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Research Article

A Generalization of a Greguš Fixed Point Theorem in Metric Spaces

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We first introduce a new class of contractive mappings in the setting of metric spaces and then we present certain Greguš type fixed point theorems for such mappings. As an application, we derive certain Greguš type common fixed theorems. Our results extend Greguš fixed point theorem in metric spaces and generalize and unify some related results in the literature. An example is also given to support our main result.

1. Introduction and Preliminaries

Let X be a Banach space and let C be a closed convex subset of X. In 1980 Greguš [1] proved the following result.

Theorem 1. *Let $T : C \to C$ be a mapping satisfying the inequality*

$$\|Tx - Ty\| \le a\|x - y\| + b\|x - Tx\| + c\|y - Ty\|, \tag{1}$$

for all $x, y \in C$, where $0 < a < 1$, $b, c \ge 0$, and $a + b + c = 1$. Then T has a unique fixed point.

Fisher and Sessa [2], Jungck [3], and Hussain et al. [4] obtained common fixed point generalizations of Theorem 1. In recent years, many theorems which are closely related to Greguš's Theorem have appeared (see [1–21]). Very recently, Moradi and Farajzadeh [21] extended Greguš fixed point theorem in complete metric spaces.

Theorem 2 ([21, Theorem 2.4]). *Let (X, d) be a complete metric space and let $T : X \to X$ be a mapping such that, for all $x, y \in X$,*

$$\begin{aligned} d(Tx, Ty) \le{}& ad(x, y) + bd(x, Tx) \\ &+ cd(y, Ty) + ed(y, Tx) + fd(x, Ty), \end{aligned} \tag{2}$$

where $0 < a < 1$, $b, c, e, f \ge 0$, $b + c > 0$, $e + f > 0$, and $a + b + c + e + f = 1$. Then T has a unique fixed point.

Let I and T be self-maps of X. A point $x \in X$ is a coincidence point (resp., common fixed point) of I and T if $Ix = Tx$ (resp., $x = Ix = Tx$). The pair $\{I, T\}$ is called (1) commuting if $TIx = ITx$ for all $x \in X$; (2) weakly commuting [2] if, for all $x \in X$, $d(ITx, TIx) \le d(Ix, Tx)$; (3) compatible [3] if $\lim_n d(TIx_n, ITx_n) = 0$ whenever $\{x_n\}$ is a sequence such that $\lim_n Tx_n = \lim_n Ix_n = t$ for some t in X; (4) weakly compatible if they commute at their coincidence points, that is, if $ITx = TIx$ whenever $Ix = Tx$. Clearly, commuting maps are weakly commuting, and weakly commuting maps are compatible. References [2, 3] give examples which show that neither implication is reversible.

The purpose of this paper is to define and to investigate a class of new generalized contractive mappings (not necessarily continuous) on metric spaces. We will prove certain fixed point and common fixed results which are generalizations of the above mentioned theorems.

2. Fixed Point Results

We denote by $\mathbb{R}_+$ the set of all nonnegative real numbers and by $\mathcal{U}$ the set of all functions $u : \mathbb{R}_+^5 \to \mathbb{R}_+$ satisfying the following conditions:

(C_1) u is continuous,

(C_2) $u(t_1, t_2, t_3, t_4, t_5)$ is nondecreasing in t_1, t_2, t_3, and t_5,

(C_3) $s < t \Rightarrow u(s, s, t, 0, s + t) < t$, for each $s, t > 0$,

(C_4) $u(t, t, t, 0, u(2t, t, t, t, 3t)) < t$, for each $t > 0$,

(C_5) $u(t, 0, 0, t, t) < t$ for each $t > 0$.

Example 3. If $u(t_1, t_2, t_3, t_4, t_5) = at_1 + bt_2 + bt_3 + et_4 + et_5$ for $t_i \in \mathbb{R}_+$, where $0 < a < 1$, $b, e > 0$, and $a + 2b + 2e = 1$, then $u \in \mathcal{U}$.

Example 4. If $u(t_1, t_2, t_3, t_4, t_5) = k \max\{t_1, t_2, t_3, (t_4 + t_5)/2\}$ for $t_i \in \mathbb{R}_+$, where $k \in [0, 1)$, then $u \in \mathcal{U}$.

Example 5. Let $u \in \mathcal{U}$. Then it is easy to see that $w \in \mathcal{U}$ where

$$w(t_1, t_2, t_3, t_4, t_5) = u(t_1, t_2, t_3, t_4, t_5) + L \min\{t_1, t_2, t_3, t_4, t_5\}, \tag{3}$$

for each $L \geq 0$.

Now we are ready to state our main result.

Theorem 6. *Let (X, d) be a complete metric space and let $T : X \to X$ be a mapping satisfying*

$$d(Tx, Ty) \leq u(d(x, y), d(x, Tx), d(y, Ty), d(y, Tx), d(x, Ty)), \tag{4}$$

for each $x, y \in X$, where $u \in \mathcal{U}$. Then T has a unique fixed point.

Proof. We first show that $\alpha = \inf_{x \in X} d(x, Tx) = 0$. If $d(x, Tx) = 0$ for some $x \in X$, then x is a fixed point of T and we are done. So, we may assume that $d(x, Tx) > 0$ for each $x \in X$. From (4), (C_2), and (C_3), we have

$$\begin{aligned} d(Tz, T^2z) &\leq u(d(z, Tz), d(z, Tz), d(Tz, T^2z), 0, d(z, T^2z)) \\ &\leq u(d(z, Tz), d(z, Tz), d(Tz, T^2z), 0, \\ &\qquad d(z, Tz) + d(Tz, T^2z)), \end{aligned} \tag{5}$$

and so

$$d(Tz, T^2z) \leq d(z, Tz), \quad \text{for each } z \in X. \tag{6}$$

Now let $\{x_n\}$ be a sequence such that

$$\alpha = \inf_{x \in X} d(x, Tx) = \lim_{n \to \infty} d(x_n, Tx_n). \tag{7}$$

From (4), (6), and (C_2), we get

$$\begin{aligned} d(Tx_n, T^3x_n) &\leq u(d(x_n, T^2x_n), d(x_n, Tx_n), d(T^2x_n, T^3x_n), \\ &\qquad d(T^2x_n, Tx_n), d(x_n, T^3x_n)) \\ &\leq u(d(x_n, Tx_n) + d(Tx_n, T^2x_n), d(x_n, Tx_n), \\ &\qquad d(T^2x_n, T^3x_n), d(T^2x_n, Tx_n), d(x_n, Tx_n) \\ &\qquad + d(Tx_n, T^2x_n) + d(T^2x_n, T^3x_n)) \\ &\leq u(2d(x_n, Tx_n), d(x_n, Tx_n), d(x_n, Tx_n), \\ &\qquad d(x_n, Tx_n), 3d(x_n, Tx_n)), \end{aligned} \tag{8}$$

for each $n \in \mathbb{N}$. From (4), (6), (8), and (C_2), we have

$$\begin{aligned} d(T^2x_n, T^3x_n) &\leq u(d(Tx_n, T^2x_n), d(Tx_n, T^2x_n), \\ &\qquad d(T^2x_n, T^3x_n), 0, d(Tx_n, T^3x_n)) \\ &\leq u(d(x_n, Tx_n), d(x_n, Tx_n), d(T^2x_n, T^3x_n), 0, \\ &\qquad u(2d(x_n, Tx_n), d(x_n, Tx_n), d(x_n, Tx_n), \\ &\qquad d(x_n, Tx_n), 3d(x_n, Tx_n))), \end{aligned} \tag{9}$$

for each $n \in \mathbb{N}$. From (6) and (7), we get

$$\alpha \leq d(T^2x_n, T^3x_n) \leq d(x_n, Tx_n), \tag{10}$$

and so by (7)

$$\lim_{n \to \infty} d(T^2x_n, T^3x_n) = \alpha. \tag{11}$$

From (7), (9), (11), and (C_1), we obtain

$$\alpha \leq u(\alpha, \alpha, \alpha, 0, u(2\alpha, \alpha, \alpha, \alpha, 3\alpha)). \tag{12}$$

Hence by (C_4)

$$\alpha = \inf_{x \in X} d(x, Tx) = 0. \tag{13}$$

Now, let

$$C_n = \left\{x \in X : d(x, Tx) \leq \frac{1}{n}\right\}, \quad \text{for each } n \in \mathbb{N}. \tag{14}$$

Notice that, by (13), $C_n \neq \emptyset$ for each $n \in \mathbb{N}$. We show that

$$\lim_{n \to \infty} \operatorname{diam}(\overline{C_n}) = 0. \tag{15}$$

On the contrary, assume that there are sequences $\{x_n\}$ and $\{y_n\}$ with $x_n, y_n \in C_n$ satisfying

$$\rho = \lim_{n \to \infty} d(x_n, y_n) > 0. \tag{16}$$

From (4), (14), and (C_2), we have

$$\begin{aligned} d(x_n,y_n) &\le d(x_n,Tx_n)+d(Tx_n,Ty_n)+d(y_n,Ty_n) \\ &\le \frac{2}{n}+u\big(d(x_n,y_n),d(x_n,Tx_n),d(y_n,Ty_n), \\ &\qquad d(y_n,Tx_n),d(x_n,Ty_n)\big) \\ &\le \frac{2}{n}+u\big(d(x_n,y_n),d(x_n,Tx_n),d(y_n,Ty_n), \\ &\qquad d(x_n,y_n)+d(x_n,Tx_n), \\ &\qquad d(x_n,y_n)+d(y_n,Ty_n)\big), \end{aligned} \tag{17}$$

for each $n \in \mathbb{N}$. From (14), (16), (17), and (C_1), we get

$$\rho \le u(\rho,0,0,\rho,\rho), \tag{18}$$

which contradicts (C_5). Thus (15) holds. Hence $\{\overline{C_n}\}$ is a decreasing sequence of closed nonempty sets with $\operatorname{diam}(\overline{C_n}) \to 0$ and so, by Cantor's intersection theorem,

$$\bigcap_{n=1}^{\infty}\overline{C_n}=\{\overline{x}\} \quad \text{for some } \overline{x}\in X. \tag{19}$$

We show that $\overline{x}$ is a fixed point of T. Since $\overline{x}\in\overline{C_n}$, there exists $u_n \in C_n$ such that $d(\overline{x},u_n)<1/n$ for all $n\in\mathbb{N}$. Now for each $n\in\mathbb{N}$, we have

$$\begin{aligned} d(\overline{x},T\overline{x}) &\le d(\overline{x},u_n)+d(u_n,Tu_n)+d(Tu_n,T\overline{x}) \\ &\le \frac{2}{n}+u\big(d(u_n,\overline{x}),d(u_n,Tu_n),d(\overline{x},T\overline{x}), \\ &\qquad d(\overline{x},Tu_n),d(u_n,T\overline{x})\big) \\ &\le \frac{2}{n}+u\big(d(u_n,\overline{x}),d(u_n,Tu_n),d(\overline{x},T\overline{x}), \\ &\qquad d(\overline{x},u_n)+d(u_n,Tu_n),d(u_n,\overline{x}) \\ &\qquad +d(\overline{x},T\overline{x})\big). \end{aligned} \tag{20}$$

Since u is continuous, from (20),

$$d(\overline{x},T\overline{x}) \le u\big(0,0,d(\overline{x},T\overline{x}),0,d(\overline{x},T\overline{x})\big), \tag{21}$$

and hence, by (C_3), $d(\overline{x},T\overline{x})=0$; that is, $T\overline{x}=\overline{x}$. To prove the uniqueness, note that if $\overline{y}$ is a fixed point of T, then $\overline{y}\in\bigcap_{n=1}^{\infty}C_n\subseteq\bigcap_{n=1}^{\infty}\overline{C_n}=\{\overline{x}\}$ and hence $\overline{x}=\overline{y}$. □

Remark 7. Note that, to prove Theorem 2, we may assume that $b=c$ and $f=e$ (see the proof of Theorem 2.4 in [21]). Thus, by Example 3, Theorem 6 is a generalization of the above mentioned Theorem 2 of Moradi and Farajzadeh.

If we take u as in Example 4, from Theorem 6 we get the main result of Ćirić [10].

The following corollary improves Theorem 2.4 in [6].

Corollary 8. *Let (X,d) be a complete metric space and let $T: X\to X$ be a mapping satisfying*

$$\begin{aligned} &d(Tx,Ty) \\ &\quad\le k\max\Big\{d(x,y),d(x,Tx),d(y,Ty), \\ &\qquad\qquad \frac{d(y,Tx)+d(x,Ty)}{2}\Big\} \\ &\qquad + L\min\{d(x,y),d(x,Tx),d(y,Ty), \\ &\qquad\qquad d(y,Tx),d(x,Ty)\} \end{aligned} \tag{22}$$

for each $x,y\in X$, where $k\in[0,1)$ and $L\ge 0$. Then T has a unique fixed point.

As an easy consequence of the axiom of choice, [13, page 5], AC5: for every function $f: X\to X$, there is a function g such that $D(g)=R(f)$ and for every $x\in D(g)$, $f(gx)=x$], we obtain the following lemma (see also [22]).

Lemma 9. *Let X be a nonempty set and let $g: X\to X$ be a mapping. Then, there exists a subset $E\subseteq X$ such that $g(E)=g(X)$ and $g: E\to X$ is one-to-one.*

As an application of Theorem 6, we now establish a common fixed point result.

Theorem 10. *Let (X,d) be a metric space and let $g,T: X\to X$ be mappings satisfying*

$$\begin{aligned} d(Tx,Ty) \le u\big(&d(gx,gy),d(gx,Tx), \\ &d(gy,Ty),d(gy,Tx),d(gx,Ty)\big), \end{aligned} \tag{23}$$

for each $x,y\in X$, where $u\in\mathcal{U}$. Suppose that $TX\subseteq gX$ and gX is complete subspace of X. Then g and T have a unique coincidence point. Further, if g and T are weakly compatible, then they have a unique common fixed point.

Proof. By Lemma 9, there exists $E\subseteq X$ such that $g(E)=g(X)$ and $g: E\to X$ is one-to-one. We define a mapping $G: g(E)\to g(E)$ by

$$G(gx)=T(x), \tag{24}$$

for all $gx\in g(E)$. As g is one-to-one on $g(E)$ and $T(X)\subseteq g(X)$, G is well defined. Thus, it follows from (23) and (24) that

$$\begin{aligned} d(G(gx),G(gy)) &= d(Tx,Ty) \\ &\le u\big(d(gx,gy),d(gx,Tx),d(gy,Ty), \\ &\qquad d(gy,Tx),d(gx,Ty)\big), \end{aligned} \tag{25}$$

for all $gx,gy\in g(X)=g(E)$. Thus the function $G: g(E)\to g(E)$ satisfies all conditions of Theorem 6, so G has a unique fixed point $z\in g(X)$. As $z\in g(X)$, there exists $w\in X$ such that $z = g(w)$. Thus $T(w) = G(gw) = G(z) = z = g(w)$ which implies that g and T have a unique coincidence point. Further if g and T are weakly compatible, then they have a unique common fixed point. □

If we take u as in Example 5, then from Theorem 10 we obtain the following result which extends many related results in the literature (see [16, 17]).

Theorem 11. *Let (X, d) be a metric space and let $g, T : X \to X$ be mappings satisfying*

$$\begin{aligned} d(Tx, Ty) &\le k \max\left\{d(gx, gy), d(gx, Tx), d(gy, Ty), \frac{d(gy, Tx) + d(gx, Ty)}{2}\right\} \\ &\quad + L \min\{d(gx, gy), d(gx, Tx), d(gy, Ty), d(gy, Tx), d(gx, Ty)\} \end{aligned} \tag{26}$$

for each $x, y \in X$, where $k \in [0, 1)$ and $L \ge 0$. Suppose that $TX \subseteq gX$ and gX is a complete subspace of X. Then g and T have a unique coincidence point. Further, if g and T are weakly compatible, then they have a unique common fixed point.

Corollary 12. *Let (X, d) be a metric space and let $g, T : X \to X$ be mappings satisfying*

$$d(Tx, Ty) \le k \max\left\{d(gx, gy), d(gx, Tx), d(gy, Ty), \frac{d(gy, Tx) + d(gx, Ty)}{2}\right\}, \tag{27}$$

for each $x, y \in X$, where $k \in [0, 1)$. Suppose that $TX \subseteq gX$ and gX is a complete subspace of X. Then g and T have a unique coincidence point. Further if g and T are weakly compatible, then they have a unique common fixed point.

As a linear continuous operator defined on a closed subset of a normed space is closed operator, we obtain the following new common fixed point results as corollaries to Theorem 11.

Corollary 13 (see Fisher and Sessa [2]). *Let T and g be two weakly commuting mappings on a closed convex subset C of a Banach space X into itself satisfying the inequality*

$$\begin{aligned} \|Tx - Ty\| &\le a \max\{\|Tx - gx\|, \|Ty - gy\|\} \\ &\quad + L \min\{\|gx - gy\|, \|Tx - gx\|, \|Ty - gy\|, \|gy - Tx\|, \|gx - Ty\|\}, \end{aligned} \tag{28}$$

for all $x, y \in C$, where $a \in (0, 1)$ and $L \ge 0$. If g is linear and nonexpansive on C and $T(C) \subseteq g(C)$, then T and g have a unique common fixed point in C.

Corollary 14 (see Jungck [3]). *Let T and g be compatible self-maps of a closed convex subset C of a Banach space X. Suppose that g is continuous and linear and that $T(C) \subset g(C)$. If T and g satisfy inequality* (28)*, then T and g have a unique common fixed point in C.*

Now, we illustrate our main result by the following example.

Example 15. Let $X = \{1, 2, 3, 4\}$ and let $d(1, 2) = 5/4$, $d(1, 3) = 1$, $d(1, 4) = 7/4$, and $d(2, 3) = d(2, 4) = d(3, 4) = 2$. Then (X, d) is a complete metric space. Let $T : X \to X$ be given by $T1 = 1$, $T2 = 4$, $T3 = 4$, and $T4 = 1$. Then it is straightforward to show that

$$d(Tx, Ty) \le \frac{7}{8} \max\left\{d(x, y), d(x, Tx), d(y, Ty), \frac{d(x, Ty) + d(y, Tx)}{2}\right\} \tag{29}$$

for each $x, y \in X$. Then by Corollary 8, T has a unique fixed point ($x = 1$ is the unique fixed point of T).

Now, we show that T does not satisfy the condition of Theorem 2 of Moradi and Farajzadeh. On the contrary, assume that there exist nonnegative numbers $0 < a < 1$, $b, c, e, f \ge 0$, $b + c > 0$, $e + f > 0$ such that

$$\begin{aligned} d(Tx, Ty) &\le ad(x, y) + bd(x, Tx) + cd(y, Ty) \\ &\quad + ed(y, Tx) + fd(x, Ty) \end{aligned} \tag{30}$$

for all $x, y \in X$. Let $(x_1, y_1) = (1, 2)$ and $(x_2, y_2) = (2, 1)$. Then from (30), we have

$$\begin{aligned} \frac{7}{4} &\le \frac{5}{4}a + 2c + \frac{5}{4}e + \frac{7}{4}f, \\ \frac{7}{4} &\le \frac{5}{4}a + 2b + \frac{7}{4}e + \frac{5}{4}f, \end{aligned} \tag{31}$$

which yield

$$7 \le 5a + 4b + 4c + 6e + 6f \le 6(a + b + c + e + f) \le 6, \tag{32}$$

a contradiction. Thus we cannot invoke the above mentioned theorem of Moradi and Farajzadeh (Theorem 2), to show the existence of a fixed point of T.

Remark 16. The technique of proof of Theorem 6 is in line with the proof of Theorem 2.4 in [23]. Therefore the reader interested in fixed point results for generalized contractions/nonexpansive mappings in the general setup of uniformly convex metric spaces is referred to [23].

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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