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Research Article

Haar Wavelet Method for the System of Integral Equations

Hassan A. Zedan[1,2] and Eman Alaidarous[1]

[1] *Mathematics Department, Faculty of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia*
[2] *Mathematics Department, Faculty of Science, Kafrelsheikh University, Kafrelsheikh, Egypt*

Correspondence should be addressed to Hassan A. Zedan; hassanzedan2003@yahoo.com

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We employed the Haar wavelet method to find numerical solution of the system of Fredholm integral equations (SFIEs) and the system of Volterra integral equations (SVIEs). Five test problems, for which the exact solution is known, are considered. Comparison of the results is obtained by the Haar wavelet method with the exact solution.

1. Introduction

The Haar wavelet was initiated and independently developed by Haar [1] in the early nineteen tens. In recent years, many different methods and different basis functions have been used to estimate the solution of the system of integral equations, such as Adomian decomposition method [1, 2], Taylor's expansion method [3, 4], homotopy perturbation method [5, 6], projection method and Nystrom method [7], Spline collocation method [8], Runge-Kutta method [9], sinc method [10], Tau method [11], block-pulse functions, hat basis functions method [12], and operational matrices [13, 14].

In the present paper, we use Haar wavelet method to solve the system of linear Fredholm integral equations (SLFIEs) of the second kind:

$$\mathbf{U}(x) - \int_0^1 \mathbf{K}(x,t)\,\mathbf{U}(t)\,dt = \mathbf{F}(x), \quad 0 < x < 1, \tag{1}$$

and the system of linear Volterra integral equations (SLVIEs) of the second kind:

$$\mathbf{U}(x) - \int_0^x \mathbf{K}(x,t)\,\mathbf{U}(t)\,dt = \mathbf{F}(x), \quad 0 < x < 1, \tag{2}$$

where

$$\begin{aligned} \mathbf{U}(x) &= \left[u_1(x), u_2(x), \ldots, u_n(x)\right]^T, \\ \mathbf{F}(x) &= \left[f_1(x), f_2(x), \ldots, f_n(x)\right]^T, \\ \mathbf{K}(x,t) &= k_{ij}(x,t), \quad i, j = 1, 2 \ldots, n. \end{aligned} \tag{3}$$

In (1) and (2), the functions $\mathbf{K}$ and $\mathbf{F}$ are given, and $\mathbf{U}$ is the vector function of the solution of systems (1) and (2) that will be determined. Also, we assume that (1) and (2) have a unique solution.

2. Haar Wavelet Method

Let us confine to the time interval $t \in [0, 1]$. The Haar wavelet family is

$$h_i(x) = \begin{cases} 1 & \text{for } t \in [\tau_1, \tau_2] \\ -1 & \text{for } t \in [\tau_2, \tau_3] \\ 0 & \text{elsewhere.} \end{cases} \tag{4}$$

Here the notations are as follows:

$$\tau_1 = \frac{k}{m}, \qquad \tau_2 = \frac{k + (1/2)}{m}, \qquad \tau_3 = \frac{k+1}{m}. \tag{5}$$

The integer $m = 2^j$, $j = 0, 1, \ldots, J$, indicates the level of the wavelet; $k = 0, 1, \ldots, m-1$ is the translation parameter. The integer J determines the maximal level of resolution. The index i is calculated from the formula $i = m + k + 1$; the minimal value for which (4) holds is $i = 2$ (then $m = 1$, $k = 0$); the maximal value is $i = 2M$, where $M = 2^J$. The index $i = 1$ corresponds to the scaling function of the Haar wavelet $h_1(t) \equiv 1$.

Simple calculations show the following.

We have that

$$\int_0^1 h_i(t)\, h_l(t)\, dt = \begin{cases} \dfrac{1}{m} & \text{for } i = l \\ 0 & \text{for } i \neq l; \end{cases} \tag{6}$$

consequently, the functions $h_i(t)$ are orthogonal.

Next we discretize the functions $h_i(x)$ by dividing the interval $x \in [0, 1]$ into $2M$ parts of equal length $\Delta x = 1/(2M)$ and introduce the collocation points:

$$x_l = \frac{l - (1/2)}{2M}, \quad l = 1, 2, \ldots, 2M. \tag{7}$$

Following Chen and Hsiao [15, 16] the coefficients matrix $H_{il} = h_i(t_l)$ is introduced (this is a $2M \times 2M$ matrix). A function $u(t)$ which is defined in the interval $x \in [0, 1]$ can be expanded into the Haar wavelet series:

$$u(x) = \sum_{i=1}^{2M} a_i h_i(x), \tag{8}$$

where a is the wavelet coefficients. The discrete form of this equation is

$$u(x_l) = \sum_{i=1}^{2M} a_i h_i(x_l) = \sum_{i=1}^{2M} a_i H_{il} \tag{9}$$

or in a matrix presentation $u = aH$, where u and a are $2M$ dimensional row vectors.

3. System of Fredholm Integral Equation

Let us consider SLFIEs (1) at $(i, j, n = 1, 2)$:

$$\begin{aligned} u(x) - \int_0^1 k_{11}(x,t)\, u(t)\, dt - \int_0^1 k_{12}(x,t)\, v(t)\, dt &= f_1(x), \\ v(x) - \int_0^1 k_{21}(x,t)\, u(t)\, dt - \int_0^1 k_{22}(x,t)\, v(t)\, dt &= f_2(x), \\ x &\in [0, 1]. \end{aligned} \tag{10}$$

Let

$$u(x) = \sum_{i=1}^{2M} a_i h_i(x), \qquad v(x) = \sum_{i=1}^{2M} b_i h_i(x). \tag{11}$$

Replacing (12) into (10) and (11) we find

$$\begin{aligned} \sum_{i=1}^{2M} a_i h_i(x) - \sum_{i=1}^{2M} a_i G_{1i}(x) - \sum_{i=1}^{2M} b_i G_{2i}(x) &= f_1(x), \\ \sum_{i=1}^{2M} b_i h_i(x) - \sum_{i=1}^{2M} a_i G_{3i}(x) - \sum_{i=1}^{2M} b_i G_{4i}(x) &= f_2(x), \end{aligned} \tag{12}$$

where

$$\begin{aligned} G_{1i}(x) &= \int_0^1 k_{11}(x,t)\, h_i(t)\, dt, \\ G_{2i}(x) &= \int_0^1 k_{12}(x,t)\, h_i(t)\, dt, \\ G_{3i}(x) &= \int_0^1 k_{21}(x,t)\, h_i(t)\, dt, \\ G_{4i}(x) &= \int_0^1 k_{22}(x,t)\, h_i(t)\, dt. \end{aligned} \tag{13}$$

Next we will evaluate the wavelet coefficients a_i and b_i in the following way.

Collocation Method. Satisfying (12) only at the collocation points (7) we get a system of linear equations:

$$\begin{aligned} \sum_{i=1}^{2M} \left[a_i \left(h_i(t_l) - G_{1i}(x_l)\right) - b_i G_{2i}(x_l)\right] &= f_1(x_l), \\ \sum_{i=1}^{2M} \left[b_i \left(h_i(t_l) - G_{4i}(x_l)\right) - a_i G_{3i}(x_l)\right] &= f_2(x_l), \\ l &= 1, 2, \ldots, 2M. \end{aligned} \tag{14}$$

Therefore the matrix form of this system is given by

$$\begin{gathered} a\left(H - G_1\right) - bG_2 = F_1, \quad b\left(H - G_4\right) - aG_3 = F_2, \\ \text{where } \mathbf{G}_{il} = \mathbf{G}_i(x_l), \ \mathbf{F}_l = \mathbf{f}(x_l). \end{gathered} \tag{15}$$

Now, we can present the following problems.

Example 1. Consider the following SLFIEs [3, 16]:

$$\begin{aligned} &u(x) - \int_0^1 (x-t)^3 u(t)\, dt - \int_0^1 (x-t)^2 v(t)\, dt \\ &\quad = \frac{1}{20} - \frac{11}{30}x + \frac{5}{3}x^2 - \frac{1}{3}x^3, \\ &v(x) - \int_0^1 (x-t)^4 u(t)\, dt - \int_0^1 (x-t)^3 v(t)\, dt \\ &\quad = -\frac{1}{30} - \frac{41}{60}x + \frac{3}{20}x^2 + \frac{23}{12}x^3 - \frac{1}{3}x^4. \end{aligned} \tag{16}$$

Carrying out the integration in (13) we obtain

$$G_1(x) = \begin{cases} x\left(1+x^2\right) - \dfrac{1+6x^2}{4} & \text{for } i=1 \\ \dfrac{7-24mx+24\left(k+k^2-2kmx+m^2x^2\right)}{32m^4} & \text{for } i>1, \end{cases}$$

$$G_2(x) = \begin{cases} \dfrac{1}{3} + (x-1)\,x & \text{for } i=1 \\ -\dfrac{1+2k-2mx}{4m^3} & \text{for } i>1, \end{cases}$$

$$G_3(x) = \begin{cases} \dfrac{1}{5} + x(x-1)(1+x(x-1)) & \text{for } i=1 \\ -\left((1+2k-2mx)\left(3+8k^2+k(8-16mx) + 8mx(-1+mx)\right)\right) \times \left(16m^5\right)^{-1} & \text{for } i>1, \end{cases}$$

$$G_4(x) = \begin{cases} x\left(1+x^2\right) - \dfrac{1+6x^2}{4} & \text{for } i=1 \\ \dfrac{7-24mx+24\left(k+k^2-2kmx+m^2x^2\right)}{32m^4} & \text{for } i>1. \end{cases} \quad (17)$$

We apply collocation method, and vectors a_i and b_i can be calculated from (15); the functions $u(x)$ and $v(x)$ are evaluated from (11).

Computations were carried out for different values of J. These results were compared with the exact solution $u(x) = x^2$ and $v(x) = -x + x^2 + x^3$.

The accuracy of the results (see Table 1 and Figure 1) was estimated by the error function for $u(x)$ and $v(x)$:

$$e_J = \max_{1\le l\le 2M} \left|u(x_l) - u_{ex}(x_l)\right|,$$
$$e_J = \max_{1\le l\le 2M} \left|v(x_l) - v_{ex}(x_l)\right|, \quad (18)$$

where x_l is defined by (7).

Example 2. Consider the following SLFIEs given by [10, 12]:

$$u(x) + \int_0^1 e^{x-t} u(x)\,dt + \int_0^1 e^{(x+2)t} v(x)\,dt = 2e^x + \frac{e^{x+1}-1}{x+1},$$
$$v(x) + \int_0^1 e^{xt} u(x)\,dt + \int_0^1 e^{x+t} v(x)\,dt = e^x + e^{-x} + \frac{e^{x+1}-1}{x+1}. \quad (19)$$

Table 1: Error of $u(x)$ and $v(x)$ of Example 1.

J	$2M$	Error of function $u(x)$	Error of function $v(x)$
2	8	$1.45E-3$	$1.57E-3$
3	16	$3.65E-4$	$4.16E-4$
4	32	$9.19E-5$	$1.06E-4$
5	64	$2.29E-5$	$2.68E-5$

Carrying out the integration in (13) we obtain

$$G_1(x) = \begin{cases} (-1+e)\,e^{-1+x} & \text{for } i=1 \\ e^{-((1+k)/m)+x}\left(-1+e^{1/2m}\right)^2 & \text{for } i>1, \end{cases}$$

$$G_2(x) = \begin{cases} \dfrac{-1+e^{2+x}}{2+x} & \text{for } i=1 \\ -\dfrac{e^{k(2+x)/m}\left(-1+e^{(2+x)/2m}\right)^2}{2+x} & \text{for } i>1, \end{cases}$$

$$G_3(x) = \begin{cases} \dfrac{-1+e^{x}}{x} & \text{for } i=1 \\ -\dfrac{x-e^{kx/m}\left(-1+e^{x/2m}\right)^2/x}{4m^2} & \text{for } i>1, \end{cases} \quad (20)$$

$$G_4(x) = \begin{cases} (-1+e)\,e^{x} & \text{for } i=1 \\ -e^{(k/m)+x}\left(-1+e^{1/2m}\right)^2 & \text{for } i>1. \end{cases}$$

We apply collocation method, and vectors a_i and b_i can be calculated from (15); the functions $u(x)$ and $v(x)$ are evaluated from (11).

Computations were carried out for different values of J. These results were compared with the exact solution (see Table 2 and Figure 2) $u(x) = e^x$ and $v(x) = e^{-x}$.

Example 3. Consider the following SLFIEs given by [10]:

$$u(x) - \int_0^1 \frac{x+t}{3} u(t)\,dt - \int_0^1 \frac{x+t}{3} v(t)\,dt = \frac{x}{18} + \frac{17}{36},$$
$$v(x) - \int_0^1 xtu(t)\,dt - \int_0^1 xtv(t)\,dt = x^2 - \frac{19}{12} + 1. \quad (21)$$

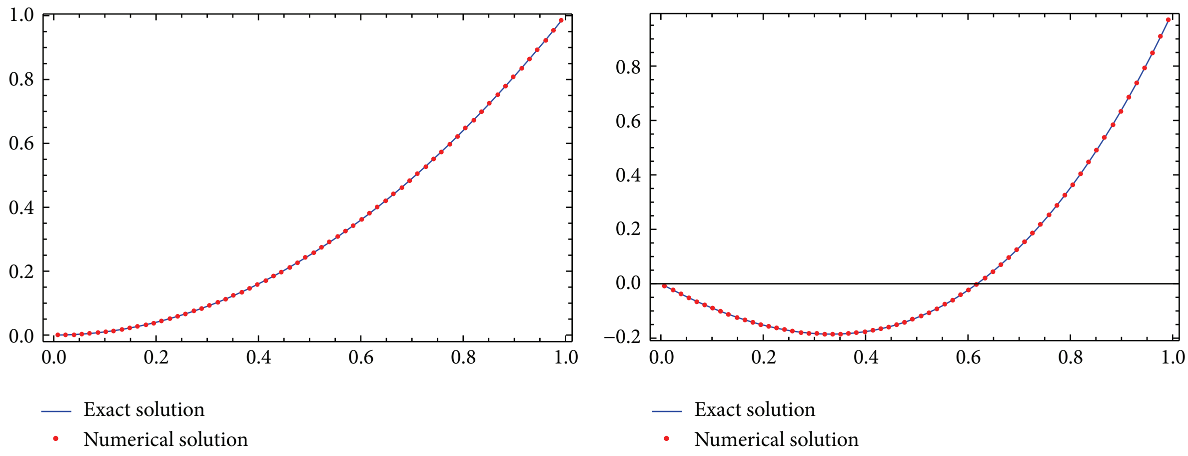
Carrying out the integration in (13) we obtain

$$G_j(x) = \begin{cases} \dfrac{1}{6} + \dfrac{x}{3} & \text{for } i=1 \\ -\dfrac{1}{12m^2} & \text{for } i>1 \end{cases} \quad j = 1,2,3,4\ . \quad (22)$$

We apply collocation method, and vectors a_i and b_i can be calculated from (15); the functions $u(x)$ and $v(x)$ are evaluated from (11).

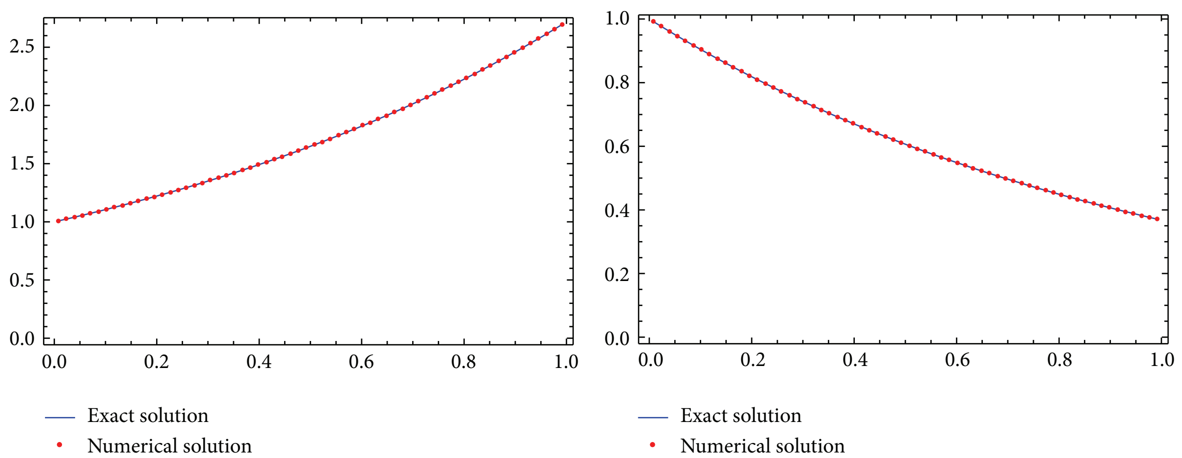
Computations were carried out for different values of J (see Table 3 and Figure 3). These results were compared with the exact solution $u(x) = x + 1$ and $v(x) = x^2 + 1$.

By comparing the results obtained with the results found in [3, 10, 12], we find that the results we have obtained are accurate and error rate, which is much less but almost



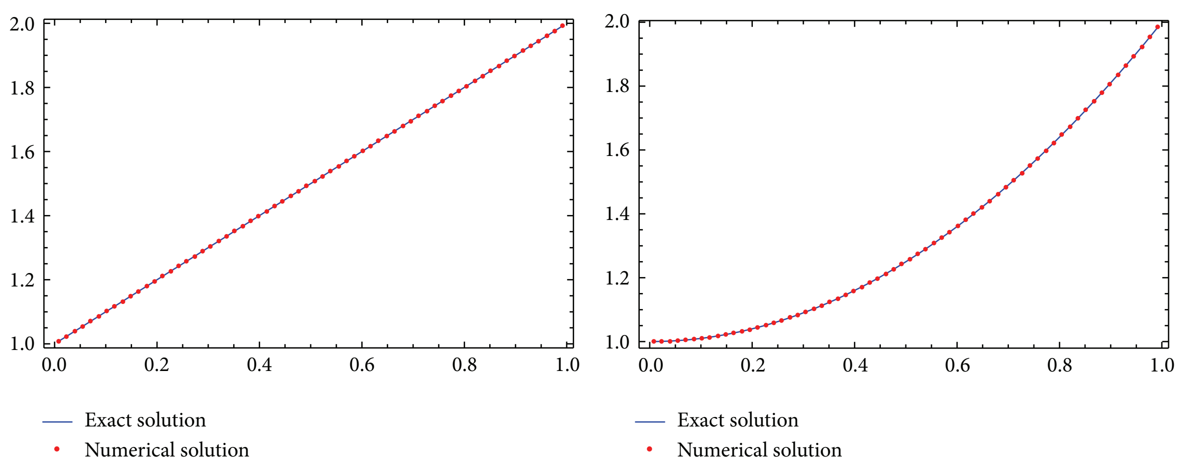
(a) (b)

Figure 1: (a) Exact and Haar wavelet solution of $u(x)$ at $J = 5$. (b) Exact and Haar wavelet solution of $v(x)$ at $J = 5$.



(a) (b)

Figure 2: (a) Exact and Haar wavelet solution of $u(x)$ at $J = 5$. (b) Exact and Haar wavelet solution of $v(x)$ at $J = 5$.



(a) (b)

Figure 3: (a) Exact and Haar wavelet solution of $u(x)$ at $J = 5$. (b) Exact and Haar wavelet solution of $v(x)$ at $J = 5$.

Table 2: Error of $u(x)$ and $v(x)$ of Example 2.

J	$2M$	Error of function $u(x)$	Error of function $v(x)$
2	8	$2.28E-2$	$7.30E-3$
3	16	$5.79E-3$	$1.82E-3$
4	32	$1.45E-3$	$4.56E-4$
5	64	$2.95E-4$	$4.62E-5$

Table 3: Error of $u(x)$ and $v(x)$ of Example 3.

J	$2M$	Error of function $u(x)$	Error of function $v(x)$
2	8	$6.15E-3$	$8.22E-3$
3	16	$1.57E-3$	$2.13E-3$
4	32	$3.96E-4$	$5.42E-4$
5	64	$9.95E-5$	$1.36E-4$

nonexistent compared to other methods, is also shown in tables of error attached and this shows the importance of the method used.

4. System of Volterra Integral Equations

Let us consider system of linear Volterra integral equations (SLVIEs) (2) at ($i, j = 1, 2, n = 2$):

$$\begin{gathered} u(x) - \int_0^x k_{11}(x,t)\, u(t)\, dt - \int_0^x k_{12}(x,t)\, v(t)\, dt = f_1(x), \\ v(x) - \int_0^x k_{21}(x,t)\, u(t)\, dt - \int_0^x k_{22}(x,t)\, v(t)\, dt = f_2(x), \\ x \in [0,1]. \end{gathered} \tag{23}$$

Its discrete form is

$$\begin{aligned} u(x_l) &- \int_0^{x_l} k_{11}(x_l,t)\, u(t)\, dt \\ &- \int_0^{x_l} k_{12}(x_l,t)\, v(t)\, dt = f_1(x_l), \\ v(x_l) &- \int_0^{x_l} k_{21}(x_l,t)\, u(t)\, dt \\ &- \int_0^{x_l} k_{22}(x_l,t)\, v(t)\, dt = f_2(x_l), \\ & x \in [0,1], \end{aligned} \tag{24}$$

where x_l defined in (7) are the collocation points.

Let

$$u(x) = \sum_{i=1}^{2M} a_i h_i(x), \qquad v(x) = \sum_{i=1}^{2M} b_i h_i(x). \tag{25}$$

Replacing (25) with (24) we find

$$\begin{gathered} \sum_{i=1}^{2M} a_i h_i(x_l) - \sum_{i=1}^{2M} a_i (G_{11})_i(x_l) - \sum_{i=1}^{2M} b_i (G_{12})_i(x_l) = f_1(x_l), \\ \sum_{i=1}^{2M} b_i h_i(x_l) - \sum_{i=1}^{2M} a_i (G_{21})_i(x_l) - \sum_{i=1}^{2M} b_i (G_{22})_i(x_l) = f_2(x_l). \end{gathered} \tag{26}$$

The matrices $(G_{sr})_{il} = (G_s)_i(x_l)$, $s, r = 1, 2$, are now defined as

$$\begin{aligned} (G_{11})_{il} &= \int_0^{x_l} k_{11}(x_l,t)\, h_i(t)\, dt, \\ (G_{12})_{il} &= \int_0^{x_l} k_{12}(x_l,t)\, h_i(t)\, dt, \\ (G_{21})_{il} &= \int_0^{x_l} k_{21}(x_l,t)\, h_i(t)\, dt, \\ (G_{22})_{il} &= \int_0^{x_l} k_{22}(x_l,t)\, h_i(t)\, dt. \end{aligned} \tag{27}$$

By computing these integrals the following cases should be distinguished:

$$(G_{sr})_{il} = \begin{cases} 0 & \text{if } x_l < \tau_1 \\ \displaystyle\int_{\tau_1}^{x_l} k_{sr}(x_l,t)\, dt & \text{if } \tau_1 \le x_l < \tau_2 \\ \displaystyle\int_{\tau_1}^{\tau_2} k_{sr}(x_l,t)\, dt - \int_{\tau_2}^{x_l} k_{sr}(x_l,t)\, dt & \text{if } \tau_2 \le x_l < \tau_3 \\ \displaystyle\int_{\tau_1}^{\tau_2} k_{sr}(x_l,t)\, dt - \int_{\tau_2}^{\tau_3} k_{sr}(x_l,t)\, dt & \text{if } x_l \ge \tau_3. \end{cases} \tag{28}$$

Next we will evaluate the wavelet coefficients a_i and b_i in the following way.

Collocation Method. Satisfying (27) and (28) only at the collocation points (7) we get a system of linear equations

$$\begin{gathered} \sum_{i=1}^{2M} [a_i (h_i(t_l) - (G_{11})_i(x_l)) - b_i (G_{12})_i(x_l)] = f_1(x_l), \\ \sum_{i=1}^{2M} [b_i (h_i(t_l) - (G_{22})_i(x_l)) - a_i (G_{21})_i(x_l)] = f_2(x_l), \\ l = 1, 2, \ldots, 2M. \end{gathered} \tag{29}$$

The matrix form of this system is

$$a(H - G_{11}) - bG_{12} = F_1, \qquad b(H - G_{22}) - aG_{21} = F_2. \tag{30}$$

Example 4. Consider the following (SLVIEs) [17]:

$$
\begin{aligned}
&u(x) - \int_0^x xtu(t)\,dt - \int_0^x (x+t)\,v(t)\,dt \\
&\quad = e^{2x}\left(-\frac{1}{2}x^2 + \frac{1}{4}x + 1\right) + e^{-2x}\left(x + \frac{1}{4}\right) - \frac{3}{4}x - \frac{1}{4}, \\
&v(x) - \int_0^x (x-t)\,u(t)\,dt - \int_0^x (x+t)^2 v(t)\,dt \\
&\quad = e^{-2x}\left(2x^2 + x + \frac{5}{4}\right) - \frac{1}{4}e^{2x} - \frac{1}{2}x^2.
\end{aligned}
\tag{31}
$$

Carrying out the integration in (28) we obtain

$$
\begin{aligned}
&(G_{11})_{il} = \begin{cases} 0 & \text{for } x_l < \tau_1 \\ \frac{1}{2}\left(x_l^3 - x_l\tau_1^2\right) & \text{for } \tau_1 \le x_l < \tau_2 \\ -\frac{1}{2}x_l\left(x_l^2 + \tau_1^2 - 2\tau_2^2\right) & \text{for } \tau_2 \le x_l < \tau_3 \\ -\frac{1}{2}x_l\left(\tau_1^2 - 2\tau_2^2 + \tau_3^2\right) & \text{for } x_l \ge \tau_3, \end{cases} \\
&(G_{12})_{il} = \begin{cases} 0 \quad \text{for } x_l < \tau_1 \\ \frac{1}{2}(x_l - \tau_1)(3x_l + \tau_1) \\ \quad \text{for } \tau_1 \le x_l < \tau_2 \\ -\frac{3x_l^2}{2} - \frac{\tau_1^2}{2} - x_l(\tau_1 - 2\tau_2) + \tau_2^2 \\ \quad \text{for } \tau_2 \le x_l < \tau_3 \\ -\frac{\tau_1^2}{2} + \tau_2^2 - \frac{\tau_3^2}{2} - x_l(\tau_1 - 2\tau_2 + \tau_3) \\ \quad \text{for } x_l \ge \tau_3, \end{cases} \\
&(G_{21})_{il} = \begin{cases} 0 & \text{for } x_l < \tau_1 \\ \frac{1}{2}(x_l - \tau_1)^2 & \text{for } \tau_1 \le x_l < \tau_2 \\ \frac{1}{2}\left(-x_l^2 + \tau_1^2 - 2x_l \right. \\ \quad \left. \times(\tau_1 - 2\tau_2) - 2\tau_2^2\right) & \text{for } \tau_2 \le x_l < \tau_3 \\ \frac{1}{2}\left(\tau_1^2 - 2\tau_2^2 + \tau_3^2 \right. \\ \quad \left. - 2x_l(\tau_1 - 2\tau_2 + \tau_3)\right) & \text{for } x_l \ge \tau_3, \end{cases} \\
&(G_{22})_{il} = \begin{cases} 0 \quad \text{for } x_l < \tau_1 \\ \frac{8x_l^3}{3} - \frac{1}{3}(x_l + \tau_1)^3 \\ \quad \text{for } \tau_1 \le x_l < \tau_2 \\ \frac{1}{3}\left(-8x_l^3 - (x_l + \tau_1)^3 + 2(x_l + \tau_2)^3\right) \\ \quad \text{for } \tau_2 \le x_l < \tau_3 \\ \frac{1}{3}\left(-(x_l + \tau_1)^3 + 2(x_l + \tau_2)^3 - (x_l + \tau_3)^3\right) \\ \quad \text{for } x_l \ge \tau_3. \end{cases}
\end{aligned}
\tag{32}
$$

We apply collocation method, and vectors a_i and b_i can be calculated from (30); the functions $u(x)$ and $v(x)$ are evaluated from (25).

Computations were carried out for different values of J (see Table 4 and Figure 4). These results were compared with the exact solution $u(x) = e^{2x}$ and $v(x) = e^{-2x}$.

The accuracy of the results was estimated by the error function for $u(x)$ and $v(x)$:

$$
\begin{aligned}
e_J &= \max_{1\le l\le 2M} \left|u(x_l) - u_{ex}(x_l)\right|, \\
e_J &= \max_{1\le l\le 2M} \left|v(x_l) - v_{ex}(x_l)\right|,
\end{aligned}
\tag{33}
$$

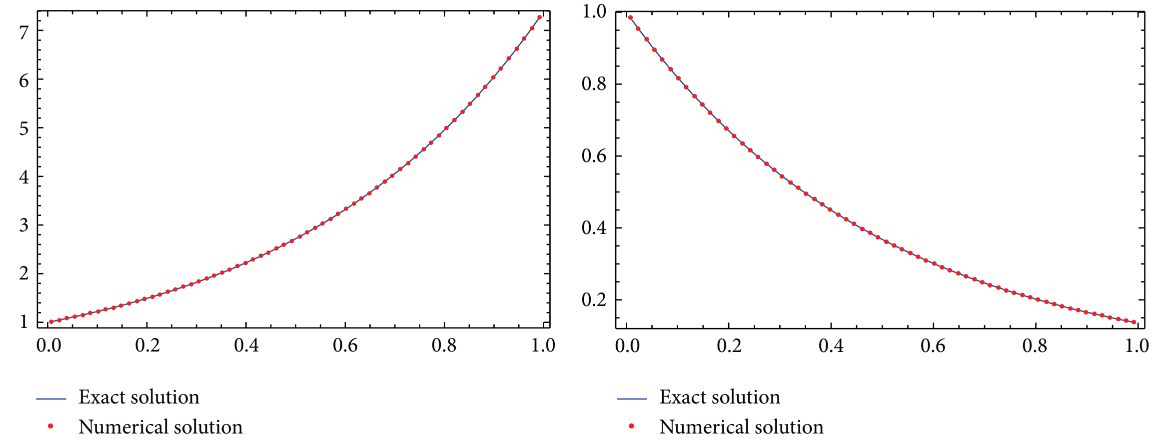
where x_l is defined by (7).

Example 5. Consider the following (SLVIEs) [4, 18]:

$$
\begin{aligned}
&u(x) \\
&\quad = -1 - x\cos(x)^2 + 2\cos(x) + \sin(x)\left(1 - \frac{1}{2}x + \cos(x)\right) \\
&\qquad + \int_0^x \left((\sin(x-t) - 1)\,u(t) + (1 - t\cos(x))\,v(t)\right)dt, \\
&v(x) = -x + \sin(x) + \int_0^x \left(u(t) + (x-t)\,v(t)\right)dt.
\end{aligned}
\tag{34}
$$

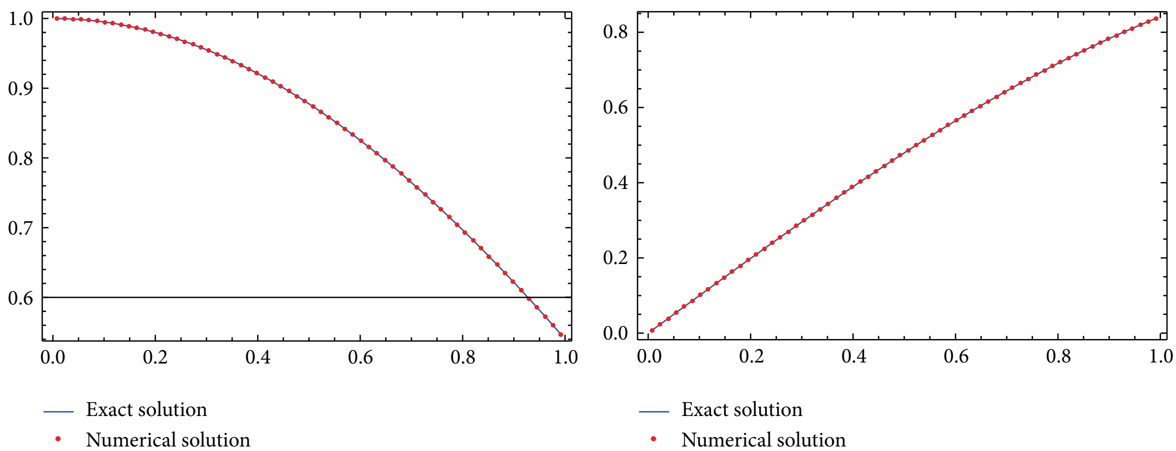
Carrying out the integration in (28) we obtain

$$
\begin{aligned}
&(G_{11})_{il} = \begin{cases} 0 & \text{for } x_l < \tau_1 \\ 1 - \cos[x_l - \tau_1] - x_l + \tau_1 & \text{for } \tau_1 \le x_l < \tau_2 \\ -1 - \cos[x_l - \tau_1] + 2\cos[x_l - \tau_2] \\ \quad + x_l + \tau_1 - 2\tau_2 & \text{for } \tau_2 \le x_l < \tau_3 \\ -\cos[x_l - \tau_1] + 2\cos[x_l - \tau_2] \\ \quad -\cos[x_l - \tau_3] + \tau_1 - 2\tau_2 + \tau_3 & \text{for } x_l \ge \tau_3, \end{cases} \\
&(G_{12})_{il} = \begin{cases} 0 & \text{for } x_l < \tau_1 \\ x_l - \tau_1 + \frac{1}{2}\cos[x_l]\left(-x_l^2 + \tau_1^2\right) & \text{for } \tau_1 \le x_l < \tau_2 \\ -x_l - \tau_1 + 2\tau_2 \\ +\frac{1}{2}\cos[x_l]\left(x_l^2 + \tau_1^2 - 2\tau_2^2\right) & \text{for } \tau_2 \le x_l < \tau_3 \\ \frac{1}{2}\left(-2(\tau_1 - 2\tau_2 + \tau_3)\right. \\ \quad \left. + \cos[x_l]\left(\tau_1^2 - 2\tau_2^2 + \tau_3^2\right)\right) & \text{for } x_l \ge \tau_3, \end{cases} \\
&(G_{21})_{il} = \begin{cases} 0 & \text{for } x_l < \tau_1 \\ x_l - \tau_1 & \text{for } \tau_1 \le x_l < \tau_2 \\ -x_l - \tau_1 + 2\tau_2 & \text{for } \tau_2 \le x_l < \tau_3 \\ -\tau_1 + 2\tau_2 - \tau_3 & \text{for } x_l \ge \tau_3, \end{cases}
\end{aligned}
$$



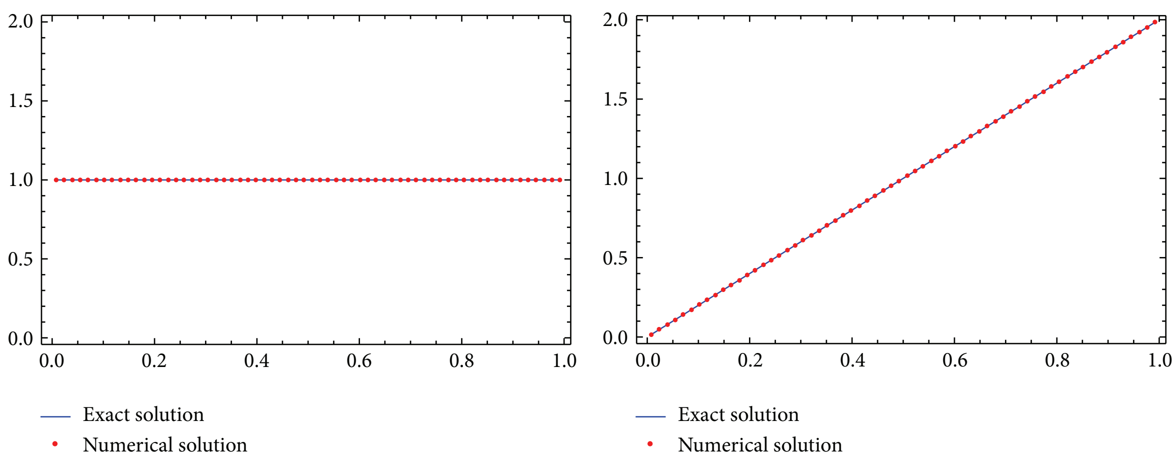
(a) (b)

Figure 4: (a) Exact and Haar wavelet solution of $u(x)$ at $J = 5$. (b) Exact and Haar wavelet solution of $v(x)$ at $J = 5$.



(a) (b)

Figure 5: (a) Exact and Haar wavelet solution of $u(x)$ at $J = 5$. (b) Exact and Haar wavelet solution of $v(x)$ at $J = 5$.



(a) (b)

Figure 6: (a) Exact and Haar wavelet solution of $u(x)$ at $J = 5$. (b) Exact and Haar wavelet solution of $v(x)$ at $J = 5$.

$$(G_{22})_{il} = \begin{cases} 0 & \text{for } x_l < \tau_1 \\ \frac{1}{2}(x_l - \tau_1)^2 & \\ \quad \text{for } \tau_1 \leq x_l < \tau_2 & \\ \frac{1}{2}\left(-x_l^2 + \tau_1^2 - 2x_l(\tau_1 - 2\tau_2) - 2\tau_2^2\right) & \\ \quad \text{for } \tau_2 \leq x_l < \tau_3 & \\ \frac{1}{2}\left(\tau_1^2 - 2\tau_2^2 + \tau_3^2 - 2x_l(\tau_1 - 2\tau_2 + \tau_3)\right) & \\ \quad \text{for } x_l \geq \tau_3. & \end{cases} \tag{35}$$

We apply collocation method, and vectors a_i and b_i can be calculated from (29); the functions $u(x)$ and $v(x)$ are evaluated from (25).

Computations were carried out for different values of J (see Table 5 and Figure 5). These results were compared with the exact solution $u(x) = \cos(x)$ and $v(x) = \sin(x)$.

Example 6. Consider the following (SLVIEs):

$$u(x) = 1 - x^2 + \int_0^x v(t)\,dt, \qquad v(x) = x + \int_0^x u(t)\,dt. \tag{36}$$

Carrying out the integration in (28) we obtain

$$(G_{12})_{il} = \begin{cases} 0 & \text{for } x_l < \tau_1 \\ x_l - \tau_1 & \text{for } \tau_1 \leq x_l < \tau_2 \\ -x_l - \tau_1 + 2\tau_2 & \text{for } \tau_2 \leq x_l < \tau_3 \\ -\tau_1 + 2\tau_2 - \tau_3 & \text{for } x_l \geq \tau_3, \end{cases} \tag{37}$$

$$(G_{21})_{il} = \begin{cases} 0 & \text{for } x_l < \tau_1 \\ x_l - \tau_1 & \text{for } \tau_1 \leq x_l < \tau_2 \\ -x_l - \tau_1 + 2\tau_2 & \text{for } \tau_2 \leq x_l < \tau_3 \\ -\tau_1 + 2\tau_2 - \tau_3 & \text{for } x_l \geq \tau_3. \end{cases}$$

We apply collocation method, and vectors a_i and b_i can be calculated from (29); the functions $u(x)$ and $v(x)$ are evaluated from (25).

Computations were carried out for different values of J (see Table 6 and Figure 6). These results were compared with the exact solution $u(x) = 1$ and $v(x) = 2x$.

5. Conclusion

In this work the Haar wavelet method for solution of linear systems integral equations is proposed. A method of solution which is applicable for different kind of integral equations, Fredholm and Volterra systems of integral equations, is worked out. The solution is based on the collocation techniques which are proposed. The elaborated method is very simple and, as it follows from the test problems, high precision of results can be obtained with a small number of calculation points. The calculations show that by doubling the number of the calculation points the error function decreases. This result follows also from analytical considerations. It should be noted that in the case of Haar wavelets we have to solve systems of linear equations with a smaller condition number as for other methods based on piecewise constant approximation.

Table 4: Error of $u(x)$ and $v(x)$ of Example 4.

J	$2M$	Error of function $u(x)$	Error of function $v(x)$
2	8	$1.51E-2$	$9.43E-3$
3	16		
4	32		
5	64	$3.14E-4$	$1.91E-4$

Table 5: Error of $u(x)$ and $v(x)$ of Example 5.

J	$2M$	Error of function $u(x)$	Error of function $v(x)$
2	8	$1.74E-3$	$1.94E-3$
3	16	$4.78E-4$	$5.07E-4$
4	32	$1.20E-4$	$1.29E-4$
5	64	$30.3E-5$	$3.27E-5$

Table 6: Error of $u(x)$ and $v(x)$ of Example 6.

J	$2M$	Error of function $u(x)$	Error of function $v(x)$
2	8	$5.76E-3$	$4.23E-3$
3	16	$1.47E-3$	$1.10E-3$
4	32	$2.72E-4$	$2.82E-4$
5	64	$9.36E-5$	$7.09E-5$

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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