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Research Article

Application of Sumudu Decomposition Method to Solve Nonlinear System Volterra Integrodifferential Equations

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We develop a method to obtain approximate solutions for nonlinear systems of Volterra integrodifferential equations with the help of Sumudu decomposition method (SDM). The technique is based on the application of Sumudu transform to nonlinear coupled Volterra integrodifferential equations. The nonlinear term can easily be handled with the help of Adomian polynomials. We illustrate this technique with the help of three examples and results of the present technique have close agreement with approximate solutions which were obtained with the help of Adomian decomposition method (ADM).

1. Introduction

The linear and nonlinear Volterra integral equations arise in many scientific fields such as the population dynamics, spread of epidemics, and semiconductor devices; for more details, see [1]. The scientists in different branches of science have been trying to solve this kind of problems; however, finding an exact solution is not easy due to the nonlinear part of these type groups of equations. Different analytical methods have been developed and applied to find a solution. For example, Adomian has introduced a so-called decomposition method for solving algebraic, differential, integrodifferential, differential-delay, and partial differential equations. In the nonlinear case for ordinary differential equations and partial differential equations, the method has the advantage of dealing directly with the problem [2, 3]. These equations are solved without transforming them to equivalent form which is more simple. The method avoids linearization, perturbation, discretization, or any unrealistic assumptions; see [4, 5]. It was also suggested in [6] that the noise terms appear always for inhomogeneous equations. Thus, most recently, Wazwaz [7] established a necessary condition that is essentially needed to ensure the appearance of "noise terms" in the inhomogeneous equations.

The integral transform has been used to solve many different types of differential and integrodifferential equations. For similar problems, Sumudu transform was introduced and further applied to several ODEs as well as PDEs. For example, in [8], this transform was applied to the one-dimensional neutron transport equation. In [9], Sumudu transform was extended to the distributions and some of their properties were also studied in [10]. Recently, Kılıçman et al. applied this transform to solve the system of differential equations (see [11]), since there are some interesting properties that Sumudu transform satisfies such as if

$$f(t) = \sum_{n=0}^{\infty} a_n t^n, \quad \text{then } F(u) = \sum_{n=0}^{\infty} n! a_n u^n; \tag{1}$$

see [12].

In the present paper, the intimate connection between Sumudu transform theory and decomposition method arising in the solution of nonlinear Volterra integrodifferential equation is demonstrated.

During the study, we use Sumudu transform which is defined over the set of the following functions:

$$A = \left\{ f(t) : \exists M, \tau_1, \tau_2 > 0,\ |f(t)| < Me^{t/\tau_j},\ \text{if } t \in (-1)^j \times [0,\infty) \right\} \tag{2}$$

by the following formula:

$$G(u) = S[f(t); u] := \int_0^\infty f(ut)\, e^{-t} dt, \quad u \in (-\tau_1, \tau_2). \tag{3}$$

Theorem 1. *Let $f(t)$ be in A, and let $G^n(u)$ denote Sumudu transform of nth derivative, $f^n(t)$ of $f(t)$; then, for $n \geq 1$,*

$$G^n(u) = \frac{G(u)}{u^n} - \sum_{k=0}^{n-1} \frac{f^{(k)}(0)}{u^{n-k}}. \tag{4}$$

For more details, see [13].

Recall that Sumudu transform of the convolution product $(f * g)(x)$ is given by

$$S[(f * g)(x; u)] = S\left[\int_0^x f(x-t)\, g(t)\, dt\right] = uF(u)\, G(u). \tag{5}$$

We consider general nonlinear Volterra integrodifferential equation:

$$\frac{d^n U}{dx^n} = f(x) + \int_0^x K(x-t)\, F(U(t))\, dt. \tag{6}$$

To solve the nonlinear Volterra integrodifferential equations by using Sumudu transform method, it is essential to use Sumudu transforms of the derivatives of $U(x)$. We can easily show that

$$S\left[\frac{d^n U}{dx^n}\right] = \frac{1}{u^n} S[U(x)] - \frac{1}{u^n} U(0) - \frac{1}{u^{n-1}} U'(0) - \cdots - \frac{U^{n-1}(0)}{u}. \tag{7}$$

Applying Sumudu transform to both sides of (7) gives

$$\frac{1}{u^n} S[U(x)] - \frac{1}{u^n} U(0) - \frac{1}{u^{n-1}} U'(0) - \cdots - \frac{U^{(n-1)}(0)}{u} = S[f(x)] + uS(K(x-t))\, S(F(u(t))); \tag{8}$$

by arrangement, we have

$$S[U(x)] = u^n S[f(x)] + U(0) + uU'(0) + \cdots + u^{n-1} U^{(n-1)}(0) + u^{n+1} S(K(x-t))\, S(F(u(t))). \tag{9}$$

The second step in Sumudu decomposition method is that we represent the solution as an infinite series given by

$$U(x, \lambda) = \sum_{i=0}^{\infty} U_i(x) \tag{10}$$

and the nonlinear term can be decomposed as

$$F(u(t)) = \sum_{i=0}^{\infty} A_i, \tag{11}$$

where A_i are Adomian polynomials [7] of $U_0, U_1, U_2, \ldots, U_n$ and they can be calculated by the following formula:

$$A_i = \frac{1}{i!} \frac{d^i}{d\lambda^i} \left[F\left(\sum_{i=0}^{\infty} \lambda^i u_i \right) \right]_{\lambda=0}, \quad i = 0, 1, 2, \ldots, \tag{12}$$

where the so-called Adomian polynomials A_n can be evaluated for all forms of nonlinearity. General formula (12) can be easily used as follows.

Assuming that the nonlinear function is $F(u(x))$, therefore, by using (12), Adomian polynomials are given by

$$\begin{aligned} A_0 &= F(U_0), \qquad A_1 = U_1 F'(U_0) \\ A_2 &= U_2 F'(U_0) + \frac{1}{2!} U_1^2 F''(U_0) \\ A_3 &= U_3 F'(U_0) + U_1 U_2 F''(U_0) + \frac{1}{3!} U_1^3 F'''(U_0) \\ A_4 &= U_4 F'(U_0) + \left(\frac{1}{2!} U_2^2 + U_1 U_3 \right) F''(U_0) \\ &\quad + \frac{1}{2!} U_1^2 U_2 F'''(U_0) + \frac{1}{4!} U_1^4 U_2 F^{(iv)}(U_0). \end{aligned} \tag{13}$$

Substitution of (10) and (11) into (9) yields

$$\begin{aligned} S\left[\sum_{i=0}^{\infty} U_i(x)\right] &= u^n S[f(x)] + U(0) + uU'(0) \\ &\quad + \cdots + u^{n-1} U^{(n-1)}(0) \\ &\quad + u^{n+1} S(K(x-t))\, S\left(\left[\sum_{i=0}^{\infty} A_i\right]\right). \end{aligned} \tag{14}$$

On comparing both sides of (14) and by using standard ADM we have

$$S[U_0(x)] = u^n S[f(x)] + U(0) + uU'(0) + \cdots + u^{n-1} U^{(n-1)}(0). \tag{15}$$

Then it follows that

$$\begin{aligned} S[U_1(x)] &= u^{n+1} S(K(x-t))\, S(A_0(x)), \\ S[U_2(x,t)] &= u^{n+1} S(K(x-t))\, S(A_1(x)). \end{aligned} \tag{16}$$

More generally way, we have

$$S\left[U_{i+1}(x)\right]=u^{n+1}S(K(x-t))S\left(A_i(x)\right) \quad i\geq 0. \tag{17}$$

Combined Sumudu transform-Adomian decomposition method for solving nonlinear Volterra integrodifferential equations of the second kind will be illustrated by studying the following example.

Example 2. Consider solving the nonlinear Volterra integrodifferential equation by using combined Sumudu transform-Adomian decomposition method:

$$\begin{aligned} U''(x) = &-1-\frac{1}{3}(\sin(x)+\sin(2x)) \\ &+\cos(x)+\int_0^x \sin(x-t)U^2(t)\,dt, \\ U'(0) = &-1, \qquad U(0)=1. \end{aligned} \tag{18}$$

By applying Sumudu transform to both sides of (18) we obtain

$$\begin{aligned} &\frac{1}{u^2}S[U(x)]-\frac{1}{u^2}U(0)-\frac{1}{u}U'(0) \\ &\quad=-1-\frac{1}{3}\left(\frac{u}{1+u^2}+\frac{2u}{1+4u^2}+\frac{1}{1+u^2}\right) \\ &\quad+\frac{u^2}{1+u^2}S\left[U^2(t)\right] \end{aligned} \tag{19}$$

or equivalently

$$\begin{aligned} S[U(x)]=&1-u-u^2 \\ &-\frac{1}{3}\left(\frac{u^3}{1+u^2}+\frac{2u^3}{1+4u^2}+\frac{u^2}{1+u^2}\right) \\ &+\frac{u^4}{1+u^2}S\left[U^2(t)\right]. \end{aligned} \tag{20}$$

Substituting the series assumption for $U(s)$ and Adomian polynomials for $u^2(x)$ as given above in (10) and (12), respectively, by using the recursive relation equation (14), we obtain

$$\begin{aligned} &S\left[U_0(x)\right] \\ &\quad=1-u-u^2-\frac{1}{3}\left(\frac{u^3}{1+u^2}+\frac{2u^3}{1+4u^2}+\frac{u^2}{1+u^2}\right). \end{aligned} \tag{21}$$

Recall that Adomian polynomials for $F(u(x))=u^2(x)$ are given by

$$\begin{aligned} &A_0=U_0^2, \qquad A_1=2U_0U_1, \\ &A_2=2U_0U_2+U_1^2, \qquad A_3=2U_0U_3+2U_1U_2. \end{aligned} \tag{22}$$

Taking the inverse Sumudu transform of both sides of the first part of (21) and using the recursive relation equation (21) give

$$\begin{aligned} U_0(x)=&-1+x+\frac{1}{2}x^2-\frac{1}{6}x^3-\frac{1}{12}x^4 \\ &+\frac{1}{40}x^5+\frac{1}{360}x^6-\frac{11}{5040}x^7+\cdots, \\ U_1(x)=&\frac{1}{24}x^4-\frac{1}{60}x^5-\frac{1}{720}x^6+\frac{1}{504}x^7+\cdots. \end{aligned} \tag{23}$$

By using (10), we obtain the series solution as follows:

$$\begin{aligned} U(x)=&\left(x-\frac{1}{3!}x^3+\frac{1}{5!}x^5-\frac{1}{7!}x^7+\cdots\right) \\ &-\left(1-\frac{1}{2!}x^2+\frac{1}{4!}x^4-\frac{1}{6!}x^6+\cdots\right). \end{aligned} \tag{24}$$

The exact solution is given by

$$U(x)=\sin(x)-\cos(x). \tag{25}$$

In the next problem, we apply the combined Sumudu transform-Adomian decomposition method. The standard form of the nonlinear Volterra integrodifferential equation of the first kind is given by

$$\begin{aligned} &\int_0^x k_1(x-t)F(U(t))\,dt \\ &\quad+\int_0^x k_2(x-t)U^n(t)\,dt=f(x). \end{aligned} \tag{26}$$

On using Sumudu transform for both sides of (26) and using (5), we get

$$S\left(k_1(x)*F(U(x))\right)+S\left(k_2(x)*U^{(n)}(x)\right)=S\left(f(x)\right). \tag{27}$$

So we have

$$uK_1(u)S(F(U(x)))+uK_2(u)S\left(U^{(n)}(x)\right)=F(u) \tag{28}$$

$$\begin{aligned} &S[U(x)] \\ &\quad=u^{n-1}\left(\frac{F(u)+K_2(u)\Psi(u)-uK_1(u)S(F(U(x)))}{K_2(u)}\right), \end{aligned} \tag{29}$$

where

$$\Psi(u)=\frac{1}{u^{n-1}}U(0)+\frac{1}{u^{n-2}}U'(0)+\cdots+U^{(n-1)}(0). \tag{30}$$

We now use Adomian decomposition method to handle (29). Substituting (10) and (11) into (29),

$$\begin{aligned} &S\left[\sum_{i=0}^{\infty}U_i(x)\right] \\ &\quad=\frac{u^{n-1}S\left[f(x)\right]}{K_2(u)}+U(0)+uU'(0) \\ &\quad+\cdots+u^{n-1}U^{(n-1)}(0)-u^n\frac{K_1(u)}{K_2(u)}S\left(\left[\sum_{i=0}^{\infty}A_i\right]\right). \end{aligned} \tag{31}$$

The Adomian decomposition method admits the use of the following recursive relation:

$$\begin{aligned} U_0(x) &= \frac{u^{n-1}S[f(x)]}{K_2(u)} + U(0) + uU'(0) \\ &\quad + \cdots + u^{n-1}U^{(n-1)}(0), \end{aligned} \tag{32}$$

$$U_{k+1}(x) = -u^n \frac{K_1(u)}{K_2(u)} S(A_k), \qquad k \geq 0.$$

Example 3. Solve the following nonlinear Volterra integrodifferential equation of the first kind by the combined Sumudu transform-Adomian decomposition method:

$$\begin{aligned} &\int_0^x (x-t) U^2(t)\,dt + \int_0^x e^{(x-t)} U'(t)\,dt \\ &\quad = -\frac{1}{4} - \frac{1}{2}x + xe^x + \frac{1}{4}e^{2x}, \quad U(0) = 1. \end{aligned} \tag{33}$$

By applying Sumudu transforms to both sides of (33), we have

$$\begin{aligned} S(U(x)) &= 1 - \frac{(1-u)}{4} - \frac{1}{2}u(1-u) \\ &\quad + \frac{u}{1-u} + \frac{1-u}{4(1-2u)} - u^2(1-u) S\left[U^2(x)\right]. \end{aligned} \tag{34}$$

Substituting the series assumption for $U(u)$ and Adomian polynomials for $U^2(x)$ and using (15) and (17), we obtain

$$\begin{aligned} &S(U_0(x)) \\ &\quad = 1 - \frac{(1-u)}{4} - \frac{1}{2}u(1-u) + \frac{u}{1-u} + \frac{1-u}{4(1-2u)}, \end{aligned} \tag{35}$$

$$S(U_{k+1}(x)) = -u^2(1-u) S[A_k(x)]. \tag{36}$$

Taking the inverse Sumudu transform of both sides of (35) and using the recursive relation equation (36) give

$$\begin{aligned} U_0(x) &= 1 + x + x^2 + \frac{1}{3}x^3 + \frac{1}{8}x^4 + \frac{1}{24}x^5 + \cdots, \\ U_1(x) &= -\frac{1}{2}x^2 - \frac{1}{6}x^3 - \frac{1}{6}x^4 - \frac{1}{12}x^5 - \cdots, \\ U_2(x) &= \frac{1}{12}x^4 + \frac{1}{20}x^5 + \cdots. \end{aligned} \tag{37}$$

The series solution is given by

$$U(x) = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \cdots, \tag{38}$$

and the exact solution is

$$U(x) = e^x. \tag{39}$$

2. Systems of Nonlinear Volterra Integrodifferential Equations

In this section, we will study systems of nonlinear Volterra integrodifferential equations of the second kind by combined Sumudu transform-Adomian decomposition method.

Consider systems of nonlinear Volterra integrodifferential equations of the second kind as follows:

$$\begin{aligned} U^{(n)} &= f_1(x) \\ &\quad + \int_0^x \left(K_1(x-t) F_1(u(t)) + R_1(x-t) G_1(v(t))\right) dt, \\ V^{(n)} &= f_2(x) \\ &\quad + \int_0^x \left(K_2(x-t) F_2(u(t)) + R_2(x-t) G_2(v(t))\right) dt. \end{aligned} \tag{40}$$

Applying Sumudu transforms to both sides of (40), we have

$$\begin{aligned} &\frac{1}{u^n} S[U(x)] - \frac{1}{u^n} U(0) - \frac{1}{u^{n-1}} U'(0) - \cdots - \frac{U^{n-1}(0)}{u} \\ &\quad = S(f_1(x)) \\ &\qquad + S\left(\left(K_1(x) * F_1(u(x)) + R_1(x) * G_1(v(x))\right)\right), \\ &\frac{1}{u^n} S[V(x)] - \frac{1}{u^n} V(0) - \frac{1}{u^{n-1}} V'(0) - \cdots - \frac{V^{n-1}(0)}{u} \\ &\quad = S(f_2(x)) \\ &\qquad + S\left(\left(K_2(x) * F_2(u(x)) + R_2(x) * G_2(v(x))\right)\right). \end{aligned} \tag{41}$$

After rearrangement, we get

$$\begin{aligned} &S[U(x)] \\ &\quad = U(0) + uU'(0) + \cdots + u^{n-1}U^{(n-1)}(0) + u^n S(f_1(x)) \\ &\qquad + u^n S\left(\left(K_1(x) * F_1(u(x)) + R_1(x) * G_1(v(x))\right)\right), \end{aligned} \tag{42}$$

$$\begin{aligned} &S[V(x)] \\ &\quad = V(0) + uV'(0) + \cdots + u^{n-1}V^{(n-1)}(0) \\ &\qquad + u^n S(f_2(x)) \\ &\qquad + u^n S\left(\left(K_2(x) * F_2(u(x)) + R_2(x) * G_2(v(x))\right)\right). \end{aligned} \tag{43}$$

To overcome the difficulty of the nonlinear terms $F_i(u(x))$, $i = 1, 2$, we apply Adomian decomposition method for handling (42) and (43). To achieve this goal, we first represent the linear terms $u(x)$ and $v(x)$ at the left side by an infinite series of components given by

$$U(x) = \sum_{i=0}^{\infty} U_i(x), \qquad V(x) = \sum_{i=0}^{\infty} V_i(x) \tag{44}$$

and the nonlinear terms $F_i(u(x))$ at the right side of (42) and (43) by

$$F(u(t)) = \sum_{n=0}^{\infty} A_n,$$
$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[F\left(\sum_{i=0}^{n}\lambda^i u_i\right)\right]_{\lambda=0}, \quad n = 0, 1, 2, \ldots, \tag{45}$$

where Adomian polynomials A_n, $n \geq 0$, can be obtained for all forms of nonlinearity. Substituting (44) and (45) into (42) and (43) leads to

$$S\left[\sum_{n=0}^{\infty} U_n(x)\right] = U(0) + uU'(0) + \cdots + u^{n-1}U^{(n-1)}(0) + u^n S(f_1(x)) + u^n S(K_1(x)) S\left(\left[\sum_{n=0}^{\infty} A_n\right]\right) + u^n S(R_1(x)) S\left(\left[\sum_{n=0}^{\infty} \widetilde{A}_n\right]\right), \tag{46}$$

$$S\left[\sum_{n=0}^{\infty} V_n(x)\right] = V(0) + uV'(0) + \cdots + u^{n-1}V^{(n-1)}(0) + u^n S(f_2(x)) + u^n S(K_2(x)) S\left(\left[\sum_{n=0}^{\infty} B_n\right]\right) + u^n S(R_2(x)) S\left(\left[\sum_{n=0}^{\infty} B_n\right]\right). \tag{47}$$

Adomian decomposition method admits the use of the following recursive relations:

$$\begin{aligned} S[U_0(x)] &= U(0) + uU'(0) + \cdots + u^{n-1}U^{(n-1)}(0) + u^n S(f_1(x)), \\ S[U_{k+1}(x)] &= u^n S(K_1(x)) S(A_k) + u^n S(R_1(x)) S(\widetilde{A}_k), \\ S[V_0(x)] &= V(0) + uV'(0) + \cdots + u^{n-1}V^{(n-1)}(0) + u^n S(f_2(x)), \\ S[V_{k+1}(x)] &= u^n S(K_2(x)) S(B_k) + u^n S(R_1(x)) S(B_k). \end{aligned} \tag{48}$$

The combined Sumudu transform-Adomian decomposition method for solving systems of nonlinear Volterra integrodifferential equations of the second kind will be illustrated by studying the following example.

Example 4. Solve the system of nonlinear Volterra integrodifferential equation by using the combined Sumudu transform-Adomian decomposition method:

$$\begin{aligned} U''(x) &= \frac{7}{3}e^x - e^{2x} - \frac{1}{3}e^{4x} + \int_0^x e^{x-t}\left(U^2(t) + V^2(t)\right)dt, \\ V''(x) &= \frac{2}{3}e^x + 3e^{2x} + \frac{1}{3}e^{4x} + \int_0^x e^{x-t}\left(U^2(t) - V^2(t)\right)dt, \\ U(0) &= 1, \quad U'(0) = 1, \quad V(0) = 1, \\ V'(0) &= 2. \end{aligned} \tag{49}$$

Taking Sumudu transforms of both sides of (49), we obtain

$$\begin{aligned} U(u) &= 1 + u + \frac{7u^2}{3(1-u)} - \frac{u^2}{(1-2u)} - \frac{u^2}{3(1-4u)} + \left(\frac{u^3}{1-u}\right) S\left[U^2(x) + V^2(x)\right], \\ V(u) &= 1 + 2u + \frac{2u^2}{3(1-u)} + \frac{3u^2}{(1-2u)} + \frac{u^2}{3(1-4u)} + \left(\frac{u^3}{1-u}\right) S\left[U^2(x) - V^2(x)\right]. \end{aligned} \tag{50}$$

By using (48), we have

$$\begin{aligned} U_0(u) &= 1 + u + \frac{7u^2}{3(1-u)} - \frac{u^2}{(1-2u)} - \frac{u^2}{3(1-4u)}, \\ U_{k+1} &= \left(\frac{u^3}{1-u}\right) S[A_k(x) + B_k(x)], \\ V_0(u) &= 1 + 2u + \frac{2u^2}{3(1-u)} + \frac{3u^2}{(1-2u)} + \frac{u^2}{3(1-4u)}, \\ V_{k+1} &= \left(\frac{u^3}{1-u}\right) S[A_k(x) - B_k(x)]. \end{aligned} \tag{51}$$

Recall that Adomian polynomials for $U^2(x)$ and $V^2(x)$ are given by

$$\begin{aligned} &A_0(x) = U_0^2, \qquad A_1(x) = 2U_0U_1, \\ &A_2 = 2U_0U_2 + U_1^2, \\ &A_3(x) = 2U_0U_3 + 2U_1U_2, \\ &B_0(x) = V_0^2, \qquad B_1(x) = 2V_0V_1, \\ &B_2 = 2V_0V_2 + V_1^2, \qquad B_3(x) = 2V_0V_3 + 2V_1V_2. \end{aligned} \tag{52}$$

Taking the inverse Sumudu transform of both sides of (48) and using the recursive relation equations (48), we obtain the solution as follows:

$$U(x) = \left(1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \dots\right),$$
$$V(x) = \left(1 + 2x + \frac{1}{2!}(2x)^2 + \frac{1}{3!}(2x)^3 + \frac{1}{4!}(2x)^4 + \dots\right). \quad (53)$$

Then the solution for the above system is given by

$$U(x) = e^x, \qquad V(x) = e^{2x}. \quad (54)$$

3. Conclusion

Sumudu transform-Adomian decomposition method has been applied to a system of nonlinear Volterra integrodifferential equations. Three examples have been presented; the method is very useful and reliable for any type of Volterra integrodifferential equations systems. Therefore, this method can even be applied to many complicated linear and nonlinear Volterra integrodifferential equations.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References

[1] A.-M. Wazwaz, *Linear and Nonlinear Integral Equations Methods and Applications*, Springer, Higher Education Press, 2011.

[2] E. Alizadeh, M. Farhadi, K. Sedighi, H. R. Ebrahimi-Kebria, and A. Ghafourian, "Solution of the Falkner-Skan equation for wedge by Adomian Decomposition Method," *Communications in Nonlinear Science and Numerical Simulation*, vol. 14, no. 3, pp. 724–733, 2009.

[3] A. M. Wazwaz, "The modified decomposition method and Pade approximants for a boundary layer equation in unbounded domain," *Applied Mathematics and Computation*, vol. 177, no. 2, pp. 737–744, 2006.

[4] N. Bellomo and R. Monaco, "A Comparison between Adomian decomposition method and pertubation techniques for nonlinear random differential equations," *Journal of Mathematical Analysis and Applications*, vol. 110, no. 2, pp. 495–502, 1985.

[5] R. Race, "On the Adomian decomposition method and comparison with Picard's method," *Journal of Mathematical Analysis and Applications*, vol. 128, no. 2, pp. 480–483, 1987.

[6] G. Adomian and R. Race, "Noise terms in decomposition solution series," *Computers & Mathematics with Applications*, vol. 24, no. 11, pp. 61–64, 1992.

[7] A. M. Wazwaz, "Necessary conditions for the appearance of noise terms in decomposition solution series," *Applied Mathematics and Computation*, vol. 81, no. 2-3, pp. 265–274, 1997.

[8] A. Kadem, *Solving the One-Dimensional Neutron Transport Equation Using Chebyshev Polynomials and the Sumudu Transform*, Analele Universitatii Din Oradea, Fascicola Matematica, 2005.

[9] H. Eltayeb, A. K. Kılıçman, and B. Fisher, "A new integral transform and associated distributions," *Integral Transforms and Special Functions*, vol. 21, no. 5, pp. 367–379, 2010.

[10] A. Kiliçman and H. E. Gadain, "On the applications of Laplace and Sumudu transforms," *Journal of the Franklin Institute*, vol. 347, no. 5, pp. 848–862, 2010.

[11] A. Kiliçman, H. Eltayeb, and P. R. Agarwal, "On sumudu transform and system of differential equations," *Abstract and Applied Analysis*, vol. 2010, Article ID 598702, 11 pages, 2010.

[12] A. Kiliçman, H. Eltayeb, and K. A. M. Atan, "A note on the comparison between laplace and sumudu transforms," *Bulletin of the Iranian Mathematical Society*, vol. 37, no. 1, pp. 131–141, 2011.

[13] F. B. M. Belgacem, A. A. Karaballi, and S. L. Kalla, "Analytical investigations of the sumudu transform and applications to integral production equations," *Mathematical Problems in Engineering*, vol. 2003, no. 3-4, pp. 103–118, 2003.