alpha)}{dz^\alpha} &= E_\alpha(z^\alpha), \\ \frac{d^\alpha \sin_\alpha z^\alpha}{dz^\alpha} &= \cos_\alpha z^\alpha, \\ \frac{d^\alpha \cos_\alpha z^\alpha}{dz^\alpha} &= -\sin_\alpha z^\alpha, \end{aligned} \tag{6}$$

where

$$\begin{aligned} E_\alpha(z^\alpha) &= \sum_{k=0}^{\infty} \frac{z^{\alpha k}}{\Gamma(1+k\alpha)}, \\ \sin_\alpha z^\alpha &= \sum_{k=0}^{\infty} \frac{(-1)^k z^{\alpha(2k+1)}}{\Gamma[1+\alpha(2k+1)]}, \\ \cos_\alpha z^\alpha &= \sum_{k=0}^{\infty} \frac{(-1)^k z^{2\alpha k}}{\Gamma(1+2\alpha k)}. \end{aligned} \tag{7}$$

Definition 3 (see [40, 46–50, 60]). The local fractional integral of complex function $f(z)$ of order α along the closed contour C is defined as

$$I_C^\alpha f(z) = \frac{1}{\Gamma(1+\alpha)} \oint_C f(z)(dz)^\alpha, \quad \alpha \in (0,1]. \tag{8}$$

The properties of the local fractional integrals of some complex functions are suggested as follows [40]:

$$\begin{aligned} &\frac{1}{\Gamma(1+\alpha)} \int_C (f(z)+g(z))(dz)^\alpha \\ &\quad = \frac{1}{\Gamma(1+\alpha)} \int_C f(z)(dz)^\alpha + \frac{1}{\Gamma(1+\alpha)} \int_C g(z)(dz)^\alpha, \\ &\frac{1}{\Gamma(1+\alpha)} \int_{C_1+C_2} f(z)(dz)^\alpha \\ &\quad = \frac{1}{\Gamma(1+\alpha)} \int_{C_1} f(z)(dz)^\alpha + \frac{1}{\Gamma(1+\alpha)} \int_{C_2} f(z)(dz)^\alpha, \\ &\frac{1}{\Gamma(1+\alpha)} \int_{C_1} f(z)(dz)^\alpha = -\frac{1}{\Gamma(1+\alpha)} \int_{-C_1} f(z)(dz)^\alpha. \end{aligned} \tag{9}$$

Theorem 4 (see [40]). *If $f(z)$ is local fractional analytic within and on a simple closed contour C and a is any point interior to C, then we have*

$$\frac{1}{(2\pi)^\alpha i^\alpha} \cdot \left\{ \frac{1}{\Gamma(1+\alpha)} \oint_C \frac{f(z)}{(z-z_0)^\alpha} (dz)^\alpha \right\} = \frac{f(z_0)}{\Gamma(1+\alpha)}. \tag{10}$$

Proof. See [40]. □

Definition 5 (see [40, 60]). If z_0 is an isolated singular point of $f(z)$, then we have a local fractional Laurent series of $f(z)$ at $C : |z - z_0| < r$ given by

$$f(z) = \sum_{i=-\infty}^{\infty} a_k (z - z_0)^{k\alpha}. \quad (11)$$

The coefficient a_{-1} of $(z - z_0)^{-\alpha}$ is called the local fractional residue of $f(z)$ at $z = z_0$ and is frequently written as

$$\mathrm{Re}_{z=z_0} s \{f(z)\} = a_{-1}. \quad (12)$$

Theorem 6 (see [40]). *If $f(z)$ is local fractional analytic within and on the boundary C of a region $\mathfrak{R}$ except at a number of poles a within $\mathfrak{R}$, having a residue a_{-1}, then*

$$\frac{1}{(2\pi)^{\alpha} i^{\alpha}} \cdot \left\{ \frac{1}{\Gamma(1+\alpha)} \oint_C f(z) (dz)^{\alpha} \right\} = \mathrm{Re}_{z=z_0} s \{f(z)\}. \quad (13)$$

Proof. See [40]. □

Theorem 7 (see [40]). *If $f(z)$ is local fractional analytic within and on the boundary C of a region $\mathfrak{R}$ except at a number of poles a within $\mathfrak{R}$, having numbers of residues, then*

$$\frac{1}{(2\pi)^{\alpha} i^{\alpha}} \cdot \left\{ \frac{1}{\Gamma(1+\alpha)} \oint_C f(z) (dz)^{\alpha} \right\} = \sum_{i=1}^{n} \mathrm{Re}_{z=z_k} s \{f(z)\}. \quad (14)$$

Proof. See [40]. □

3. Local Fractional Z-Transforms and Their Properties

In this section, we give the local fractional Z-transforms and their properties.

Definition 8 (see [60]). Local fractional Z-transform of $f(n)$ of order α is defined as

$$Z_{\alpha} \{f(n)\} = F_{\alpha}(z) = \sum_{n=\infty}^{\infty} f(n) z^{-n\alpha}, \quad (15)$$

where the above formula is convergent.

For a given sequence, the set $\mathfrak{R}$ of values of z for which its local fractional Z-transform converges is called the region of convergence (ROC), namely,

$$\sum_{n=\infty}^{\infty} \left| f(n) z^{-n\alpha} \right| < \infty. \quad (16)$$

The inverse formula of local fractional Z-transform of $f(n)$ of order α reads as follows (see [60]):

$$\begin{aligned} Z_{\alpha}^{-1} \{F_{\alpha}(z)\} &= f(n) \\ &= \frac{1}{(2\pi i)^{\alpha} \Gamma(1+\alpha)} \oint_C F_{\alpha}(z) z^{(n-1)\alpha} (dz)^{\alpha}, \end{aligned} \quad (17)$$

where C is a counterclockwise closed fractal path encircling the origin and entirely in the region of convergence.

Let $Z_{\alpha}\{f(n)\} = F_{\alpha}(z)$ within the region of convergence $\mathfrak{R}_1$ and let $Z_{\alpha}\{g(n)\} = G_{\alpha}(z)$ within the region of convergence $\mathfrak{R}_2$.

Property 1 (linearity). We have

$$Z_{\alpha} \{f(n) + g(n)\} = F_{\alpha}(z) + G_{\alpha}(z) \quad (18)$$

within the region of convergence $\mathfrak{R}_1 \cap \mathfrak{R}_2$.

Proof. From (15) we have

$$\begin{aligned} Z_{\alpha} \{f(n) + g(n)\} &= \sum_{n=\infty}^{\infty} (f(n) + g(n)) z^{-n\alpha} \\ &= Z_{\alpha} \{f(n)\} + Z_{\alpha} \{g(n)\} \end{aligned} \quad (19)$$

within the region of convergence $\mathfrak{R}_1 \cap \mathfrak{R}_2$. □

Property 2 (time shifting). If the variable z has a useful interpretation in terms of time delay, then we have

$$Z_{\alpha} \{f(n-k)\} = z^{-z\alpha} F_{\alpha}(z). \quad (20)$$

Proof. From (15), we have

$$\begin{aligned} Z_{\alpha} \{f(n-k)\} &= \sum_{n=\infty}^{\infty} f(n-k) z^{-n\alpha} = \sum_{n=\infty}^{\infty} f(n) z^{-(n+k)\alpha} \\ &= z^{-k\alpha} \sum_{n=\infty}^{\infty} f(n) z^{-n\alpha} = z^{-k\alpha} Z_{\alpha} \{f(n)\}. \end{aligned} \quad (21)$$

□

Property 3 (frequency modulation). We have

$$Z_{\alpha} \{z_0^{n\alpha} f(n)\} = F_{\alpha}\left(\frac{z}{z_0}\right). \quad (22)$$

Proof. From (15), we have

$$\begin{aligned} Z_{\alpha} \{z_0^{n\alpha} f(n)\} &= \sum_{n=\infty}^{\infty} f(n) z_0^{n\alpha} z^{-n\alpha} \\ &= \sum_{n=\infty}^{\infty} f(n) \left(\frac{z}{z_0}\right)^{-n\alpha} = F_{\alpha}\left(\frac{z}{z_0}\right). \end{aligned} \quad (23)$$

□

4. Some Illustrative Examples

In this section, we give some samples for nondifferentiable signals defined on Cantor sets.

Example 1. Let us consider the following signal in the form:

$$f(n) = \delta_{\alpha}(n) = \begin{cases} 1, & n = 0, \\ 0, & n \neq 0. \end{cases} \quad (24)$$

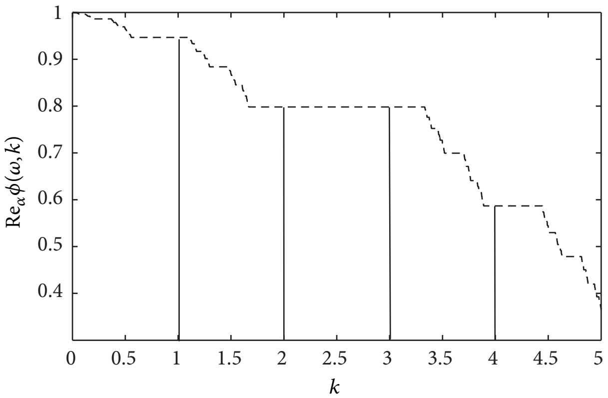

FIGURE 2: The graph of $\mathrm{Re}_\alpha\phi(\omega,k)$ with premasters $\omega = 1$ and $\alpha = \ln 2/\ln 3$.

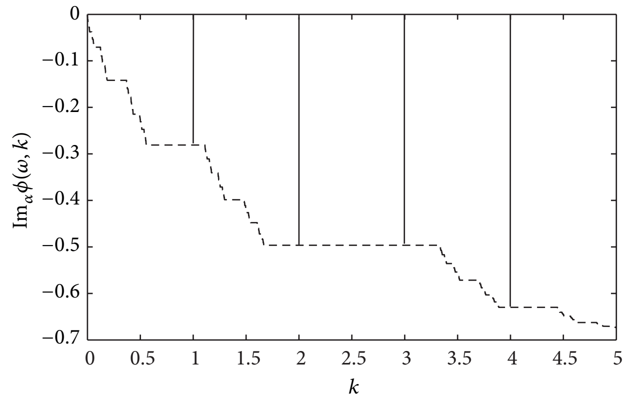

FIGURE 3: The plot of $\mathrm{Im}_\alpha\phi(\omega,k)$ with premasters $\omega = 1$ and $\alpha = \ln 2/\ln 3$.

Taking local fractional Z-transform, we have

$$Z_\alpha\{f(n)\} = Z_\alpha\{\delta_\alpha(n)\} = \sum_{n=\infty}^{\infty}\delta_\alpha(n)z^{-n\alpha} = 1. \quad (25)$$

Example 2. We now suggest the following signal in the form:

$$f(n) = \delta_\alpha(n-k) = \begin{cases}1, & n = k,\\ 0, & n \neq k.\end{cases} \quad (26)$$

Taking local fractional Z-transform, we obtain

$$Z_\alpha\{f(n)\} = Z_\alpha\{\delta_\alpha(n-k)\} = \sum_{n=\infty}^{\infty}\delta_\alpha(n-k)z^{-n\alpha} = z^{-k\alpha}. \quad (27)$$

When $z^\alpha = E_\alpha(j^\alpha\omega^\alpha)$ with the imaginary unit j^α [40, 44–50], we get

$$\begin{aligned}\phi(\omega,k) &= Z_\alpha\{f(n)\} = E_\alpha(-j^\alpha\omega^\alpha k^\alpha)\\ &= \cos_\alpha(\omega^\alpha k^\alpha) - j^\alpha\sin_\alpha(\omega^\alpha k^\alpha).\end{aligned} \quad (28)$$

Hence, from (28), we get

$$\begin{aligned}\mathrm{Re}_\alpha\phi(\omega,k) &= \cos_\alpha(\omega^\alpha k^\alpha),\\ \mathrm{Im}_\alpha\phi(\omega,k) &= -\sin_\alpha(\omega^\alpha k^\alpha)\end{aligned} \quad (29)$$

with the real part graph in Figure 2 and imaginary part graph in Figure 3.

Example 3. There is the signal in the following form:

$$f(n) = \delta_\alpha(n-k) + \delta_\alpha(n+k) = \begin{cases}1, & n = \pm k,\\ 0, & n \neq \pm k.\end{cases} \quad (30)$$

Taking local fractional Z-transform, we have

$$\begin{aligned}Z_\alpha\{f(n)\} &= Z_\alpha\{\delta_\alpha(n-k) + \delta_\alpha(n+k)\}\\ &= \sum_{n=\infty}^{\infty}(\delta_\alpha(n-k) + \delta_\alpha(n+k))z^{-n\alpha} = z^{-k\alpha} + z^{k\alpha}.\end{aligned} \quad (31)$$

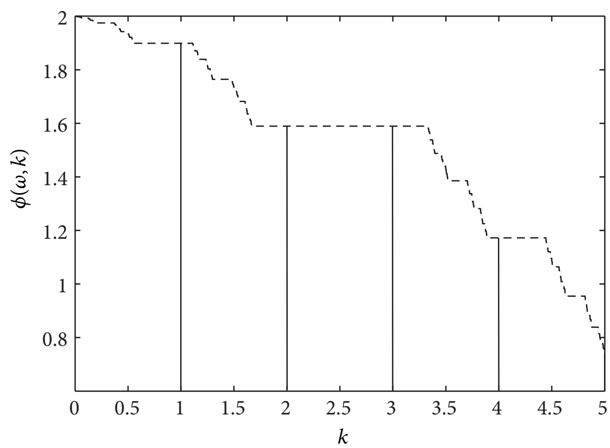

FIGURE 4: The map of $\phi(\omega,k)$ with premasters $\omega = 1$ and $\alpha = \ln 2/\ln 3$.

When $z^\alpha = E_\alpha(j^\alpha\omega^\alpha)$, we get

$$\phi(\omega,k) = Z_\alpha\{f(n)\} = E_\alpha(j^\alpha\omega^\alpha k^\alpha) = 2\cos_\alpha(\omega^\alpha k^\alpha) \quad (32)$$

with the graph of $\phi(\omega,k)$ shown in Figure 4.

Example 4. We have the following signal in the form:

$$f(n) = \begin{cases}a^{n\alpha}, & a \geq 0,\\ 0, & a < 0.\end{cases} \quad (33)$$

Local fractional Z-transform gives the following form:

$$Z_\alpha\{f(n)\} = Z_\alpha\{a^{n\alpha}\} = \sum_{n=0}^{\infty}a^{n\alpha}z^{-n\alpha} = \sum_{n=0}^{\infty}\left(\frac{z}{a}\right)^{-n\alpha} \quad (34)$$

with the region of convergence $|z| > |a|$.

Example 5. We consider the following signal in the form:

$$f(n) = \begin{cases}0, & a \geq 0,\\ a^{n\alpha}, & a < 0.\end{cases} \quad (35)$$

Taking local fractional Z-transform, we arrive at the following form:

$$Z_\alpha \{f(n)\} = Z_\alpha \{a^{n\alpha}\} = \sum_{n=\infty}^{0} a^{n\alpha} z^{-n\alpha} = \sum_{n=\infty}^{0} \left(\frac{z}{a}\right)^{-n\alpha} \tag{36}$$

with the region of convergence $|z| < |a|$.

Example 6. We present the following signal in the form:

$$f(n) = \begin{cases} b^{n\alpha}, & a \geq 0, \\ a^{n\alpha}, & a < 0. \end{cases} \tag{37}$$

Local fractional Z-transform gives the following form:

$$\begin{aligned} Z_\alpha \{f(n)\} &= Z_\alpha \{a^{n\alpha} + a^{n\alpha}\} = \sum_{n=0}^{\infty} b^{n\alpha} z^{-n\alpha} + \sum_{n=\infty}^{0} a^{n\alpha} z^{-n\alpha} \\ &= \sum_{n=0}^{\infty} \left(\frac{z}{b}\right)^{-n\alpha} + \sum_{n=\infty}^{0} \left(\frac{z}{a}\right)^{-n\alpha} \end{aligned} \tag{38}$$

with the region of convergence $|b| < |z| < |a|$.

5. Conclusions

In this work, we investigated the local fractional Z-transforms based on the local fractional complex calculus and some properties are also obtained. Some illustrative examples were also given. The obtained results show the accuracy and efficiency of the presented method.

Conflict of Interests

The authors declare that they have no competing interests in this paper.

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