

## Editorial

# Approximation Theory and Numerical Analysis

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Approximation Theory and Numerical Analysis are closely related areas of mathematics. Approximation Theory lies in the crossroads of pure and applied mathematics. It includes a wide spectrum of areas ranging from abstract problems in real, complex, and functional analysis to direct applications in engineering and industry. Therefore, Approximation Theory employs a great variety of methods, which originate in analysis, operator theory, harmonic analysis, quantum calculus, algorithms, probability theory, and further areas of mathematics.

This special issue was launched in November 2013 aiming at bringing out new developments in these subjects with main focus on the interaction between Approximation Theory and Numerical Analysis. All papers submitted to the issue have been refereed by experts in their respective fields and examined by the academic editors. Thoroughly selected papers falling into the scope of the issue have been published, and hopefully they will be of interest for the reader.

The issue contains a number of papers related to the Bernstein type operators based on the  $q$ -integers. After  $q$ -analogues of the Bernstein polynomials had been introduced by A. Lupaş in 1987 and G. Phillips in 1997, the study of various  $q$ -analogues of the classical operators became an area of intensive research. The present special issue supplies a selection of papers on the  $q$ -operators including those written by well-known specialists in the area such as N. I. Mahmudov, H. Wang, and X. Wu. In those works, distinct modifications of the  $q$ -Bernstein polynomials have been studied, for example,  $q$ -Bernstein-Durrmeyer polynomials,  $q$ -Lupaş operator,  $q$ -Szász-Mirakjan operator, and truncated  $q$ -Bernstein polynomials. The authors deal with the convergence of the operators,

both in real and complex cases, shape-preserving properties, and discuss possible generalizations of the classical results. Moreover, the paper by X. Wu presents a complete solution to the long-standing open problem on the approximation of all continuous functions on  $[0, 1]$  by the  $q$ -Bernstein polynomials in the case  $q \rightarrow 1^+$ .

Furthermore, the issue includes a paper on inequalities for real functions. To be specific, the subject of the paper of A. Qayyum et al. is Ostrowski type inequalities, that is, inequalities giving bounds for the deviation of a function from its integral mean. The authors obtain bounds for the deviation of a function from a combination of integral means over two subintervals covering the entire interval in terms of the  $L_p$ -norms of the second derivative of the function,  $1 \leq p \leq \infty$ .

The issue contains a paper on the bivariate interpolation and some related topics. Particularly, the paper by L. Zou and S. Tang studies the interpolation theorem, algorithms, and dual interpolation. It also provides many kinds of interpolation schemes.

A classical topic—solution of nonlinear equations—is also represented in the issue. F. Dubeau's paper is devoted to studying Schröder's processes which are fixed point processes for finding simple roots of nonlinear equations. The author shows that Schröder's processes of the first kind and of the second kind are related by polynomial and rational approximations, thus giving an answer to a question raised by M. Petković to find and explain a possible link between the two processes.

The papers outlined below regard the area of Numerical Analysis. The first deals with  $LM - g$  splines.  $LM - g$  splines

had been introduced by R. J. P. de Figueiredo in 1977 as a generalization of the  $L-g$  splines in terms of two differential operators  $L$  and  $M$ , and they reduce to  $L-g$  splines if  $M$  is the identity operator. In their paper, X. Liu et al. investigate the structural properties of the  $LM-g$  splines by optimization and optimal control theory as well as the relationship between  $LM-g$  splines and  $L-g$  splines.

Besides, a new interpolation spline with two parameters, called an EH interpolation spline, which is the extension of the standard cubic Hermite interpolation spline, is presented in the paper by J. Xie and X. Liu. The authors demonstrate properties and advantages of this spline for a class of interpolation problems.

In the paper by R. An and X. Wang, new models are considered which help to understand the behavior of numerical methods in presence of layers in more complex problems like the Navier-Stokes equations in fluid dynamics or convection diffusion equations in chemical reaction processes. Moreover, a new stabilized finite element method for incompressible flows based on Brezzi-Pitkäranta stabilized method is presented. The stability and error estimates of finite element solutions are derived for the classical one-level method. Authors also propose a new Newton correction scheme based on the above two-level iteration methods. In addition, some numerical experiments are given to support the theoretical results and to check the efficiency of these two-level iteration methods.

A first supercloseness analysis for higher order FEM/LDG coupled method for solving singularly perturbed convection-diffusion problem is presented by S. Xie et al. Based on piecewise polynomial approximations of degree  $k$  ( $k \geq 1$ ), a supercloseness property of order  $k + 1/2$  in DG norm is established on an S-type mesh. Numerical experiments complement the theoretical results.

S. B. G. Karakoç et al. employ a septic  $B$ -spline collocation method to find numerical solutions of the modified regularized long wave (MRLW) equation for the motion of a single solitary wave, the interaction of two and three solitary waves, and the development of the Maxwellian initial condition into solitary waves. Moreover, they give a linear stability analysis of their methods.

The article by C.-X. Li et al. considers the numerical solution of complex symmetric linear systems. They introduce a generalized preconditioned modified Hermitian and skew Hermitian splitting (GPMHSS) method based on the modified Hermitian and skew-Hermitian splitting (MHSS) and preconditioned MHSS (PMHSS) methods. Numerical test results discussing the efficiency of GPMHSS and inexact GPMHSS are provided.

The paper by H. Wang and Y. Sun discusses a feasible interval of the parameter  $k$  and a general expression of the matrix  $X$  which satisfies the rank equation  $r(A - BXC) = k$ . The authors study the problems to determine the maximal and minimal ranks under this rank constraint, as well as to derive the least squares solutions of  $\|BXC - A\|_F = \min$ .

A backward perturbation analysis for the block skew circulant linear systems with skew circulant blocks has been carried out by Z. Jiang et al. First, they give a block style spectral decomposition of the coefficient matrix of the linear

system. Then, based on this decomposition, they perform structured backward perturbation analysis for the block skew circulant linear systems.

M. I. Berenguer et al. introduce a new iterative method for the numerical solution of systems of nonlinear Fredholm integrodifferential equations of the second kind by employing Banach fixed-point theorem and Schauder basis and perform the convergence analysis of their methods.

The paper by P. Hessari et al. aims to solve an elliptic interface problem with a discontinuous coefficient and a singular source term by the spectral collocation method. An algorithm for this problem has been designed. Afterwards, its efficiency has been demonstrated.

Another topic covered by the issue is the applications of numerical analysis in optimal control theory. Namely, in the paper of J. Zhou, Legendre-Galerkin spectral methods are employed to solve state-constrained optimal control problems. Explicit formulae of the constants in the a posteriori error indicator are investigated.

The paper authored by A. S. Al-Fhaid et al. aims to construct a matrix iteration for finding approximate inverses of nonsingular square matrices and also apply the new method for computing the Drazin inverse. It is proved that the method possesses the convergence rate nine. Numerical experiments are performed to support the findings.

On the whole, the issue comprises papers which cover a variety of different topics within the scope of Approximation Theory and Numerical Analysis. Hopefully, they will be useful and interesting for the reader working in those subjects.

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