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Research Article

On Properties of Class $A(n)$ and n-Paranormal Operators

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Let n be a positive integer, and an operator $T \in B(\mathscr{H})$ is called a class $A(n)$ operator if $|T^{1+n}|^{2/(1+n)} \geq |T|^2$ and n-paranormal operator if $\|T^{1+n}x\|^{1/(1+n)} \geq \|Tx\|$ for every unit vector $x \in \mathscr{H}$, which are common generalizations of class A and paranormal, respectively. In this paper, firstly we consider the tensor products for class $A(n)$ operators, giving a necessary and sufficient condition for $T \otimes S$ to be a class $A(n)$ operator when T and S are both non-zero operators; secondly we consider the properties for n-paranormal operators, showing that a n-paranormal contraction is the direct sum of a unitary and a $C_{.0}$ completely non-unitary contraction.

1. Introduction

Let $\mathscr{H}$ be a separable complex Hilbert space and let $\mathscr{C}$ be the set of complex numbers. Let $B(\mathscr{H})$ denote the C^*-algebra of all bounded linear operators acting on $\mathscr{H}$. If $T \in B(\mathscr{H})$, we will write $\ker T$ and $\operatorname{ran}T$ for the null space and range of T, respectively. Also let $\alpha(T) = \dim\ker T$, $\beta(T) = \dim\ker T^*$ and let $\sigma(T)$, $\sigma_p(T)$ denote the spectrum, point spectrum of T. Let $p = p(T)$ be the ascent of T, that is, the smallest nonnegative integer p such that $\ker T^p = \ker T^{p+1}$. If such integer does not exist, we put $p(T) = \infty$. Analogously, let $q = q(T)$ be the descent of T, that is, the smallest nonnegative integer q such that $\operatorname{ran}T^q = \operatorname{ran}T^{q+1}$, and if such integer does not exist, we put $q(T) = \infty$. An operator $T \in B(\mathscr{H})$ is called upper (lower, resp.) semi-Fredholm if $\operatorname{ran}T$ is closed and $\alpha(T) < \infty$ ($\beta(T) < \infty$, resp.). If $T \in B(\mathscr{H})$ is either an upper semi-Fredholm operator or a lower semi-Fredholm operator, then T is called a semi-Fredholm operator, and the index of a semi-Fredholm operator $T \in B(\mathscr{H})$, denoted by $\operatorname{ind}(T)$, is given by the integer $\operatorname{ind}(T) = \alpha(T) - \beta(T)$. If both $\alpha(T)$ and $\beta(T)$ are finite, then T is called a Fredholm operator. An operator $T \in B(\mathscr{H})$ is called Weyl if it is Fredholm of index zero and Browder if it is Fredholm of finite ascent and descent. The essential spectrum $\sigma_e(T)$, the Weyl spectrum $\sigma_w(T)$, and the Browder spectrum $\sigma_b(T)$ of $T \in B(\mathscr{H})$ are defined by $\sigma_e(T) = \{\lambda \in \mathscr{C} : T - \lambda \text{ is not Fredholm}\}$, $\sigma_w(T) = \{\lambda \in \mathscr{C} : T - \lambda \text{ is not Weyl}\}$, and $\sigma_b(T) = \{\lambda \in \mathscr{C} : T - \lambda \text{ is not Browder}\}$.

Let $\mathscr{H}$, $\mathscr{K}$ be complex Hilbert spaces and $\mathscr{H} \otimes \mathscr{K}$ the tensor product of $\mathscr{H}$, $\mathscr{K}$, that is, the completion of the algebraic tensor product of $\mathscr{H}$, $\mathscr{K}$ with the inner product $\langle x_1 \otimes y_1, x_2 \otimes y_2\rangle = \langle x_1, x_2\rangle\langle y_1, y_2\rangle$ for $x_1, x_2 \in \mathscr{H}$, $y_1, y_2 \in \mathscr{K}$. Let $T \in B(\mathscr{H})$ and $S \in B(\mathscr{K})$. $T \otimes S \in B(\mathscr{H} \otimes \mathscr{K})$ denotes the tensor product of T and S; that is, $(T \otimes S)(x \otimes y) = Tx \otimes Sy$ for $x \in \mathscr{H}$, $y \in \mathscr{K}$.

A contraction is an operator T such that $\|T\| \leq 1$; equivalently, $\|Tx\| \leq \|x\|$ for every $x \in \mathscr{H}$. A contraction T is said to be a proper contraction if $\|Tx\| < \|x\|$ for every nonzero $x \in \mathscr{H}$. A strict contraction is an operator T such that $\|T\| < 1$. A strict contraction is a proper contraction, but a proper contraction is not necessarily a strict contraction, although the concepts of strict and proper contraction coincide for compact operators. A contraction T is of class $C_{0.}$ if $\|T^n x\| \to 0$ when $n \to \infty$ for every $x \in \mathscr{H}$ (i.e., T is a strongly stable contraction) and it is said to be of class $C_{1.}$ if $\lim_{n\to\infty}\|T^n x\| > 0$ for every nonzero $x \in \mathscr{H}$. Classes $C_{.0}$ and $C_{.1}$ are defined by considering T^* instead of T and we define the class $C_{\alpha\beta}$ for $\alpha, \beta = 0, 1$ by $C_{\alpha\beta} = C_{\alpha.} \bigcap C_{.\beta}$. An isometry is a contraction for which $\|Tx\| = \|x\|$ for every $x \in \mathscr{H}$.

Recall that $T \in B(\mathscr{H})$ is called p-hyponormal for $p > 0$ if $(T^*T)^p - (TT^*)^p \geq 0$ [1]; when $p = 1$, T is called hyponormal.

And T is called paranormal if $\|Tx\|^2 \le \|T^2x\|\|x\|$ for all $x \in \mathscr{H}$ [2, 3]. And T is called normaloid if $\|T^n\| = \|T\|^n$ for all $n \in \mathbb{N}$ (equivalently, $\|T\| = r(T)$, the spectral radius of T). In order to discuss the relations between paranormal operators and p-hyponormal and log-hyponormal operators (T is invertible and $\log T^*T \ge \log TT^*$), Furuta et al. [4] introduced a very interesting class of operators: class A defined by $|T^2| - |T|^2 \ge 0$, where $|T| = (T^*T)^{1/2}$ which is called the absolute value of T, and they showed that class A is a subclass of paranormal and contains p-hyponormal and log-hyponormal operators. Recently Yuan and Gao [5] introduced class $A(n)$ (i.e., $|T^{1+n}|^{2/(1+n)} \ge |T|^2$) operators and n-paranormal operators (i.e., $\|T^{1+n}x\|^{1/(1+n)} \ge \|Tx\|$ for every unit vector $x \in \mathscr{H}$) for some positive integer n.

For more interesting properties on class $A(n)$ and n-paranormal operators, see [5–8].

In general, the following implications hold:

$$p\text{-hyponormal} \subseteq \text{class } A \subseteq \text{paranormal} \subseteq n\text{-paranormal},$$
$$p\text{-hyponormal} \subseteq \text{class } A \subseteq \text{class } A(n) \subseteq n\text{-paranormal}. \tag{1}$$

In this paper, firstly we consider the tensor products for class $A(n)$ operators, giving a necessary and sufficient condition for $T \otimes S$ to be a class $A(n)$ operator when T and S are both nonzero operators; secondly we consider the properties for n-paranormal operators, showing that a n-paranormal contraction is the direct sum of a unitary and a $C_{.0}$ completely nonunitary contraction.

2. Tensor Products for Class $A(n)$ Operators

Let $T \otimes S$ denote the tensor product on the product space $\mathscr{H} \otimes \mathscr{K}$ for nonzero $T \in B(\mathscr{H})$ and $S \in B(\mathscr{K})$. The operation of taking tensor products $T \otimes S$ preserves many properties of $T \in B(\mathscr{H})$ and $S \in B(\mathscr{K})$, but it was not always this way. For example, the normaloid property is invariant under tensor products, the spectraloid property is not (see [9, pp. 623 and 631]), and $T \otimes S$ is normal if and only if T and S are normal [10, 11]; however, there exist paranormal operators $T \in B(\mathscr{H})$ and $S \in B(\mathscr{K})$ such that $T \otimes S$ is not paranormal [12]. Duggal [13] showed that for nonzero $T \in B(\mathscr{H})$ and $S \in B(\mathscr{K})$, $T \otimes S$ is p-hyponormal if and only if T, S are p-hyponormal. This result was extended to p-quasihyponormal operators, class A operators, log-hyponormal operators, and class $A(s,t)$ operators ($(|T^*|^t|T|^{2s}|T^*|^t)^{t/(s+t)} \ge |T^*|^{2t}$, $s, t > 0$) in [14–16], respectively. The following theorem gives a necessary and sufficient condition for $T \otimes S$ to be a class $A(n)$ operator when T and S are both nonzero operators.

Theorem 1. *Let $T \in B(\mathscr{H})$ and $S \in B(\mathscr{K})$ be nonzero operators. Then $T \otimes S \in B(\mathscr{H} \otimes \mathscr{K})$ is a class $A(n)$ operator if and only if T and S are class $A(n)$ operators.*

Proof. It is clear that $T \otimes S$ is a class $A(n)$ operator if and only if

$$\begin{aligned} &\left|(T\otimes S)^{1+n}\right|^{2/(1+n)} \ge |T\otimes S|^2 \\ &\Longleftrightarrow \left|T^{1+n}\otimes S^{1+n}\right|^{2/(1+n)} \ge |T|^2 \otimes |S|^2 \\ &\Longleftrightarrow \left(\left|T^{1+n}\right|^{2/(1+n)} - |T|^2\right) \\ &\quad \otimes \left|S^{1+n}\right|^{2/(1+n)} + |T|^2 \\ &\quad \otimes \left(\left|S^{1+n}\right|^{2/(1+n)} - |S|^2\right) \ge 0. \end{aligned} \tag{2}$$

Therefore, the sufficiency is clear.

Conversely, suppose that $T \otimes S$ is a class $A(n)$ operator. Let $x \in \mathscr{H}$ and $y \in \mathscr{K}$ be arbitrary. Then we have

$$\begin{aligned} &\left\langle\left(\left|T^{1+n}\right|^{2/(1+n)} - |T|^2\right)x, x\right\rangle \left\langle \left|S^{1+n}\right|^{2/(1+n)} y, y\right\rangle \\ &\quad + \left\langle |T|^2 x, x\right\rangle \left\langle\left(\left|S^{1+n}\right|^{2/(1+n)} - |S|^2\right) y, y\right\rangle \ge 0. \end{aligned} \tag{3}$$

On the contrary, assume that T is not a class $A(n)$ operator; then there exists $x_0 \in \mathscr{H}$ such that

$$\begin{aligned} \left\langle\left(\left|T^{1+n}\right|^{2/(1+n)} - |T|^2\right)x_0, x_0\right\rangle &= \alpha < 0, \\ \left\langle |T|^2 x_0, x_0\right\rangle &= \beta > 0. \end{aligned} \tag{4}$$

From (3), we have

$$\alpha \left\langle \left|S^{1+n}\right|^{2/(1+n)} y, y\right\rangle + \beta \left\langle\left(\left|S^{1+n}\right|^{2/(1+n)} - |S|^2\right) y, y\right\rangle \ge 0 \tag{5}$$

for all $y \in \mathscr{K}$; that is,

$$(\alpha+\beta)\left\langle \left|S^{1+n}\right|^{2/(1+n)} y, y\right\rangle \ge \beta \left\langle |S|^2 y, y\right\rangle \tag{6}$$

for all $y \in \mathscr{K}$. Therefore, S is a class $A(n)$ operator. We have

$$\begin{aligned} \left\langle |S|^2 y, y\right\rangle &= \|Sy\|^2, \\ \left\langle \left|S^{1+n}\right|^{2/(1+n)} y, y\right\rangle &= \left\|\left|S^{1+n}\right|^{1/(1+n)} y\right\|^2. \end{aligned} \tag{7}$$

So we have

$$(\alpha+\beta)\left\|\left|S^{1+n}\right|^{1/(1+n)} y\right\|^2 \ge \beta\|Sy\|^2 \tag{8}$$

for all $y \in \mathscr{K}$ by (6). By (8), we have

$$(\alpha+\beta)\left\|\left|S^{1+n}\right|^{1/(1+n)}\right\|^2 \ge \beta\|S\|^2. \tag{9}$$

Since self-adjoint operators are normaloid, we have

$$\left\|\left|S^{1+n}\right|^{1/(1+n)}\right\|^{1+n} = \left\|\left(\left|S^{1+n}\right|^{1/(1+n)}\right)^{1+n}\right\| = \left\|S^{1+n}\right\| \le \|S\|^{1+n}. \tag{10}$$

Hence, we have

$$\left\| \left| S^{1+n} \right|^{1/(1+n)} \right\| \le \|S\|. \tag{11}$$

By (9) and (11), we have

$$(\alpha + \beta) \|S\|^2 \ge \beta \|S\|^2. \tag{12}$$

This implies that $S = 0$. This contradicts the assumption $S \ne 0$. Hence T must be a class $A(n)$ operator. A similar argument shows that S is also a class $A(n)$ operator. The proof is complete. □

3. On n-Paranormal Operators

An operator $T \in B(\mathscr{H})$ is said to have the single valued extension property (SVEP) at $\lambda \in \mathbb{C}$ if, for every open neighborhood $\mathscr{G}$ of λ, the only function $f \in H(\mathscr{G})$ such that $(T - \mu) f(\mu) = 0$ on G is $0 \in H(\mathscr{G})$, where $H(\mathscr{G})$ means the space of all analytic functions on G. When T has SVEP at each $\lambda \in \mathbb{C}$, say that T has SVEP.

In the following, we consider the properties of n-paranormal operators. References [17, 18] showed that paranormal contractions and $*$-paranormal contractions in $B(\mathscr{H})$ are the direct sum of a unitary and a $C_{.0}$ contraction. In the following theorem, we extend this result to n-paranormal operators.

Theorem 2 (see [19]). *Let T be a contraction of n-paranormal operators for a positive integer n. Then T is the direct sum of a unitary and a $C_{.0}$ completely nonunitary contraction.*

Proof. If T is a contraction, then the sequence $\{T^k T^{*k}\}$ is a decreasing sequence of self-adjoint operators, converging strongly to a contraction. Let $A = (\lim_{k\to\infty} T^k T^{*k})^{1/2}$. A is self-adjoint and $0 \le A \le I$ and $TA^2T^* = A^2$. By [20] we have that there exists an isometry $V: \overline{\operatorname{ran}(A)} \to \overline{\operatorname{ran}(A)}$ such that $VA = AT^*$ on $\overline{\operatorname{ran}(A)}$ and $\|AV^n x\| \to \|x\|$ for every $x \in \overline{\operatorname{ran}(A)}$. V can be extended to a bounded linear operator on $\mathscr{H}$; we still denote it by V. Let $x_k = AV^k x$, $k \in \mathbb{N} \cup \{0\}$. Then for all nonnegative integers m,

$$T^m x_{m+k} = T^m A V^{k+m} x = A V^{*m} V^{k+m} x = A V^k x = x_k. \tag{13}$$

So we have, for all $m \le k$, $T^m x_k = x_{k-m}$. The sequence $\{\|x_n\|\}$ is a bounded above increasing sequence. In the following, we will prove that if T is n-paranormal for a positive integer n, then A is a projection. Firstly we prove that $\{x_k\}$ is a constant sequence. Suppose that T is a n-paranormal operator for a positive integer n. Then, for all $k \ge 1$ and nonzero $x \in \overline{\operatorname{ran}(A)}$,

$$\begin{aligned} \|x_k\|^2 = \|Tx_{k+1}\|^2 &\le \left\| T^{1+n} x_{k+1} \right\|^{2/(1+n)} \left\| x_{k+1} \right\|^{2n/(1+n)} \\ &= \left\| x_{k+1-(1+n)} \right\|^{2/(1+n)} \left\| x_{k+1} \right\|^{2n/(1+n)} \\ &= \left\| x_{k-n} \right\|^{2/(1+n)} \left\| x_{k+1} \right\|^{2n/(1+n)}, \end{aligned} \tag{14}$$

so we have

$$\begin{aligned} \|x_k\| &\le \left\| x_{k-n} \right\|^{1/(n+1)} \left\| x_{k+1} \right\|^{n/(n+1)} \\ &\le \frac{1}{n+1} \left(\left\| x_{k-n} \right\| + n \left\| x_{k+1} \right\| \right). \end{aligned} \tag{15}$$

Hence,

$$\begin{aligned} n \left(\|x_{k+1}\| - \|x_k\| \right) \ge \|x_k\| - \|x_{k-n}\| \\ = \left(\|x_k\| - \|x_{k-1}\| \right) + \left(\|x_{k-1}\| \, \| - \|x_{k-2}\| \right) \\ + \cdots + \left(\|x_{k-n+1}\| - \|x_{k-n}\| \right). \end{aligned} \tag{16}$$

Putting $b_k = \|x_k\| - \|x_{k-1}\|$, we have that

$$n b_{k+1} \ge b_k + b_{k+1} + \cdots + b_{k-n+1}, \tag{17}$$

where $b_k \ge 0$ and $b_k \to 0$ as $k \to \infty$. Suppose that there exists an integer $i \ge 1$ such that $b_i > 0$; then $b_{i+1} \ge (b_i/n) > 0$, and we have that $b_k \ge (b_i/n) > 0$, for all $k > i$ by an induction argument. This is contradictory with the fact that $b_k \to 0$ as $k \to \infty$. Consequently, we have that $b_k = 0$ for all k, which implies that $\|x_{k-1}\| = \|x_k\|$ for all $k \ge 1$. This means that for all $x \in \overline{\operatorname{ran}(A)} \|AV^k x\| = \|Ax\| = \|x\|$. So we have that $A^2 = I$ on $\operatorname{ran}(A)$, and so $A = I$ on $\overline{\operatorname{ran}(A)}$. Therefore, we have that $A = \left(\begin{smallmatrix} I & 0 \\ 0 & 0 \end{smallmatrix}\right)$ on $\mathscr{H} = \overline{\operatorname{ran}(A)} \oplus \ker(A)$. Hence A is a projection. By [21], we have that if A is a projection, then T has a decomposition:

$$T = T_u \oplus T_c, \qquad T_c = S^* \oplus T_0, \tag{18}$$

where T_u is unitary and the completely nonunitary part T_c of T is the direct sum of backward unilateral shift S^* and a $C_{.0}$-contraction T_0. We will prove that S^* is missing from the direct sum. It is well known that an operator $B = B_1 \oplus B_2$ has SVEP at a point λ if and only if B_1 and B_2 have SVEP at the point λ. Since n-paranormal operators have SVEP by [6, Corollary 3.4], it follows that if S^* is present in the direct sum of T, then it has SVEP. This contradicts the fact that the backward unilateral shift does not have SVEP anywhere on its spectrum except for the boundary point of its spectrum. Therefore, $T = T_u \oplus T_0$. The proof is complete. □

In the following, we give a sufficient condition for a n-paranormal contraction to be proper.

Theorem 3. *Let T be a contraction of n-paranormal operators for a positive integer n. If T has no nontrivial invariant subspace, then T is a proper contraction.*

Proof. Suppose that T is a n-paranormal operator, then $\|T^{1+n}x\|\|x\|^n \ge \|Tx\|^{n+1}$ for all $x \in \mathscr{H}$. By [22, Theorem 3.6], we have that

$$T^*Tx = \|T\|^2 x \quad \text{if and only if} \quad \|Tx\| = \|T\| \, \|x\|. \tag{19}$$

Put $\mathscr{U} = \{x \in \mathscr{H} : \|Tx\| = \|T\|\|x\|\} = \ker(|T|^2 - \|T\|^2)$, which is a subspace of $\mathscr{H}$. In the following, we will show that

$\mathscr{U}$ is an invariant subspace of T. For every $x \in \mathscr{U}$, if T is a n-paranormal operator, we have

$$\|T\|^{n+1}\|x\|^{n+1} = \|Tx\|^{1+n} \le \left\|T^{1+n}x\right\| \|x\|^{n} \le \|T\|^{n-1}\left\|T^2x\right\| \|x\|^{n}. \quad (20)$$

By (20) we have $\|T^2x\| \ge \|T\|^2\|x\|$. So we have that

$$\|T(Tx)\| = \|T\| \|Tx\|. \quad (21)$$

That is, $\mathscr{U}$ is an invariant subspace of T. Now suppose that T is a contraction of n-paranormal operators. If T is a strict contract, then it is trivially a proper contraction. If T is not a strict contraction (i.e., $\|T\| = 1$) and T has no nontrivial invariant subspace, then $\mathscr{U} = \{x \in \mathscr{H} : \|Tx\| = \|x\|\} = \{0\}$ (actually, if $\mathscr{U} = \mathscr{H}$, then T is an isometry, and isometries have nontrivial invariant subspaces). Thus for every nonzero $x \in \mathscr{H}$, $\|Tx\| < \|x\|$, so T is a proper contraction. The proof is complete. □

Uchiyama [23] showed that if T is paranormal and $w(T) = 0$, then T is compact and normal. Now we extend this result to n-paranormal operators.

Theorem 4. *Let T be a n-paranormal operator for a positive integer n and $\sigma_w(T) = \{0\}$. Then T is compact and normal.*

Proof. By [5, Theorem 2.1], we have that

$$\frac{\sigma(T)}{w(T)} = \frac{\sigma(T)}{\{0\}} \subseteq \pi_{00}(T), \quad (22)$$

where $\pi_{00}(T)$ is the set of all isolated points which are eigenvalues of T with finite multiplicities. This implies that $\sigma(T)\backslash w(T)$ is a finite set or a countable infinite set with 0 as its only accumulation point. Let $\sigma(T)\backslash\{0\} = \{\lambda_n\}$, where $\lambda_n \ne \lambda_m$ whenever $n \ne m$ and $\{|\lambda_n|\}$ is a nonincreasing sequence. By [8, Proposition 1], we have that T is normaloid. So we have $|\lambda_1| = \|T\|$. By the general theory, $(T-\lambda_1)x = 0$ implies $(T-\lambda_1)^*x = 0$. In fact,

$$\left\|\left(\|T\|^2 - T^*T\right)^{1/2}x\right\|^2 = \|T\|^2\|x\|^2 - \|Tx\|^2 = \|T\|^2\|x\|^2 - \left\|\lambda_1 x\right\|^2 = 0. \quad (23)$$

Thus $\lambda_1 T^*x = T^*Tx = \|T\|^2x = |\lambda_1|^2x$ and $T^*x = \overline{\lambda_1}x$. Therefore, $\ker(T-\lambda_1)$ is a reducing subspace of T. Let E_1 be the orthogonal projection onto $\ker(T-\lambda_1)$. Then $T = \lambda_1 \oplus T_1$ on $\mathscr{H} = E_1\mathscr{H} \oplus (I-E_1)\mathscr{H}$. Since T_1 is n-paranormal and $\sigma_p(T) = \sigma_p(T_1) \cup \{\lambda_1\}$, we have that $\lambda_2 \in \sigma_p(T_1)$. By the same argument as above, $\ker(T-\lambda_2) = \ker(T_1-\lambda_2)$ is a finite dimensional reducing subspace of T which is included in $(I-E_1)\mathscr{H}$. Let E_2 be the orthogonal projection onto $\ker(T-\lambda_2)$. Then $T = \lambda_1 E_1 \oplus \lambda_2 E_2 \oplus T_2$ on $\mathscr{H} = E_1\mathscr{H} \oplus E_2\mathscr{H} \oplus (I-E_1-E_2)\mathscr{H}$. By the same argument, each $\ker(T-\lambda_n)$ is a reducing subspace of T and $\|T - \bigoplus_{k=1}^{n}\lambda_k E_k\| = \|T_n\| = |\lambda_{n+1}| \to 0$ as $n \to \infty$. Here E_k is the orthogonal projection onto $\ker(T-\lambda_k)$ and $T = (\bigoplus_{k=1}^{n}\lambda_k E_k) \oplus T_n$ on $\mathscr{H} = (\lambda_1 E_1 \bigoplus_{k=1}^{n} E_k\mathscr{H}) \oplus (I - \Sigma_{k=1}^{n}E_k)\mathscr{H}$. Hence $T = \bigoplus_{k=1}^{\infty}\lambda_k E_k$ is compact and normal because each E_k is a finite rank orthogonal projection which satisfies $E_kE_l = 0$ whenever $k \ne l$ by [5, Lemma 2.5] and $\lambda_n \to 0$ as $n \to \infty$. The proof is complete. □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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