Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2014, Article ID 486384, 6 pages
http://dx.doi.org/10.1155/2014/486384

Research Article

On Common Coupled Fixed Point Theorems for Comparable Mappings in Ordered Partially Metric Spaces

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Received 13 September 2013; Accepted 31 October 2013; Published 29 January 2014

Academic Editor: Abdon Atangana

Common coupled fixed point theorems are examined in this paper for comparable mappings ensuring nonlinear contraction in ordered partial metric spaces. Given theorems enlarge and universalize some conclusions of Gnana Bhaskar and Lakshmikantham (2006).

1. Introduction

The contraction method presented the fixed point theory on partially metric spaces. It is enlarged to nonlinear contraction mapping, which is attributed by many authors. (cf. [1–25]). Particularly a partial metric space is a universalized metric space. Some further generalizations of the conclusions in [16] are demonstrated by Valero [25], Oltra and Valero [18], Shatanawi et al. [23], and Altun and Erduran [5]. Additionally, Caristi type fixed point theorem on a partial metric space was introduced by Romaguera [21].

Existence of fixed points was introduced in ordered metric spaces by Ran and Reurings [19]. Some applications of fixed points are also shown for linear and nonlinear equations. Fixed and common fixed point theorems are searched recently by many authors on this topic. Moreover coupled coincidence and coupled fixed point theorems for two mappings F and g such that F has to be mixed g-monotone property are stated by Lakshmikantham and Ćirić [15].

The authors propose to give more information about couple fixed point theory exists in the theory [1–25] for the reader.

Let us give some necessarily definitions related to mixed monotone maps and common coupled fixed point of a mapping.

Definition 1. Suppose that $(X, \leq)$ is a partially ordered set and also $F : X \times X \rightarrow X$. Assume that $F(x, y)$ is monotone nondecreasing pursuant x and also is monotone nonincreasing according to y, for any $x, y \in X$, at the time the map F is named to have mixed monotone property:

$$x_1, x_2 \in X,\ x_1 \leq x_2 \text{ implies } F(x_1, y) \leq F(x_2, y),$$
$$y_1, y_2 \in X,\ y_1 \leq y_2 \text{ implies } F(x, y_1) \geq F(x, y_2) \quad (\text{see [15]}). \tag{1}$$

Definition 2. If $F(y, x) = y$ and $F(x, y) = x$, then $(x, y) \in X \times X$ is defined as an a coupled fixed point of a mapping $F : X \times X \rightarrow X$ [15].

Definition 3. Suppose that X is a nonempty set. A partial metric on X is a real function of d of ordered pairs of elements of X which satisfies the following four conditions:

(pms_1) $x = y \Leftrightarrow d(x, x) = d(x, y) = d(y, y)$,
(pms_2) $d(x, x) \leq d(x, y)$,
(pms_3) $d(x, y) = d(y, x)$,
(pms_4) $d(x, y) \leq d(x, z) + d(z, y) - d(z, z)$ [16].

A metric space consists of two objects: a set $X \neq \emptyset$ and partial metric d on X, and also the elements of X are called the point of the metric space (X, d) (see [16]).

Notice that the span of any point to itself need not be null; so universalizing metrics, namely, a metric on a set $X \neq \emptyset$, are named to be a partial metric d on X providing $d(x, x) = 0$ for any $x \in X$. We refer the reader to check some results and related examples on partial metric spaces in the theory [1–25].

Each partial metric d on X generates a T_0 topology τ_p on X, which has a base of the family of open p-balls $\{B_p(x, \varepsilon), x \in X, \varepsilon > 0\}$, where

$$B_p(x, \varepsilon) = \{y \in X : d(x, y) < d(x, x) + \varepsilon\}. \quad (2)$$

If d is a partial metric on X, then the function $d^s : X \times X \to \mathbb{R}^+$ given by

$$d^s(x, y) = 2d(x, y) - d(x, x) - d(y, y) \quad (3)$$

is a metric on X.

Definition 4. Assume that (X, d) is a partial metric space and also $\{x_n\}$ is a sequence in X.

At the time,

(i) $d(x, x) = \lim_{n\to\infty} d(x_n, x) \Leftrightarrow \{x_n\}$ converges to a point $x \in X$,

(ii) if there exists $\lim_{n,m\to\infty} d(x_n, x_m)$, then $\{x_n\}$ is a Cauchy sequence [5].

Definition 5. A partial metric space (X, d) is named to be complete if every Cauchy sequence $\{x_n\}$ in X converges,

in accordance with τ_p, to a point $x \in X$, with $d(x, x) = \lim_{n,m\to\infty} d(x_n, x_m)$ [5].

Lemma 6. *Suppose that (X, d) is a partial metric space. At the time*

(i) *the sequence $\{x_n\}$ is Cauchy sequence in (X, d) $\Leftrightarrow$ it is a Cauchy sequence in the metric space (X, d^s),*

(ii) *(X, d) is complete $\Leftrightarrow$ the metric space (X, d^s) is complete. Besides, $\lim_{n\to\infty} d^s(x_n, x) = 0 \Leftrightarrow d(x, x) = \lim_{n\to\infty} d(x, x_n) = \lim_{n,m\to\infty} d(x_n, x_m)$ [16].*

Theorem 7. *Assume that (X, d) is a complete partial metric space and also suppose that $f : X \to X$ is a mapping to itself. Then there exists a constant $c \in [0, 1)$ providing*

$$d(fx, fy) \leq c\, d(x, y), \quad (4)$$

for all $x, y \in X$. So f has an individual fixed point [16].

Recently, Gnana Bhaskar and Lakshmikantham [8] obtained the following nice result for possessing the mixed monotone property mapping, which universalizes Theorem 7 of Matthews [16].

Theorem 8. *Suppose that $F : X \times X \to X$ is a continuous mapping possessing the mixed monotone property on X. There exists a $k \in [0, 1)$ such that*

$$d(F(x, y), F(u, v)) \leq \frac{k}{2}[d(x, u) + d(y, v)], \quad \forall x \geq u,\ y \leq v. \quad (5)$$

If there exist $x_0, y_0 \in X$ with

$$x_0 \leq F(x_0, y_0), \quad y_0 \geq F(y_0, x_0), \quad (6)$$

then, there exist $x, y \in X$ with

$$x = F(x, y), \quad y = F(y, x) \quad (\textit{see } [8, 15]). \quad (7)$$

The goal of the paper is to build coupled and common fixed point theorems in partially ordered partial metric spaces with a function φ providing conditions $\varphi(t) < t$, nonincreasing, and $\lim_{r\to t^+}\varphi(r) < t$ for each $t > 0(t)$. Offered theorems universalize and enlarge to a pair of mappings which are conclusions of Gnana Bhaskar and Lakshmikantham [8] and some other theorems related to them.

2. Main Result

Definition 9. Assume that $(X, \leq)$ is a partially ordered set and $F, G : X \times X \to X$. F and G mappings have the following properties:

if n is even, then $F(x_n, y_n) \geq G(x_{n-1}, y_{n-1})$ and $F(y_n, x_n) \leq G(y_{n-1}, x_{n-1})$;

if n is odd, then $G(x_n, y_n) \geq F(x_{n-1}, y_{n-1})$ and $G(y_n, x_n) \leq F(y_{n-1}, x_{n-1})$.

Theorem 10. *Suppose that $(X, \leq)$ is a partially ordered set and d is a partial metric on X with (X, d) being a complete partial metric space. Assume that $F, G : X \times X \to X$ are satisfied by Definition 2 and also are continuous mappings possessing the mixed monotone property on X. Let there be a non-increasing function $\varphi : \mathbb{R}^+ \to \mathbb{R}$ such that $\varphi(t) < t$, and $\lim_{r\to t^+}\varphi(r) < t$ for all $t > 0$ and also having $x \leq u$ and $y \geq v$, with*

$$d(F(x, y), G(u, v)) \leq \varphi\left(\frac{d(x, u) + d(y, v) + d(x, v) + d(y, u)}{2}\right), \quad (8)$$

for $x, y, z, u, v \in X$. If there exists $(x_0, y_0) \in X \times X$ with $x_0 \leq F(x_0, y_0)$ and $y_0 \geq F(y_0, x_0)$, at the time $\exists x, y \in X$ with $x = F(x, y) = G(x, y)$ and $y = F(y, x) = G(y, x)$.

Proof. Suppose $x_0, y_0 \in X$ with $x_0 \leq F(x_0, y_0)$ and $y_0 \geq F(y_0, x_0)$. Define sequences $\{x_n\}$ and $\{y_n\}$ in X in the following way:

$$x_{2n+1} = F(x_{2n}, y_{2n}), \qquad y_{2n+1} = F(y_{2n}, x_{2n}),$$
$$x_{2n+2} = G(x_{2n+1}, y_{2n+1}), \qquad y_{2n+2} = G(y_{2n+1}, x_{2n+1}). \quad (9)$$

We are to prove that $\{x_n\}$ sequence is nondecreasing and $\{y_n\}$ sequence is nonincreasing. That is, for all $n \geq 0$

$$x_{2n} \leq x_{2n+1}, \quad y_{2n} \geq y_{2n+1}. \quad (10)$$

For this, mathematical induction method is used.

Firstly suppose $n = 0$. Having $x_0 \leq x_1$ and $y_0 \geq y_1$, because $x_0 \leq F(x_0, y_0)$ and $y_0 \geq F(y_0, x_0)$ and as $x_1 = F(x_0, y_0)$ and $y_1 = F(y_0, x_0)$, so (10) is verified for $n = 0$.

Assume that (10) is satisfied for a constant $n \geq 0$; then, because $x_{2n} \leq x_{2n+1}$ and $y_{2n} \geq y_{2n+1}$, from Definition 9 we have

$$x_{2n+1} = F(x_{2n}, y_{2n}) \leq G(x_{2n+1}, y_{2n+1}) = x_{2n+2},$$
$$y_{2n+1} = F(y_{2n}, x_{2n}) \geq G(y_{2n+1}, y_{2n+1}) = y_{2n+2}. \quad (11)$$

Thus we get $x_{2n} \leq x_{2n+1}$ and $y_{2n} \geq y_{2n+1}$.

Hereby, by the induction method we conclude that (10) hold for all $n \geq 0$. Thereof,

$$x_0 \leq x_1 \leq x_2 \leq \cdots \leq x_n \leq x_{n+1} \leq \cdots, \quad (12)$$

$$y_0 \geq y_1 \geq y_2 \geq \cdots \geq y_n \geq y_{n+1} \geq \cdots. \quad (13)$$

Denote

$$\delta_n = d(x_n, x_{n+1}) + d(y_n, y_{n+1}), \quad (14)$$

showing $\{\delta_n\}$ sequence is nonincreasing. From (10) and (8) we have

$$\begin{aligned} d(x_{2n+1}, x_{2n+2}) &= d(F(x_{2n}, y_{2n}), G(x_{2n+1}, y_{2n+1})) \\ &\leq \varphi\big((d(x_{2n}, x_{2n+1}) + d(y_{2n}, y_{2n+1}) \\ &\qquad + d(x_{2n}, y_{2n+1}) + d(y_{2n}, x_{2n+1})) \\ &\qquad \times 2^{-1}\big) \\ &\leq \varphi\left(\frac{d(x_{2n}, x_{2n+1}) + d(y_{2n}, y_{2n+1})}{2}\right). \end{aligned} \quad (15)$$

Similarly, we can obtain

$$d(y_{2n+1}, y_{2n+2}) \leq \varphi\left(\frac{d(x_{2n}, x_{2n+1}) + d(y_{2n}, y_{2n+1})}{2}\right). \quad (16)$$

Thus, using properties of φ function we get

$$\begin{aligned} d(x_{2n+1}, x_{2n+2}) &+ d(y_{2n+1}, y_{2n+2}) \\ &\leq 2\varphi\left(\frac{d(x_{2n}, x_{2n+1}) + d(y_{2n}, y_{2n+1})}{2}\right) \\ &\leq 2\frac{d(x_{2n}, x_{2n+1}) + d(y_{2n}, y_{2n+1})}{2} \\ &= d(x_{2n}, x_{2n+1}) + d(y_{2n}, y_{2n+1}). \end{aligned} \quad (17)$$

Similarly one can show that

$$\begin{aligned} d(x_{2n+2}, x_{2n+3}) &+ d(y_{2n+2}, y_{2n+3}) \\ &\leq d(x_{2n+1}, x_{2n+2}) + d(y_{2n+1}, y_{2n+2}). \end{aligned} \quad (18)$$

Then, we obtain

$$\begin{aligned} d(x_{n+1}, x_{n+2}) &+ d(y_{n+1}, y_{n+2}) \\ &\leq d(x_n, x_{n+1}) + d(y_n, y_{n+1}) \\ &\leq \cdots \leq d(x_0, x_1) + d(y_0, y_1). \end{aligned} \quad (19)$$

Thus a sequence $\{\delta_n\}$ is nonincreasing. Thence, there is a $\delta \geq 0$ is obtained with

$$\lim_{n\to\infty} \delta_n = \delta. \quad (20)$$

Now, we claim that

$$\lim_{n\to\infty} \delta_n = 0. \quad (21)$$

we substitute $n = 2k$ in (14). Then we can get

$$\begin{aligned} \delta_n = \delta_{2k} &= d(x_{2k}, x_{2k+1}) + d(y_{2k}, y_{2k+1}) \\ &\leq 2\varphi\left(\frac{d(x_{2k-1}, x_{2k}) + d(y_{2k-1}, y_{2k})}{2}\right) \\ &= 2\varphi\left(\frac{\delta_{n-1}}{2}\right). \end{aligned} \quad (22)$$

Letting $n \to \infty$ in (22), we get

$$\delta = \lim \delta_n \leq 2 \lim \varphi\left(\frac{\delta_{n-1}}{2}\right) \leq 2\frac{\delta}{2} = \delta. \quad (23)$$

Hence $\delta = 0$. That is

$$\lim_{n\to\infty} d(y_n, y_{n+1}) + d(x_n, x_{n+1}) = 0. \quad (24)$$

Now we show that

$$\lim_{n\to\infty} d(y_n, y_m) + d(x_n, x_m) = 0. \quad (25)$$

Suppose the contrary. At the time there exists $\varepsilon > 0$ when obtaining two subsequences $\{x_{2n(i)}\}$ and $\{x_{2m(i)}\}$ of $\{x_n\}$ with $2n(i)$ is the smallest index where

$$2n(i) > 2m(i) > i, \quad d(x_{2m(i)}, x_{2n(i)}) + d(y_{2m(i)}, y_{2n(i)}) \geq \varepsilon. \quad (26)$$

This means that

$$d(x_{2m(i)}, x_{2n(i)-1}) + d(y_{2m(i)}, y_{2n(i)-1}) < \varepsilon. \quad (27)$$

By (pms_4) in Definition 3 and (27), we have

$$\begin{aligned} d(x_{2m(i)}, x_{2n(i)}) &\leq d(x_{2m(i)}, x_{2m(i)+1}) + d(x_{2m(i)+1}, x_{2n(i)}) \\ &\qquad - d(x_{2m(i)+1}, x_{2m(i)+1}) \\ &\leq d(x_{2m(i)}, x_{2m(i)+1}) + d(x_{2m(i)+1}, x_{2n(i)}) \\ &\leq d(x_{2m(i)}, x_{2m(i)+1}) + d(x_{2m(i)+1}, x_{2m(i)}) \\ &\qquad + d(x_{2m(i)}, x_{2n(i)}) - d(x_{2m(i)}, x_{2m(i)}) \\ &\leq 2d(x_{2m(i)}, x_{2m(i)+1}) + d(x_{2m(i)}, x_{2n(i)}) \\ &\leq 2d(x_{2m(i)}, x_{2m(i)+1}) + d(x_{2m(i)}, x_{2n(i)-1}) \\ &\qquad + d(x_{2n(i)-1}, x_{2n(i)}). \end{aligned} \quad (28)$$

Similarly, we can obtain that

$$d(y_{2m(i)}, y_{2n(i)}) \leq 2d(y_{2m(i)}, y_{2m(i)+1}) + d(y_{2m(i)}, y_{2n(i)-1}) + d(y_{2n(i)-1}, y_{2n(i)}). \tag{29}$$

Adding (28) and (29) and also from (27) and (26) we get

$$\begin{aligned} \varepsilon &\leq d(x_{2m(i)}, x_{2n(i)}) + d(y_{2m(i)}, y_{2n(i)}) \\ &\leq 2[d(x_{2m(i)}, x_{2m(i)+1}) + d(y_{2m(i)}, y_{2m(i)+1})] + \varepsilon \\ &+ d(x_{2n(i)-1}, x_{2n(i)}) \\ &+ d(y_{2n(i)-1}, y_{2n(i)}). \end{aligned} \tag{30}$$

Taking the limit as $i \to \infty$ in (30) and by (26) we get

$$\lim_{i\to\infty} d(x_{2m(i)}, x_{2n(i)}) + d(y_{2m(i)}, y_{2n(i)}) = \varepsilon. \tag{31}$$

Employing the triangle inequality,

$$\begin{aligned} &d(x_{2m(i)}, x_{2n(i)}) + d(y_{2m(i)}, y_{2n(i)}) \\ &\leq d(x_{2m(i)}, x_{2n(i)-1}) + d(y_{2m(i)}, y_{2n(i)-1}) \\ &+ d(x_{2n(i)-1}, x_{2n(i)}) \\ &+ d(y_{2n(i)-1}, y_{2n(i)}). \end{aligned} \tag{32}$$

Similarly, we get

$$\begin{aligned} &d(x_{2m(i)}, x_{2n(i)-1}) + d(y_{2m(i)}, y_{2n(i)-1}) \\ &\leq d(x_{2m(i)}, x_{2n(i)}) + d(y_{2m(i)}, y_{2n(i)}) \\ &+ d(x_{2n(i)}, x_{2n(i)-1}) \\ &+ d(y_{2n(i)}, y_{2n(i)-1}). \end{aligned} \tag{33}$$

As $i \to \infty$ in (33) and (32) and from (31) and (26) we can obtain

$$\lim_{i\to\infty} d(x_{2m(i)}, x_{2n(i)-1}) + d(y_{2m(i)}, y_{2n(i)-1}) = \varepsilon. \tag{34}$$

Since from (12) we have $x_{2m(i)} \leq x_{2n(i)-1}$ and $y_{2m(i)} \geq y_{2n(i)-1}$ and also by (8) and (10),

$$\begin{aligned} &d(x_{2m(i)+1}, x_{2n(i)}) \\ &= d(F(x_{2m(i)}, y_{2m(i)}), G(x_{2n(i)-1}, y_{2n(i)-1})) \\ &\leq \varphi\left((d(x_{2m(i)}, x_{2n(i)-1}) + d(y_{2m(i)}, y_{2n(i)-1})) \times 2^{-1}\right) \\ &< \frac{d(x_{2m(i)}, x_{2n(i)-1}) + d(y_{2m(i)}, y_{2n(i)-1})}{2}. \end{aligned} \tag{35}$$

Similarly, we get

$$d(y_{2m(i)+1}, y_{2n(i)}) < \frac{d(x_{2m(i)}, x_{2n(i)-1}) + d(y_{2m(i)}, y_{2n(i)-1})}{2}. \tag{36}$$

Thus

$$\begin{aligned} &d(x_{2m(i)+1}, x_{2n(i)}) + d(y_{2m(i)+1}, y_{2n(i)}) \\ &< d(x_{2m(i)}, x_{2n(i)-1}) \\ &+ d(y_{2m(i)}, y_{2n(i)-1}). \end{aligned} \tag{37}$$

As $i \to \infty$ in (37) we get $\varepsilon = 0$, which is a contrast. Whence (25) is verified, possessing

$$\lim_{n,m\to\infty} d(x_n, x_m) = 0, \qquad \lim_{n,m\to\infty} d(y_n, y_m) = 0. \tag{38}$$

By (3), we have

$$\begin{aligned} d^s(x_n, x_m) &\leq 2d(x_n, x_m) = 0, \\ d^s(y_n, y_m) &\leq 2d(y_n, y_m) = 0. \end{aligned} \tag{39}$$

$\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in the metric space (X, d^s). Because (X, d) is complete, it is also the case for (X, d^s), then there exist $a, b \in X$ with

$$\lim_{n\to\infty} d^s(x_n, a) = 0, \qquad \lim_{n\to\infty} d^s(y_n, b) = 0. \tag{40}$$

On the other hand, we have

$$d^s(x_n, a) = 2d(x_n, a) - d(x_n, x_n) - d(a, a). \tag{41}$$

Getting the limit as $n \to \infty$ in the upward equation and utilizing (40) and (38), we attain

$$\lim_{n\to\infty} d(x_n, a) = \frac{1}{2} d(a, a), \tag{42}$$

in other words, possessing $d(a, a) \leq d(a, x_n)$ for all $n \in \mathbb{N}$. On letting $n \to \infty$, we achieve

$$d(a, a) \leq \lim_{n\to\infty} d(a, x_n). \tag{43}$$

Using (42) and (43), we get that

$$\lim_{n\to\infty} d(a, x_n) = d(a, a) = 0. \tag{44}$$

Analogously, one can show that

$$\lim_{n\to\infty} d(b, y_n) = d(b, b) = 0, \tag{45}$$

exposing $a = F(a, b)$, $a = G(a, b)$, $b = F(b, a)$, and $b = G(b, a)$. To do that we prove the following steps. □

Step 1. Demonstrate that $d(F(a, b), F(a, b)) = 0$ and $d(G(a, b), G(a, b)) = 0$.

Since $a \leq a$ and $b \leq b$, we have

$$\begin{aligned} &d(F(a,b),F(a,b)) \\ &\leq d(F(a,b),x_{2n+2}) + d(x_{2n+2},F(a,b)) - d(x_{2n+2},x_{2n+2}) \\ &\leq d(F(a,b),x_{2n+2}) + d(x_{2n+2},F(a,b)) \\ &= 2d(F(a,b),G(x_{2n+1},x_{2n+1})) \\ &\leq 2\varphi\left(\frac{d(a,x_{2n+1}) + d(b,y_{2n+1})}{2}\right) \\ &\leq d(a,x_{2n+1}) + d(b,y_{2n+1}). \end{aligned} \tag{46}$$

Letting $n \to \infty$ in (46) we get $d(F(a,b),F(a,b)) = 0$. The same one can demonstrate that $d(G(a,b),G(a,b)) = 0$.

Step 2. We show that $\lim_{n\to\infty} d(x_{2n+1},F(a,b)) = d(F(a,b),F(a,b))$ and $\lim_{n\to\infty} d(x_{2n+2},G(a,b)) = d(G(a,b),G(a,b))$.

We have $d(x_{2n+1},F(a,b)) = d(F(x_{2n},y_{2n}),F(a,b))$. Since $x_n \to a$ and $y_n \to b$ as $n \to \infty$ in (X,d) and F is continuous as $n \to \infty$ in (X,d), then we get

$$F(x_{2n},y_{2n}) \longrightarrow F(a,b). \tag{47}$$

That is,

$$\begin{aligned} \lim_{n\to\infty} d(x_{2n+1},F(a,b)) &= \lim_{n\to\infty} d(F(x_{2n},y_{2n}),F(a,b)) \\ &= d(F(a,b),F(a,b)). \end{aligned} \tag{48}$$

Similarly one can show that $G(x_{2n+1},y_{2n+1}) \to G(a,b)$.

Step 3. Indicating $a = F(a,b)$ and $a = G(a,b)$, we have

$$\begin{aligned} d(a,F(a,b)) &\leq d(a,x_{2n+1}) + d(x_{2n+1},F(a,b)) \\ &\quad - d(x_{2n+1},x_{2n+1}) \\ &\leq d(a,x_{2n+1}) + d(x_{2n+1},F(a,b)), \end{aligned} \tag{49}$$

While $n \to \infty$ in (49) and employing (46) and Steps 1 and 2, we obtain $d(a,F(a,b)) = 0$. By (pms_1) and (pms_2) in Definition 3, we have $a = F(a,b)$. Similarly one can show that $a = G(a,b)$, $b = F(b,a)$, and $b = G(b,a)$.

Theorem 11. *Intercalarily to the supposition of Theorem 10 assume that there exist x^*, y^* such that x^* is compared with y^*. Then $x^* = y^*$ for (x^*,y^*) is couple common fixed point. To wit, F and G possess a couple common fixed point and $F(x^*,x^*) = x^* = G(x^*,x^*)$.*

Proof. If x^* is comparable to y^*, at the time $F(x^*,y^*) = x^*$ is comparable to $F(y^*,x^*) = y^*$. So if we substitute $x = x^*$, $y = y^*$, $u = y^*$, and $v = x^*$ in (8), then we obtain

$$\begin{aligned} d(x^*,y^*) &= d(F(x^*,y^*),G(x^*,y^*)) \\ &\leq \varphi\Big((d(x^*,y^*) + d(y^*,x^*) \\ &\qquad + d(x^*,x^*) + d(y^*,y^*)) \times 2^{-1}\Big) \\ &\leq \varphi\left(\frac{d(x^*,y^*)}{2}\right) \\ &< \frac{d(x^*,y^*)}{2}. \end{aligned} \tag{50}$$

Therefore $x^* = y^*$. □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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