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Research Article

Existence Theorems of ε-Cone Saddle Points for Vector-Valued Mappings

Tao Chen

College of Culture and Tourism, Yunnan Open University, Kunming 650223, China

Correspondence should be addressed to Tao Chen; chentao1cq@126.com

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A new existence result of ε-vector equilibrium problem is first obtained. Then, by using the existence theorem of ε-vector equilibrium problem, a weakly ε-cone saddle point theorem is also obtained for vector-valued mappings.

1. Introduction

Saddle point problems are important in the areas of optimization theory and game theory. As for optimization theory, the main motivation of studying saddle point has been their connection with characterized solutions to minimax dual problems. Also, as for game theory, the main motivation has been the determination of two-person zero-sum games based on the minimax principle.

In recent years, based on the development of vector optimization, a great deal of papers have been devoted to the study of cone saddle points problems for vector-valued mappings and set-valued mappings, such as [1–8]. Nieuwenhuis [5] introduced the notion of cone saddle points for vector-valued functions in finite-dimensional spaces and obtained a cone saddle point theorem for general vector-valued mappings. Gong [2] established a strong cone saddle point theorem of vector-valued functions. Li et al. [4] obtained an existence theorem of lexicographic saddle point for vector-valued mappings. Bigi et al. [1] obtained a cone saddle point theorem by using an existence theorem of a vector equilibrium problem. Zhang et al. [9] established a general cone loose saddle point for set-valued mappings. Zhang et al. [8] obtained a minimax theorem and an existence theorem of cone saddle points for set-valued mappings by using Fan-Browder fixed point theorem. Some other types of existence results can be found in [3, 10–18].

On the other hand, in some situations, it may not be possible to find an exact solution for an optimization problem, or such an exact solution simply does not exist, for example, if the feasible set is not compact. Thus, it is meaningful to look for an approximate solution instead. There are also many papers to investigate the approximate solution problem, such as [19–21]. Kimura et al. [20] obtained several existence results for ε-vector equilibrium problem and the lower semicontinuity of the solution mapping of ε-vector equilibrium problem. Anh and Khanh [19] have considered two kinds of solution sets to parametric generalized ε-vector quasiequilibrium problems and established the sufficient conditions for the Hausdorff semicontinuity (or Berge semicontinuity) of these solution mappings. X. B. Li and S. J. Li [21] established some semicontinuity results on ε-vector equilibrium problem.

The aim of this paper is to characterize the ε-cone saddle point of vector-valued mappings. For this purpose, we first establish an existence theorem for ε-vector equilibrium problem. Then, by this existence result, we obtain an existence theorem for ε-cone saddle point of vector-valued mappings.

2. Preliminaries

Let X be a real Hausdorff topological vector space and let V be a real local convex Hausdorff topological vector space. Assume that S is a pointed closed convex cone in V with nonempty interior $\operatorname{int} S \neq \emptyset$. Let V^* be the topological dual space of V. Denote the dual cone of S by S^*:

$$S^* = \{s^* \in V^* : s^*(s) \geq 0, \forall s \in S\}. \quad (1)$$

Note that from Lemma 3.21 in [22] we have

$$z \in S \Longleftrightarrow \{\langle z^*, z\rangle \geq 0, \forall z^* \in S^*\},$$
$$z \in \operatorname{int} S \Longleftrightarrow \{\langle z^*, z\rangle > 0, \forall z^* \in S^* \setminus \{0\}\}. \quad (2)$$

Definition 1 (*see [7, 23]*). Let $f : X \to V$ be a vector-valued mapping. f is said to be S-upper semicontinuous on X if and only if, for each $x \in X$ and any $s \in \operatorname{int} S$, there exists an open neighborhood U_x of x such that

$$f(u) \in f(x) + s - \operatorname{int} S, \quad \forall u \in U_x. \quad (3)$$

f is said to be S-lower semicontinuous on X if and only if $-f$ is S-upper semicontinuous on X.

Lemma 2 (see [17]). *Let $f : X \times X \to V$ be a vector-valued mapping and $s^* \in S^* \setminus \{0\}$. If f is S-lower semicontinuous, then $s^* \circ f$ is lower semicontinuous.*

Definition 3 (*see [24]*). Let A and B be nonempty subsets of X and $f : A \times B \to V$ be a vector-valued mapping.

(i) f is said to be S-concavelike in its first variable on A if and only if, for all $x_1, x_2 \in A$ and $l \in [0,1]$, there exists $\overline{x} \in A$ such that

$$f(\overline{x}, y) \in lf(x_1, y) + (1-l) f(x_2, y) + S, \quad \forall y \in B. \quad (4)$$

(ii) f is said to be S-convexlike in its second variable on B if and only if, for all $y_1, y_2 \in B$ and $l \in [0,1]$, there exists $\overline{y} \in B$ such that

$$f(x, \overline{y}) \in lf(x, y_1) + (1-l) f(x, y_2) - S, \quad \forall x \in A. \quad (5)$$

(iii) f is said to be S-concavelike-convexlike on $A \times B$ if and only if f is S-concavelike in its first variable and S-convexlike in its second variable.

Definition 4. Let $A \subset V$ be a nonempty subset and $\varepsilon \in \operatorname{int} S$.

(i) A point $z \in A$ is said to be a weak ε-minimal point of A if and only if $A \cap (z - \varepsilon - \operatorname{int} S) = \emptyset$ and $\operatorname{Min}_\varepsilon A$ denotes the set of all weak ε-minimal points of A.

(ii) A point $z \in A$ is said to be a weak ε-maximal point of A if and only if $A \cap (z + \varepsilon + \operatorname{int} S) = \emptyset$ and $\operatorname{Max}_\varepsilon A$ denotes the set of all weak ε-maximal points of A.

Definition 5. Let $f : A \times B \to V$ be a vector-valued mapping and $\varepsilon \in \operatorname{int} S$. A point $(a, b) \in A \times B$ is said to be a weak ε-S-saddle point of f on $A \times B$ if

$$f(a, b) \in \operatorname{Max}_\varepsilon f(A, b) \bigcap \operatorname{Min}_\varepsilon f(a, B). \quad (6)$$

3. Existence of ε-Vector Equilibrium Problem

In this section, we deal with the following ε-vector equilibrium problem (for short VAEP). Find $\overline{x} \in E$ such that

$$f(x, y) + \varepsilon \notin -\operatorname{int} S, \quad \forall y \in E, \quad (7)$$

where $f : X \times X \to V$ is a vector-valued mapping, E is a nonempty subset of X, and $\varepsilon \in \operatorname{int} S$.

If $f(x, y) = g(y) - g(x)$, $x, y \in E$, and if $\overline{x} \in E$ is a solution of VAEP, then $\overline{x} \in E$ is a solution of ε-vector optimization of g, where g is a vector-valued mapping.

Denote the ε-solution set of (VAEP) by

$$S(\varepsilon) := \{\overline{x} \in E : f(x, y) + \varepsilon \notin -\operatorname{int} S, \forall y \in E\}. \quad (8)$$

Lemma 6 (see [20]). *Let E be a nonempty subset of X. Suppose that $f : X \times X \to V$ is a vector-valued mapping and the following conditions are satisfied:*

(i) $\operatorname{cl} E$ *is a compact set;*

(ii) $\{x \in \operatorname{cl} E : f(x, y) \notin -\operatorname{int} S, \forall y \in \operatorname{cl} E\} \neq \emptyset$;

(iii) f *is S-lower semicontinuous on* $\operatorname{cl} E \times \operatorname{cl} E$.

Then, for each $\varepsilon \in \operatorname{int} S$, $S(\varepsilon) \neq \emptyset$.

Next, we give a sufficient condition for the condition (ii) in Lemma 6.

Lemma 7. *Let E be a nonempty subset of X. Suppose that $f : X \times X \to V$ is a vector-valued mapping with $f(x, x) = 0$ for all $x \in X$ and the following conditions are satisfied:*

(i) $\operatorname{cl} E$ *is a compact set;*

(ii) f *is S-concavelike-convexlike on* $\operatorname{cl} E \times \operatorname{cl} E$;

(iii) *for each $x \in \operatorname{cl} E$, $f(x, \cdot)$ is S-lower semicontinuous on* $\operatorname{cl} E$.

Then, there exists $\overline{x} \in \operatorname{cl} E$ such that

$$f(\overline{x}, y) \notin -\operatorname{int} S, \quad \forall y \in \operatorname{cl} E. \quad (9)$$

Proof. For any $t < 0$ and $s^* \in S^* \setminus \{0\}$, we define a multifunction $G : \operatorname{cl} E \to 2^{\operatorname{cl} E}$ by

$$G(x) = \{y \in \operatorname{cl} E : s^*(f(x, y)) \leq t\}, \quad \forall x \in \operatorname{cl} E. \quad (10)$$

First, by assumptions, we must have

$$\bigcap_{x \in \operatorname{cl} E} G(x) = \emptyset. \quad (11)$$

In fact, if there exists $\overline{y} \in \operatorname{cl} E$ such that $\overline{y} \in G(x)$, for all $x \in \operatorname{cl} E$, then

$$s^*(f(x, \overline{y})) \leq t, \quad \forall x \in \operatorname{cl} E. \quad (12)$$

Particularly, taking $x = \overline{y}$, we have $0 = s^*(f(\overline{y}, \overline{y})) \leq t$, which contradicts the assumption about t.

Then, by Lemma 2, $G(x)$ is a closed set, for each $x \in \operatorname{cl} E$. By (11), for any $y \in \operatorname{cl} E$, we have

$$y \in V \setminus \bigcap_{x \in \operatorname{cl} E} G(x) = \bigcup_{x \in \operatorname{cl} E} V \setminus G(x). \quad (13)$$

Since $\operatorname{cl} E$ is compact, there exists a finite point set $\{x_1, x_2, \ldots, x_n\}$ in $\operatorname{cl} E$ such that

$$\operatorname{cl} E \subset \bigcup_{1 \leq i \leq n} V \setminus G(x_i). \quad (14)$$

Namely, for each $y \in \mathrm{cl}\, E$, there exists $i \in \{1, 2, \ldots, n\}$ such that

$$s^*(f(x_i, y)) > t. \quad (15)$$

Now, we consider the set

$$M := \{(z_1, z_2, \ldots, z_n, r) \in R^{n+1} \mid \exists y \in \mathrm{cl}\, E, \; s^*(f(x_i, y)) \le r + z_i, \forall i = 1, 2, \ldots, n\}. \quad (16)$$

Obviously, by the condition (ii), M is a convex set. By (15), we have the fact that $(0_{R^n}, t) \notin M$.

By the separation theorem of convex sets, there exists $(\lambda_1, \lambda_2, \ldots, \lambda_n, \overline{r}) \neq 0_{R^n}$ such that

$$\sum_{i=1}^{n} \lambda_i z_i + \overline{r} r \ge \overline{r} t, \quad \forall (z_1, z_2, \ldots, z_n, r) \in M. \quad (17)$$

Since $M + R^{n+1} \subset M$, we can get $\lambda_i \ge 0$ and $\overline{r} \ge 0$, for all $i = 1, 2, \ldots, n$. By the definition of M, for each $y \in \mathrm{cl}\, E$,

$$\left(0_{R^n}, 1 + \max_{1 \le i \le n} s^*(f(x_i, y))\right) \in \mathrm{int}\, M, \quad (18)$$

$$(s^*(f(x_1, y)) - r, \; s^*(f(x_2, y)) - r, \ldots, s^*(f(x_n, y)) - r, r) \in M. \quad (19)$$

By (18), $\overline{r} > 0$. Then, by (17) and (19),

$$\sum_{i=1}^{n} \frac{\lambda_i}{\overline{r}} s^*(f(x_i, y)) + r\left(1 - \sum_{i=1}^{n} \frac{\lambda_i}{\overline{r}}\right) \ge t. \quad (20)$$

By (20), $\sum_{i=1}^{n} (\lambda_i / \overline{r}) = 1$. Thus, by the condition (ii), for each $y \in \mathrm{cl}\, E$, there exists $\overline{x} \in \mathrm{cl}\, E$ such that

$$s^*(f(\overline{x}, y)) \ge \sum_{i=1}^{n} \frac{\lambda_i}{\overline{r}} s^*(f(x_i, y)) \ge t. \quad (21)$$

By the assumption about t and s^*, there exists $\overline{x} \in \mathrm{cl}\, E$ such that

$$f(\overline{x}, y) \notin -\mathrm{int}\, S, \quad \forall y \in \mathrm{cl}\, E. \quad (22)$$

This completes the proof. □

By Lemmas 6 and 7, we can get the following result.

Theorem 8. *Let E be a nonempty subset of X. Suppose that $f : X \times X \to V$ is a vector-valued mapping with $f(x, x) = 0$ for all $x \in X$ and the following conditions are satisfied:*

(i) $\mathrm{cl}\, E$ *is a compact set;*

(ii) f *is S-concavelike-convexlike on* $\mathrm{cl}\, E \times \mathrm{cl}\, E$*;*

(iii) f *is S-lower semicontinuous on* $\mathrm{cl}\, E \times \mathrm{cl}\, E$.

Then, for each $\varepsilon \in \mathrm{int}\, S$, $S(\varepsilon) \neq \emptyset$.

Remark 9. Note that the condition (i) does not require the fact that $\mathrm{cl}\, E$ is a convex set. So Theorem 8 is different from Theorem 3.2 in [20]. The following example explains this case.

Example 10. Let $X = R$, $V = R^2$, and $E = [0, 1/3] \cup [2/3, 1]$,

$$f(x, y) = \{(xy, xz) \in R^2 \mid z = 1 - y^2\}, \quad x, y \in X, \qquad S = \{(x, y) \in R^2 \mid x \ge 0, y \ge 0\}. \quad (23)$$

Obviously, $\mathrm{cl}\, E$ is a compact set. However, $\mathrm{cl}\, E$ is not a convex set. So, Theorem 3.2 in [20] is not applicable. By the definition of f, f is S-concavelike-convexlike on $\mathrm{cl}\, E \times \mathrm{cl}\, E$ and S-lower semicontinuous on $\mathrm{cl}\, E \times \mathrm{cl}\, E$. Thus, all conditions of Theorem 8 hold. Indeed, for each $\varepsilon = (\varepsilon_1, \varepsilon_2) \in \mathrm{int}\, S$,

$$f(0, y) + \varepsilon = (\varepsilon_1, \varepsilon_2) \notin -\mathrm{int}\, S, \quad \forall y \in E. \quad (24)$$

Namely, $0 \in S(\varepsilon)$.

4. Existence of ε-Cone Saddle Points

Lemma 11. *Let E be a nonempty subset of X and $E = A \times B$. Let $\varepsilon \in \mathrm{int}\, S$ and let $f : X \times X \to V$ be a vector-valued mapping with $f(x, y) = g(a, v) - g(u, b)$, where $x = (a, b)$, $y = (u, v)$, $a, u \in A$, and $v, b \in B$. If there exists $\overline{x} = (\overline{a}, \overline{b}) \in E$ such that*

$$f(\overline{x}, y) + \varepsilon \notin -\mathrm{int}\, S, \quad \forall y \in E, \quad (25)$$

then $(\overline{a}, \overline{b}) \in A \times B$ is a weak ε-S-saddle point of g on $A \times B$.

Proof. By assumptions, we have

$$f(\overline{x}, y) + \varepsilon \notin -\mathrm{int}\, S, \quad \forall y \in E. \quad (26)$$

Then,

$$g(\overline{a}, v) - g(u, \overline{b}) + \varepsilon \notin -\mathrm{int}\, S, \quad \forall (u, v) \in A \times B. \quad (27)$$

By (27), taking $u = \overline{a}$,

$$g(\overline{a}, v) - g(\overline{a}, \overline{b}) + \varepsilon \notin -\mathrm{int}\, S, \quad \forall v \in B, \quad (28)$$

which implies $g(\overline{a}, \overline{b}) \in \mathrm{Min}_{\varepsilon} g(\overline{a}, B)$. Then, by (27), taking $v = \overline{b}$,

$$g(\overline{a}, \overline{b}) - g(u, \overline{b}) + \varepsilon \notin -\mathrm{int}\, S, \quad \forall u \in A, \quad (29)$$

which implies $g(\overline{a}, \overline{b}) \in \mathrm{Max}_{\varepsilon} g(A, \overline{b})$. Thus, $(\overline{a}, \overline{b}) \in A \times B$ is a weak ε-S-saddle point of g on $A \times B$. This completes the proof. □

Theorem 12. *Let A and B be nonempty sets and $\varepsilon \in \mathrm{int}\, S$. Suppose that g is a vector-valued mapping and the following conditions are satisfied:*

(i) $\mathrm{cl}\, A$ *and* $\mathrm{cl}\, B$ *are compact sets;*

(ii) g *is S-concavelike-convexlike on* $\mathrm{cl}\, A \times \mathrm{cl}\, B$*;*

(iii) g *is S-upper semicontinuous on* $\mathrm{cl}\, A \times \mathrm{cl}\, B$*;*

(iv) g *is S-lower semicontinuous on* $\mathrm{cl}\, A \times \mathrm{cl}\, B$.

Then, g has a weak ε-S-saddle point on $A \times B$.

Proof. Let $A \times B = E$ and $f : \mathrm{cl}\, E \times \mathrm{cl}\, E \rightarrow V$ be a vector-valued mappings by

$$f(x, y) = g(a, v) - g(u, b), \quad \forall\ x = (a, b) \in \mathrm{cl}\, E, \quad y = (u, v) \in \mathrm{cl}\, E. \tag{30}$$

Next, we show that all assumptions of Theorem 8 are satisfied by g.

Clearly, by the condition (i), $\mathrm{cl}\, E$ is compact. Then, by the condition (ii), we have the fact that, for each $a_1, a_2 \in \mathrm{cl}\, A$ and $l \in [0, 1]$, there exists $a_3 \in \mathrm{cl}\, A$ such that

$$g(a_3, b) \in lg(a_1, b) + (1 - l)\, g(a_2, b) + S, \quad \forall b \in \mathrm{cl}\, B \tag{31}$$

and, for each $b_1, b_2 \in \mathrm{cl}\, B$ and $l \in [0, 1]$, there exists $b_3 \in \mathrm{cl}\, B$

$$g(a, b_3) \in lg(a, b_1) + (1 - l)\, g(a, b_2) - S, \quad \forall a \in \mathrm{cl}\, A. \tag{32}$$

By (31) and (32), for each $(a_1, b_1), (a_2, b_2) \in \mathrm{cl}\, E$ and $l \in [0, 1]$, there exists $(a_3, b_3) \in \mathrm{cl}\, E$ such that

$$\begin{aligned} g(a_3, b) - g(a, b_3) &\in l\left(g(a_1, b) - g(a, b_1)\right) \\ &\quad + (1 - l)\left(g(a_2, b) - g(a, b_2)\right) + S, \\ &\quad \forall (a, b) \in \mathrm{cl}\, E, \\ g(a, b_3) - g(a_3, b) &\in l\left(g(a, b_1) - g(a_1, b)\right) \\ &\quad + (1 - l)\left(g(a, b_2) - g(a_2, b)\right) - S, \\ &\quad \forall (a, b) \in \mathrm{cl}\, E. \end{aligned} \tag{33}$$

Namely, f is S-concavelike-convexlike on $\mathrm{cl}\, E \times \mathrm{cl}\, E$.

Now, we show that f is S-lower semicontinuous on $\mathrm{cl}\, E \times \mathrm{cl}\, E$. By the condition (iii), for each $(a, v) \in \mathrm{cl}\, A \times \mathrm{cl}\, B$ and $s \in \mathrm{int}\, S$, there exists an open neighborhood U_a of a and U_v of v such that

$$g(u_a, u_v) \in g(a, v) - \frac{s}{2} + \mathrm{int}\, S, \quad \forall u_a \in U_a,\ u_v \in U_v, \tag{34}$$

and, for each $(u, b) \in \mathrm{cl}\, A \times \mathrm{cl}\, B$ and $s \in \mathrm{int}\, S$, there exists an open neighborhood U_u of u and U_b of b such that

$$g(u_u, u_b) \in g(u, b) + \frac{s}{2} - \mathrm{int}\, S, \quad \forall u_u \in U_u,\ u_b \in U_b. \tag{35}$$

By (34) and (35), we have the fact that, for any $((a, b), (u, v)) \in \mathrm{cl}\, E \times \mathrm{cl}\, E$,

$$g(u_a, u_v) - g(u_u, u_b) \in g(a, v) - g(u, b) - s + \mathrm{int}\, S, \quad \forall\left((u_a, u_b), (u_u, u_v)\right) \in U_a \times U_b \times U_u \times U_v. \tag{36}$$

Namely, f is S-lower semicontinuous on $\mathrm{cl}\, E \times \mathrm{cl}\, E$. Therefore, by Lemma 11, g has a weak ε-S-saddle point on $A \times B$. This completes the proof. □

Remark 13. The conditions (iii) and (iv) of Theorem 12 do not imply that g is continuous (see [23]).

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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