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Research Article

Differential Subordination Results for Analytic Functions in the Upper Half-Plane

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There are many articles in the literature dealing with differential subordination problems for analytic functions in the unit disk, and only a few articles deal with the above problems in the upper half-plane. In this paper, we aim to derive several differential subordination results for analytic functions in the upper half-plane by investigating certain suitable classes of admissible functions. Some useful consequences of our main results are also pointed out.

1. Introduction

Let Δ denote the upper half-plane; that is,

$$\Delta = \{z \in \mathbb{C} : \operatorname{Im}(z) > 0\}, \quad (1)$$

and let $\mathscr{H}[\Delta]$ denote the class of functions f which are analytic in Δ and which satisfy the so-called hydrodynamic normalization (see [1–3]):

$$\lim_{\Delta \ni z \to \infty} [f(z) - z] = 0. \quad (2)$$

Also let $\mathscr{S}[\Delta]$ denote the class of all functions in $\mathscr{H}[\Delta]$ which are univalent in Δ. Various basic properties concerning functions belonging to the class $\mathscr{S}[\Delta]$ were developed in a series of articles (see, for details, [4–6]).

A function $f \in \mathscr{H}[\Delta]$, with $f(z) \neq 0$, is said to be starlike in Δ if and only if

$$\operatorname{Im}\left\{\frac{f'(z)}{f(z)}\right\} < 0. \quad (3)$$

We denote by $\mathscr{S}^*[\Delta]$ the subclass of $\mathscr{H}[\Delta]$ which consists of functions which are starlike in Δ.

A function $f \in \mathscr{H}[\Delta]$, with $f(z) \neq z$, is said to be convex in Δ if and only if

$$\operatorname{Im}\left\{\frac{f''(z)}{f'(z)}\right\} > 0. \quad (4)$$

Also, we denote by $\mathscr{K}[\Delta]$ the subclass of $\mathscr{H}[\Delta]$ which consists of functions which are convex in Δ. The classes $\mathscr{S}^*[\Delta]$ and $\mathscr{K}[\Delta]$ were introduced by Stankiewicz [3].

We first need to recall the notion of subordination in the upper half-plane.

Let f and g be members of $\mathscr{H}[\Delta]$. The function f is subordinate to g, written as $f \prec g$ or $f(z) \prec g(z)$, if there exists a function $\varphi \in \mathscr{H}[\Delta]$ with $\varphi[\Delta] \subset \Delta$ such that $f(z) = g(\varphi(z))$. Furthermore, if the function g is univalent in Δ, then we have the following equivalence (cf. [7]):

$$f(z) \prec g(z) \quad (z \in \Delta) \Longleftrightarrow f(\Delta) \subset g(\Delta). \quad (5)$$

Using methods similar to those used in the unit disk, Răducanu and Pascu [7] have extended the theory of differential subordinations to the upper half-plane. In the following, we will list some definitions and theorems, which are required to prove our main results.

Definition 1 (see [8, Definition 8.3i, p.403]). Denote by $\mathcal{Q}(\Delta)$ the set of functions $q \in \mathcal{H}[\Delta]$ that are analytic and injective on $\overline{\Delta} \setminus E(q)$, where

$$E(q) = \left\{\xi \in \partial\Delta : \lim_{z \to \xi} q(z) = \infty\right\}, \tag{6}$$

and are such that $q'(\xi) \neq 0$ for $\xi \in \partial\Delta \setminus E(q)$.

Definition 2 (see [7]). Let Ω be a set in $\mathbb{C}$ and $q \in \mathcal{Q}(\Delta)$. The class of admissible functions $\Psi_\Delta[\Omega, q]$ consists of those functions $\psi : \mathbb{C}^3 \times \Delta \to \mathbb{C}$ that satisfy the following admissibility condition:

$$\psi(r, s, t; z) \notin \Omega, \tag{7}$$

whenever

$$r = q(\xi), \quad s = kq'(\xi), \quad \operatorname{Im}\left\{\frac{t}{q'(\xi)}\right\} \geq k^2 \operatorname{Im}\left\{\frac{q''(\xi)}{q'(\xi)}\right\}, \tag{8}$$

where $z \in \Delta$, $\xi \in \partial\Delta \setminus E(q)$, and $k \geq 0$.

If $\psi : \mathbb{C}^2 \times \Delta \to \mathbb{C}$, then the admissibility condition reduces to

$$\psi\left(q(\xi), kq'(\xi); z\right) \notin \Omega, \tag{9}$$

where $z \in \Delta$, $\xi \in \partial\Delta \setminus E(q)$, and $k \geq 0$.

Theorem 3 (see [7]). *Let $\psi \in \Psi_\Delta[\Omega, q]$ and $p \in \mathcal{H}[\Delta]$. If*

$$\psi\left(p(z), p'(z), p''(z); z\right) \in \Omega, \tag{10}$$

for $z \in \Delta$, then

$$p(z) \prec q(z). \tag{11}$$

In the present paper, by making use of the differential subordination results in the upper half-plane of Răducanu and Pascu [7] (which is a generalization of results in the unit disk obtained by Miller and Mocanu [8]), we determine certain appropriate classes of admissible functions and investigate some differential subordination properties of analytic functions in the upper half-plane. It should be remarked in passing that, in recent years, several authors obtained many interesting results associated with differential subordination and superordination in the unit disk; the interested reader may refer to, for example, [9–18].

2. The Main Subordination Results

We first define the following class of admissible functions that are required in proving our first result.

Definition 4. Let Ω be a set in $\mathbb{C}$ and $q \in \mathcal{Q}(\Delta) \cap \mathcal{H}[\Delta]$. The class of admissible functions $\Phi_\Delta[\Omega, q]$ consists of those functions $\phi : \mathbb{C}^3 \times \Delta \to \mathbb{C}$ that satisfy the following admissibility condition:

$$\phi(u, v, w; z) \notin \Omega, \tag{12}$$

whenever

$$u = q(\xi), \quad v = \frac{kq'(\xi)}{q(\xi)} \quad (q(\xi) \neq 0),$$
$$\operatorname{Im}\left\{\frac{u\left(wv + v^2\right)}{q'(\xi)}\right\} \geq k^2 \operatorname{Im}\left\{\frac{q''(\xi)}{q'(\xi)}\right\} \quad (z \in \Delta; \xi \in \partial\Delta \setminus E(q); k \geq 0). \tag{13}$$

Theorem 5. *Let $\phi \in \Phi_\Delta[\Omega, q]$. If $f \in \mathcal{H}[\Delta]$ satisfies*

$$\left\{\phi\left(\frac{f'(z)}{f(z)}, \frac{f''(z)}{f'(z)} - \frac{f'(z)}{f(z)}, \frac{f(z)\left[f'''(z) f'(z) - \left(f''(z)\right)^2\right]}{f'(z)\left[f(z) f''(z) - \left(f'(z)\right)^2\right]} - \frac{f'(z)}{f(z)}; z\right) : z \in \Delta\right\} \subset \Omega, \tag{14}$$

then

$$\frac{f'(z)}{f(z)} \prec q(z) \quad (z \in \Delta). \tag{15}$$

Proof. Define the function $p(z)$ in Δ by

$$p(z) = \frac{f'(z)}{f(z)}. \tag{16}$$

A simple calculation yields

$$\frac{f''(z)}{f'(z)} - \frac{f'(z)}{f(z)} = \frac{p'(z)}{p(z)}. \tag{17}$$

Further computations show that

$$\frac{f(z)\left[f'''(z) f'(z) - \left(f''(z)\right)^2\right]}{f'(z)\left[f(z) f''(z) - \left(f'(z)\right)^2\right]} - \frac{f'(z)}{f(z)} = \frac{p''(z)}{p'(z)} - \frac{p'(z)}{p(z)}. \tag{18}$$

We now define the transformation from $\mathbb{C}^3$ to $\mathbb{C}$ by

$$u(r, s, t) = r, \quad v(r, s, t) = \frac{s}{r}, \quad w(r, s, t) = \frac{rt - s^2}{rs}. \tag{19}$$

Let

$$\psi(r, s, t; z) = \phi(u, v, w; z) = \phi\left(r, \frac{s}{r}, \frac{rt - s^2}{rs}; z\right). \tag{20}$$

Using (16)–(18), and from (20), we obtain

$$\psi\left(p(z), p'(z), p''(z); z\right) = \phi\left(\frac{f'(z)}{f(z)}, \frac{f''(z)}{f'(z)} - \frac{f'(z)}{f(z)}, \frac{f(z)\left[f'''(z)f'(z) - \left(f''(z)\right)^2\right]}{f'(z)\left[f(z)f''(z) - \left(f'(z)\right)^2\right]} - \frac{f'(z)}{f(z)}; z\right). \tag{21}$$

Hence, (14) becomes

$$\psi\left(p(z), p'(z), p''(z); z\right) \in \Omega. \tag{22}$$

From (19), we easily get

$$t = u\left(wv + v^2\right). \tag{23}$$

Thus, the admissibility condition for $\phi \in \Phi_\Delta[\Omega, q]$ in Definition 4 is equivalent to the admissibility condition for ψ as given in Definition 2. Therefore $\psi \in \Psi_\Delta[\Omega, q]$, and by Theorem 3, we have $p(z) \prec q(z)$, or, equivalently, $f'(z)/f(z) \prec q(z)$, which evidently completes the proof of Theorem 5. □

If $\Omega \neq \mathbb{C}$ is a simply connected domain, then $\Omega = h(\Delta)$ for some conformal mapping $h(z)$ of Δ onto Ω. In this case, the class $\Phi_\Delta[h(\Delta), q]$ is written as $\Phi_\Delta[h, q]$. The following result is an immediate consequence of Theorem 5.

Theorem 6. *Let $\phi \in \Phi_\Delta[h, q]$. If $f \in \mathscr{H}[\Delta]$ satisfies*

$$\phi\left(\frac{f'(z)}{f(z)}, \frac{f''(z)}{f'(z)} - \frac{f'(z)}{f(z)}, \frac{f(z)\left[f'''(z)f'(z) - \left(f''(z)\right)^2\right]}{f'(z)\left[f(z)f''(z) - \left(f'(z)\right)^2\right]} - \frac{f'(z)}{f(z)}; z\right) \prec h(z), \tag{24}$$

then

$$\frac{f'(z)}{f(z)} \prec q(z) \quad (z \in \Delta). \tag{25}$$

Our next result is an extension of Theorem 5 to the case where the behavior of $q(z)$ on $\partial\Delta$ is not known.

Theorem 7. *Let h and q be univalent in Δ with $q \in \mathbb{Q}(\Delta)$, and set $q_\rho(z) = q(\rho z)$ and $h_\rho(z) = h(\rho z)$. Let $\phi : \mathbb{C}^3 \times \Delta \to \mathbb{C}$ satisfy one of the following conditions:*

(1) $\phi \in \Phi_\Delta[h, q_\rho]$, for some $\rho \in (0, 1)$, or

(2) there exists $\rho_0 \in (0, 1)$ such that $\phi \in \Phi_\Delta[h_\rho, q_\rho]$ for all $\rho \in (\rho_0, 1)$.

If $f \in \mathscr{H}[\Delta]$ satisfies (24), then

$$\frac{f'(z)}{f(z)} \prec q(z) \quad (z \in \Delta). \tag{26}$$

Proof. The proof of Theorem 7 is similar to that of [8, Theorem 2.3d, p.30] and so we choose to omit it. □

The next theorem yields the best dominant of the differential subordination (24).

Theorem 8. *Let h be univalent in Δ and $\phi : \mathbb{C}^3 \times \Delta \to \mathbb{C}$. Suppose that the following differential equation:*

$$\phi\left(q(z), \frac{q'(z)}{q(z)}, \frac{q''(z)}{q'(z)} - \frac{q'(z)}{q(z)}; z\right) = h(z), \tag{27}$$

has a solution $q(z)$ and satisfies one of the following conditions:

(1) $q \in \mathbb{Q}(\Delta)$ and $\phi \in \Phi_\Delta[h, q]$,

(2) q is univalent in Δ and $\phi \in \Phi_\Delta[h, q_\rho]$, for some $\rho \in (0, 1)$, or

(3) q is univalent in Δ and there exists $\rho_0 \in (0, 1)$ such that $\phi \in \Phi_\Delta[h_\rho, q_\rho]$ for all $\rho \in (\rho_0, 1)$.

If $f \in \mathscr{H}[\Delta]$ satisfies (24), then

$$\frac{f'(z)}{f(z)} \prec q(z), \tag{28}$$

and q is the best dominant.

Proof. Following the same arguments as in [8, Theorem 2.3e, p.31], we deduce that q is a dominant from Theorems 6 and 7. Since q satisfies (27), it is also a solution of (24) and therefore q will be dominated by all dominants. Hence, q is the best dominant. □

In the particular case $q(z) = z$, and in view of Definition 4, the class of admissible functions $\Phi_\Delta[\Omega, q]$, denoted by $\Phi_\Delta[\Omega, z]$, is described below.

Definition 9. Let Ω be a set in $\mathbb{C}$. The class of admissible functions $\Phi_\Delta[\Omega, z]$ consists of those functions $\phi : \mathbb{C}^3 \times \Delta \to \mathbb{C}$ such that

$$\phi\left(\xi, \frac{k}{\xi}, \frac{L\xi - k^2}{k\xi}; z\right) \notin \Omega, \tag{29}$$

whenever $z \in \Delta$, $\mathrm{Im}(L) = 0$, $\xi \in \mathbb{R} \setminus \{0\}$, and $k > 0$.

Corollary 10. *Let $\phi \in \Phi_\Delta[\Omega, z]$. If $f \in \mathscr{H}[\Delta]$ satisfies*

$$\phi\left(\frac{f'(z)}{f(z)}, \frac{f''(z)}{f'(z)} - \frac{f'(z)}{f(z)}, \frac{f(z)\left[f'''(z)f'(z) - \left(f''(z)\right)^2\right]}{f'(z)\left[f(z)f''(z) - \left(f'(z)\right)^2\right]} - \frac{f'(z)}{f(z)}; z\right) \in \Omega, \tag{30}$$

then

$$\frac{f'(z)}{f(z)} \prec z \quad (z \in \Delta). \tag{31}$$

For the special case $\Omega = q(\Delta) = \{\omega : \mathrm{Im}(\omega) > 0\}$, the class $\Phi_\Delta[\Omega, z]$ is simply denoted by $\Phi_\Delta[\Delta, z]$. Corollary 10 can now be written in the following form.

Corollary 11. *Let $\phi \in \Phi_\Delta[\Delta, z]$. If $f \in \mathcal{H}[\Delta]$ satisfies*

$$\mathrm{Im}\left\{\phi\left(\frac{f'(z)}{f(z)}, \frac{f''(z)}{f'(z)} - \frac{f'(z)}{f(z)}, \frac{f(z)\left[f'''(z) f'(z) - \left(f''(z)\right)^2\right]}{f'(z)\left[f(z) f''(z) - \left(f'(z)\right)^2\right]} - \frac{f'(z)}{f(z)}; z\right)\right\} > 0, \tag{32}$$

then

$$\mathrm{Im}\left\{\frac{f'(z)}{f(z)}\right\} > 0 \quad (z \in \Delta). \tag{33}$$

Example 12. Let the functions $A, B : \Delta \to \mathbb{C}$ be analytic in Δ and satisfy $\mathrm{Im}\ A(z) \le 0$ and $\mathrm{Im}\ B(z) \le 0$. Then, the functions

$$\begin{aligned} \phi_1(u, v, w; z) &= u + v + A(z), \\ \phi_2(u, v, w; z) &= \alpha(u + v) + (1 - \alpha) u + B(z) \quad (0 \le \alpha \le 1) \end{aligned} \tag{34}$$

satisfy the admissibility condition (29) and hence Corollary 10 yields

$$\begin{aligned} &\mathrm{Im}\left\{\frac{f''(z)}{f'(z)} + A(z)\right\} > 0 \Longrightarrow \mathrm{Im}\left\{\frac{f'(z)}{f(z)}\right\} > 0, \\ &\mathrm{Im}\left\{\alpha\frac{f''(z)}{f'(z)} + (1 - \alpha)\frac{f'(z)}{f(z)} + B(z)\right\} \\ &> 0 \Longrightarrow \mathrm{Im}\left\{\frac{f'(z)}{f(z)}\right\} > 0. \end{aligned} \tag{35}$$

Next, we introduce the following class of admissible functions.

Definition 13. Let Ω be a set in $\mathbb{C}$ and $q \in \mathcal{Q}(\Delta) \cap \mathcal{H}[\Delta]$. The class of admissible functions $\Phi_{\Delta,1}[\Omega, q]$ consists of those functions $\phi : \mathbb{C}^2 \times \Delta \to \mathbb{C}$ that satisfy the admissibility condition

$$\phi\left(q(\xi), q(\xi) + \frac{kq'(\xi)}{q(\xi)}; z\right) \notin \Omega, \tag{36}$$

where $z \in \Delta$, $\xi \in \partial\Delta \setminus E(q)$, and $k \ge 0$.

Theorem 14. *Let $\phi \in \Phi_{\Delta,1}[\Omega, q]$. If $f \in \mathcal{H}[\Delta]$ satisfies*

$$\left\{\phi\left(\frac{f'(z)}{f(z)}, \frac{f''(z)}{f'(z)}; z\right) : z \in \Delta\right\} \subset \Omega, \tag{37}$$

then

$$\frac{f'(z)}{f(z)} \prec q(z) \quad (z \in \Delta). \tag{38}$$

Proof. Define the function $p(z)$ in Δ by

$$p(z) = \frac{f'(z)}{f(z)}. \tag{39}$$

A simple calculation yields

$$\frac{f''(z)}{f'(z)} = p(z) + \frac{p'(z)}{p(z)}. \tag{40}$$

Define the transformation from $\mathbb{C}^2$ to $\mathbb{C}$ by

$$u(r, s) = r, \quad v(r, s) = r + \frac{s}{r}. \tag{41}$$

Let

$$\psi(r, s; z) = \phi(u, v; z) = \phi\left(r, r + \frac{s}{r}; z\right). \tag{42}$$

The proof will make use of Theorem 3. Using (39) and (40), and from (42), we get

$$\psi\left(p(z), p'(z); z\right) = \phi\left(\frac{f'(z)}{f(z)}, \frac{f''(z)}{f'(z)}; z\right). \tag{43}$$

Hence, (37) becomes

$$\psi\left(p(z), p'(z); z\right) \in \Omega. \tag{44}$$

From (42), we see that the admissibility condition for $\phi \in \Phi_{\Delta,1}[\Omega, q]$ in Definition 13 is equivalent to the admissibility condition for ψ as given in Definition 2. Hence $\psi \in \Psi_\Delta[\Omega, q]$, and by Theorem 3, we have $p(z) \prec q(z)$ or $f'(z)/f(z) \prec q(z)$. □

We will denote the class $\Phi_{\Delta,1}[h(\Delta), q]$ by $\Phi_{\Delta,1}[h, q]$, where h is the conformal mapping of Δ onto $\Omega \ne \mathbb{C}$.

Theorem 15. *Let $\phi \in \Phi_{\Delta,1}[h, q]$. If $f \in \mathcal{H}[\Delta]$ satisfies*

$$\phi\left(\frac{f'(z)}{f(z)}, \frac{f''(z)}{f'(z)}; z\right) \prec h(z), \tag{45}$$

then

$$\frac{f'(z)}{f(z)} \prec q(z). \tag{46}$$

We extend Theorem 15 to the case in which the behavior of $q(z)$ on $\partial\Delta$ is not known.

Theorem 16. *Let $\Omega \subset \mathbb{C}$ and q be univalent in Δ with $q \in \mathcal{Q}(\Delta)$. Let $\phi \in \Phi_{\Delta,1}[h, q_\rho]$, for some $\rho \in (0,1)$, where $q_\rho(z) = q(\rho z)$. If $f \in \mathscr{H}[\Delta]$ satisfies* (37)*, then* (46) *holds.*

As a special case, when $q(z) = z$, we get the following corollary.

Corollary 17. *Let Ω be a set in $\mathbb{C}$ and let $\phi : \mathbb{C}^2 \times \Delta \to \mathbb{C}$ satisfy*

$$\phi\left(\xi, \xi + \frac{k}{\xi}; z\right) \notin \Omega, \tag{47}$$

whenever $z \in \Delta$, $\xi \in \mathbb{R} \setminus \{0\}$, and $k \geq 0$. If $f \in \mathscr{H}[\Delta]$ satisfies

$$\phi\left(\frac{f'(z)}{f(z)}, \frac{f''(z)}{f'(z)}; z\right) \in \Omega, \tag{48}$$

then

$$\operatorname{Im}\left\{\frac{f'(z)}{f(z)}\right\} > 0 \quad (z \in \Delta). \tag{49}$$

In the special case $\Omega = q(\Delta) = \{\omega : \operatorname{Im}(\omega) > 0\}$, Corollary 17 reduces to the following corollary.

Corollary 18. *Let $\phi : \mathbb{C}^2 \times \Delta \to \mathbb{C}$ satisfy*

$$\operatorname{Im}\left\{\phi\left(\xi, \xi + \frac{k}{\xi}; z\right)\right\} \leq 0, \tag{50}$$

whenever $z \in \Delta$, $\xi \in \mathbb{R} \setminus \{0\}$, and $k \geq 0$. If $f \in \mathscr{H}[\Delta]$ satisfies

$$\operatorname{Im}\left\{\phi\left(\frac{f'(z)}{f(z)}, \frac{f''(z)}{f'(z)}; z\right)\right\} > 0, \tag{51}$$

then

$$\operatorname{Im}\left\{\frac{f'(z)}{f(z)}\right\} > 0 \quad (z \in \Delta). \tag{52}$$

Example 19. Let the function $C : \Delta \to \mathbb{C}$ be analytic in Δ and satisfy $\operatorname{Im} C(z) \leq 0$. Then, the function

$$\phi(u, v; z) = uv + C(z) \tag{53}$$

satisfies the admissibility condition (47) and hence Corollary 18 becomes

$$\operatorname{Im}\left\{\frac{f''(z)}{f(z)} + C(z)\right\} > 0 \Longrightarrow \operatorname{Im}\left\{\frac{f'(z)}{f(z)}\right\} > 0. \tag{54}$$

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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