

## Research Article

# Application of Variational Iteration Method for Dropping Damage Evaluation of the Suspension Spring Packaging System

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The dropping damage evaluation for packaging system is essential for safe transportation and storage. A dynamic model of nonlinear cubic-quintic Duffing oscillator for the suspension spring packaging system was proposed. Then, a first-order approximate solution was obtained by applying He's variable iteration method. Based on the results, a damage evaluation equation was derived, which reveals the main controlling physical parameters for damage potential of drop to packaged products concretely. Finally, the dropping damage boundary curves and surfaces for the system were discussed. It was found that decreasing the suspension angle can improve the safe region of the system.

## 1. Introduction

Newton [1] proposed the concept of damage boundary for the first time in 1968, which established the foundation of present cushioning packaging dynamics. However, it can only be applied to linear packaging systems. For nonlinear packaging systems, there has been some undergoing work since then. A dropping damage evaluation for a tangent nonlinear system with a critical component was proposed by Wang et al. [2]. Wang et al. [3] proposed a three-dimensional shock spectrum for nonlinear packaging system with a critical component. They also suggested the damage boundary surfaces concept for damage evaluation of a tangent nonlinear packaging system with a critical component [4]. The fatigue damage of most packaged products is caused by dropping shock in the process of transportation. Therefore, Wang [5, 6] proposed the concept of dropping damage boundary curve with system parameter and the dimensionless dropping shock velocity as two basic evaluation quantities. These theories are all based on numerical analysis method. However, the influence of relevant parameters cannot be revealed to show their physical significance clearly. The variational iteration method (VIM) proposed by He et al. [7–9], which has been widely applied [10–19], can solve many kinds of nonlinear equations without small parameters limitation (first-order approximate

analytical solutions can achieve high precision). Wang et al. [20–22] studied the dropping response of typical nonlinear packaging systems and obtained inner-resonance conditions.

The suspension spring system with eight springs as cushioning components performs geometric nonlinearity and is suitable for protecting high precision instrument with low fragility. Wu and Yang [23] studied the natural vibration characteristics of the system under the excitation of foundational displacement. They found that the shock absorption performance of the system with pendulum springs was better than the one with vertical springs. Assuming the system was excited by rectangular pulse, the three-dimensional shock spectrum and the damage boundary surface were obtained by Wang and Chen [24–26]. They achieved the conclusion that both increasing the pulse incentive amplitude and decreasing the suspension angle could expand the safe area, and the safe area would be expanded more obviously when the angle was less than  $75^\circ$ .

In this paper, by applying the VIM, we solve the nondimensional dynamic equation of the suspension spring system under the excitation of dropping shock to obtain a first-order approximate solution and obtain the nondimensional maximum acceleration expression. Then, a damage evaluation equation presenting the relationship between physical

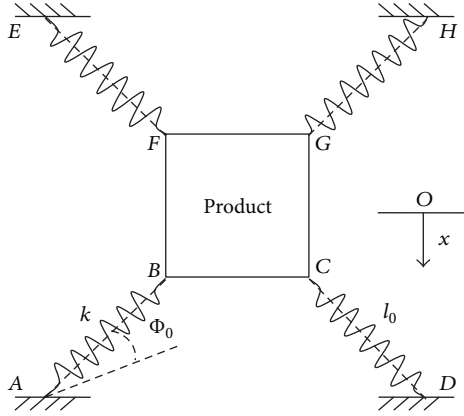


FIGURE 1: Dynamic model of the suspension spring system.

parameters and the damage boundary is suggested. Finally, the dropping damage boundary curves and surfaces of the system are discussed according to the damage evaluation equation.

## 2. Modelling and Equations

The dynamic model of the suspension spring packaging system is shown in Figure 1. A product is suspended in the middle of the container by 8 springs (four springs are on the upside, and the other four are on the downside).

Wang and Chen [24–26] proposed an approximate dynamic equation with a cubic oscillator of the suspension spring system. For acquiring a higher precision, we establish a more concrete dynamic model with nonlinear cubic-quintic Duffing oscillators of the system by using the Taylor series, which can be expressed as

$$m \frac{d^2 x}{dt^2} + 8k \left( a_0 x + \frac{b_0}{l_0^2} x^3 + \frac{c_0}{l_0^4} x^5 \right) = 0, \quad (1)$$

where

$$\begin{aligned} a_0 &= \sin^2 \phi_0, \\ b_0 &= \frac{(1 - 6\sin^2 \phi_0 + 5\sin^4 \phi_0)}{2}, \\ c_0 &= \frac{(63\sin^6 \phi_0 - 105\sin^4 \phi_0 + 45\sin^2 \phi_0 - 3)}{8}. \end{aligned} \quad (2)$$

Based on dropping shock, the initial conditions can be written as

$$\begin{aligned} x(0) &= 0, \\ \frac{dx(0)}{dt} &= \sqrt{2gH}. \end{aligned} \quad (3)$$

Here are the coefficients:  $H$  is the dropping height,  $x$  denotes the product displacement,  $m$  denotes the product mass,  $g$  is the acceleration of gravity,  $k$  denotes the coupling

stiffness,  $l_0$  denotes the original length of the springs, and  $\phi_0$  denotes the suspension angle.

By introducing new nondimensional parameters, (1) can be equivalently written in the following dimensionless form:

$$\frac{d^2 y}{d\tau^2} + a_0 y + b_0 y^3 + c_0 y^5 = 0, \quad (4)$$

where

$$\begin{aligned} y &= \frac{x}{l_0}, \\ \tau &= \frac{t}{T}, \\ \omega &= \sqrt{\frac{8k}{m}}, \\ T &= \frac{1}{\omega}. \end{aligned} \quad (5)$$

The initial conditions can be written as

$$\begin{aligned} y(0) &= 0, \\ \frac{dy(0)}{d\tau} &= V = \sqrt{\frac{2gmH}{8kl_0^2}}, \end{aligned} \quad (6)$$

where  $V$  denotes the dimensionless dropping velocity.

## 3. Variational Iteration Method

A nonlinear equation can be written as

$$Ly(t) + Ny(t) = g(t), \quad (7)$$

where  $L$  is a linear operator,  $N$  is a nonlinear operator, and  $g$  is a continuous function.

The VIM is proposed by He et al. [7–9] for the first time, and the correction functional for the nonlinear equation can be established as follows:

$$y_{n+1} = y_n(t) + \int_0^t \lambda(s) \{Ly_n(s) + N\tilde{y}_n(s) - g(s)\} ds, \quad (8)$$

where  $\lambda$  is the Laplace multiplier which can be obtained by the variational theory and  $\tilde{y}_n$  is the restriction on varying which is equal to 0 solving the Laplace multiplier.

Applying the VIM, construct the following correction functional of the system:

$$\begin{aligned} y_{n+1}(\tau) &= y_n(\tau) \\ &+ \int_0^\tau \lambda(s) \left( \frac{d^2 y_n(s)}{ds^2} + a_0 y_n(s) + b_0 \tilde{y}_n^3(s) + c_0 \tilde{y}_n^5(s) \right) ds. \end{aligned} \quad (9)$$

According to the principle of stationary, (9) can be turned into the following form:

$$\begin{aligned} &\delta y_{n+1}(\tau) \\ &= \delta y_n(\tau) \\ &\quad + \delta \int_0^\tau \lambda(s) \left( \frac{d^2 y_n(s)}{ds^2} + a_0 y_n(s) + b_0 \tilde{y}_n^3(s) + c_0 \tilde{y}_n^5(s) \right) ds \\ &= \delta y_n(\tau) + \delta \int_0^\tau \lambda(s) \ddot{y}_n(s) ds + \delta \int_0^\tau \lambda(s) a_0 y_n(s) ds \\ &= \delta y_n(\tau) + \lambda(s) \delta \dot{y}_n(s) \Big|_{s=\tau} - \dot{\lambda}(s) \delta y_n(s) \Big|_{s=\tau} \\ &\quad + \int_0^\tau [\ddot{\lambda}(s) + a_0 \lambda(s)] \cdot \delta y_n(s) ds. \end{aligned} \tag{10}$$

The conditions of stationary can be written as

$$\begin{aligned} \ddot{\lambda}(s) + a_0 \lambda(s) &= 0, \\ \lambda(s) \Big|_{s=\tau} &= 0, \\ \dot{\lambda}(s) \Big|_{s=\tau} &= 1. \end{aligned} \tag{11}$$

The Laplace multiplier is obtained as

$$\lambda(s) = \frac{1}{\sqrt{a_0}} \sin[\sqrt{a_0}(s - \tau)]. \tag{12}$$

The following iteration formula can be constructed as

$$\begin{aligned} &y_{n+1}(\tau) \\ &= y_n(\tau) \\ &\quad + \int_0^\tau \frac{1}{\sqrt{a_0}} \sin[\sqrt{a_0}(s - \tau)] \\ &\quad \cdot \left( \frac{d^2 y_n(s)}{ds^2} + a_0 y_n(s) + b_0 y_n^3(s) + c_0 y_n^5(s) \right) ds. \end{aligned} \tag{13}$$

For the nondimensional dynamic equation (4) and the initial conditions equation (6), we can take the initial solution below:

$$y_0(\tau) = A \sin \alpha \tau, \tag{14}$$

where  $\alpha$  is the frequency parameter and the amplitude  $A = V/\alpha$ . The first-order iteration approximate solution can be obtained as

$$\begin{aligned} y_1(\tau) &= \left[ \frac{3b_0 A^3}{4(\alpha^2 - a_0)} + \frac{5c_0 A^5}{8(\alpha^2 - a_0)} \right] \sin \alpha \tau \\ &\quad + \left[ -\frac{b_0 A^3}{4(9\alpha^2 - a_0)} - \frac{5c_0 A^5}{16(9\alpha^2 - a_0)} \right] \sin 3\alpha \tau \\ &\quad + \left[ \frac{c_0 A^5}{16(25\alpha^2 - a_0)} \right] \sin 5\alpha \tau \end{aligned}$$

$$\begin{aligned} &+ \left[ \frac{A\alpha}{\sqrt{a_0}(\alpha^2 - a_0)} \left( \alpha^2 - a_0 - \frac{3}{4}b_0 A^3 - \frac{5}{8}c_0 A^5 \right) \right. \\ &\quad + \frac{3\alpha b_0 A^3}{4\sqrt{a_0}(9\alpha^2 - a_0)} + \frac{15\alpha c_0 A^5}{16\sqrt{a_0}(9\alpha^2 - a_0)} \\ &\quad \left. - \frac{5\alpha c_0 A^5}{16\sqrt{a_0}(25\alpha^2 - a_0)} \right] \sin \sqrt{a_0} \tau. \end{aligned} \tag{15}$$

Let the coefficient of  $\sin \sqrt{a_0} \tau$  be equal to zero so that there is no secular term appearing in the next iteration; namely,

$$\begin{aligned} &\frac{A\alpha}{\sqrt{a_0}(\alpha^2 - a_0)} \left( \alpha^2 - a_0 - \frac{3}{4}b_0 A^3 - \frac{5}{8}c_0 A^5 \right) \\ &\quad + \frac{3\alpha b_0 A^3}{4\sqrt{a_0}(9\alpha^2 - a_0)} + \frac{15\alpha c_0 A^5}{16\sqrt{a_0}(9\alpha^2 - a_0)} \\ &\quad - \frac{5\alpha c_0 A^5}{16\sqrt{a_0}(25\alpha^2 - a_0)} = 0. \end{aligned} \tag{16}$$

As the dropping shock pulse is a half-sine pulse, the nondimensional dropping shock extended period can be obtained as

$$\tau = \frac{\pi}{\alpha}. \tag{17}$$

The first-order nondimensional displacement iteration approximate expression, namely, the first-order approximate solution, can be written as

$$\begin{aligned} y_1(\tau) &= \left[ \frac{3b_0 A^3}{4(\alpha^2 - a_0)} + \frac{5c_0 A^5}{8(\alpha^2 - a_0)} \right] \sin \alpha \tau \\ &\quad - \left[ \frac{b_0 A^3}{4(9\alpha^2 - a_0)} + \frac{5c_0 A^5}{16(9\alpha^2 - a_0)} \right] \sin 3\alpha \tau \\ &\quad + \left[ \frac{c_0 A^5}{16(25\alpha^2 - a_0)} \right] \sin 5\alpha \tau. \end{aligned} \tag{18}$$

The first-order nondimensional acceleration iteration approximate expression can be written as

$$\begin{aligned} y_1''(\tau) &= -\alpha^2 \left[ \frac{3b_0 A^3}{4(\alpha^2 - a_0)} + \frac{5c_0 A^5}{8(\alpha^2 - a_0)} \right] \sin \alpha \tau \\ &\quad + 9\alpha^2 \left[ \frac{b_0 A^3}{4(9\alpha^2 - a_0)} + \frac{5c_0 A^5}{16(9\alpha^2 - a_0)} \right] \sin 3\alpha \tau \\ &\quad - 25\alpha^2 \left[ \frac{c_0 A^5}{16(25\alpha^2 - a_0)} \right] \sin 5\alpha \tau. \end{aligned} \tag{19}$$

Substitute  $\alpha\tau = \pi/2$  into (18) and (19); the nondimensional maximum displacement can be written as

$$y_1(\tau)_m = \left[ \left[ \frac{3b_0A^3}{4(\alpha^2 - a_0)} + \frac{5c_0A^5}{8(\alpha^2 - a_0)} \right] + \left[ \frac{b_0A^3}{4(9\alpha^2 - a_0)} + \frac{5c_0A^5}{16(9\alpha^2 - a_0)} \right] + \left[ \frac{c_0A^5}{16(25\alpha^2 - a_0)} \right] \right] \quad (20)$$

and the nondimensional maximum acceleration can be written as

$$y_1''(\tau)_m = \left[ -\alpha^2 \left[ \frac{3b_0A^3}{4(\alpha^2 - a_0)} + \frac{5c_0A^5}{8(\alpha^2 - a_0)} \right] - 9\alpha^2 \left[ \frac{b_0A^3}{4(9\alpha^2 - a_0)} + \frac{5c_0A^5}{16(9\alpha^2 - a_0)} \right] - 25\alpha^2 \left[ \frac{c_0A^5}{16(25\alpha^2 - a_0)} \right] \right] \quad (21)$$

For the following amounts,  $m = 290$  kg,  $k = 2 \times 10^5$  N/m, and  $l_0 = 0.075$  m, we choose the dropping height  $H = 0.3m$ , and the suspension angle  $\phi_0 = 60^\circ$ . According to (16), we gain the frequency parameter  $\alpha = 0.8148$ . By applying the VIM the nondimensional maximum acceleration and the dropping shock extended period are obtained as  $y_{m1}'' = 0.3298$  and  $\tau_1 = 3.8555$ . Applying the Runge-Kutta (R-K) method, results are  $y_m'' = 0.3338$  and  $\tau = 3.8392$ . Comparing with the R-K method, the relative errors by using the VIM are  $E_{y_m''} = 1.20\%$  and  $E_\tau = 0.42\%$ , which can meet the requirement of the packaging design.

The impact energy under dropping shock is related not only to the maximum acceleration, but also to the whole waveform. Hence, it is necessary to prove that the overall precision of the waveform can meet the requirement. The nondimensional acceleration response curves can be obtained just as shown in Figure 2. It indicates that the curve by applying the VIM can meet very well with the one by the R-K method.

#### 4. Dropping Damage Evaluation

Wang [5, 6] proposed the concept of dropping damaging boundary, which can provide the theoretical foundation for the dropping damage evaluation of the suspension spring system.

The dropping shock acceleration of the system can be written as follows:

$$\frac{d^2x}{dt^2} = \beta \frac{d^2y}{d\tau^2}, \quad (22)$$

where the system parameter

$$\beta = \frac{l_0}{T^2} = \frac{8kl_0}{m}. \quad (23)$$

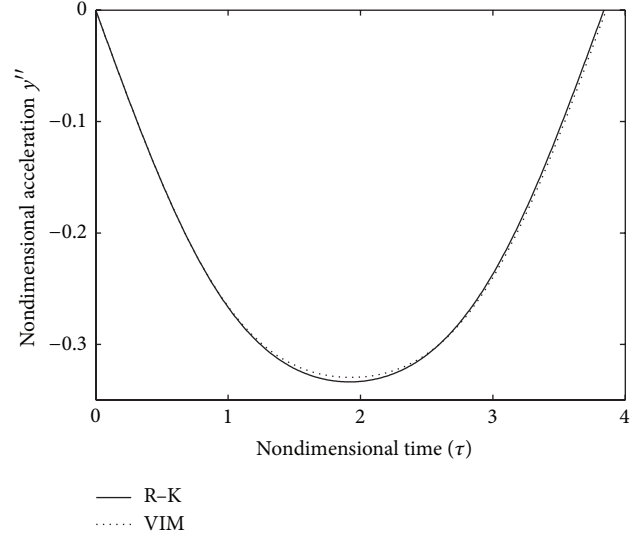


FIGURE 2: Comparison of the nondimensional acceleration  $y''$ -time  $\tau$  response of the system by the VIM with the one by the R-K method when the suspension angle  $\phi_0 = 60^\circ$ .

Set  $A_c$  as the product fragility. The relational expression about the nondimensional maximum acceleration  $y_m''$ , the product fragility  $A_c$ , and the system parameter  $\beta$  can be obtained just as follows:

$$A_c g = \left( \frac{d^2x}{dt^2} \right)_m = \beta \left( \frac{d^2y}{d\tau^2} \right)_m. \quad (24)$$

By combining (21) and (24), the system parameter can be written as

$$\begin{aligned} \beta &= A_c g \\ &\times \left( \left[ -\alpha^2 \left[ \frac{3b_0A^3}{4(\alpha^2 - a_0)} + \frac{5c_0A^5}{8(\alpha^2 - a_0)} \right] - 9\alpha^2 \left[ \frac{b_0A^3}{4(9\alpha^2 - a_0)} + \frac{5c_0A^5}{16(9\alpha^2 - a_0)} \right] - 25\alpha^2 \left[ \frac{5c_0A^5}{16(25\alpha^2 - a_0)} \right] \right] \right)^{-1}. \end{aligned} \quad (25)$$

Equation (25) includes more than one variable such as  $\beta$ ,  $A_c$ ,  $A$  (related to  $V$  and  $\alpha$ ), and  $\phi_0$ . Therefore, according to (25), we can evaluate the dropping shock characteristics of the system.

Respectively, we choose the suspension angle  $\phi_0$ , equal to  $60^\circ$ ,  $65^\circ$ ,  $70^\circ$ , and  $75^\circ$ , and select the product fragility  $A_c$ , equal to 10 and 15. The system parameter  $\beta$  and the dimensionless dropping shock velocity  $V$  are selected as two basic evaluation quantities [5, 6]. According to (25), dropping damage boundary curves of the system can be obtained just as Figure 3 indicated. The safe area is under the dropping damage boundary curve, and the product is safe when the coordinate point  $(\beta, V)$  enters the area.

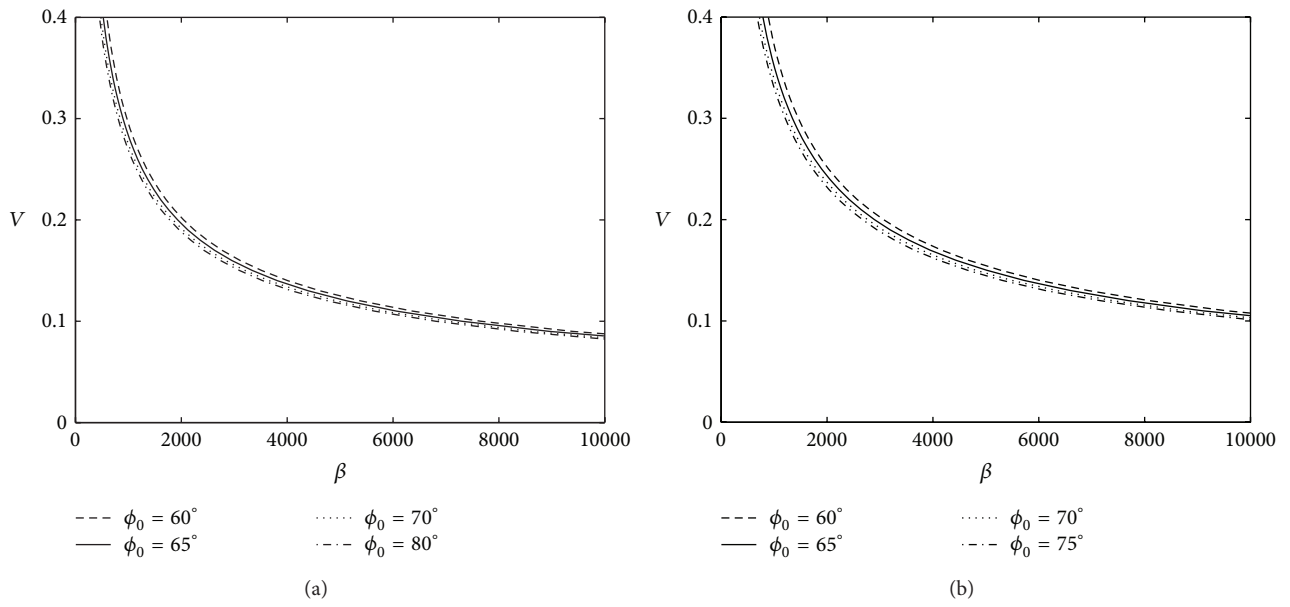


FIGURE 3: Dropping damage boundary curves of the system when the suspension angle  $\phi_0 = 60^\circ, 65^\circ, 70^\circ,$  and  $75^\circ$  and the product fragility (a)  $A_c = 10$  and (b)  $A_c = 15$ .  $V$ : the dimensionless dropping shock velocity;  $\beta$ : the system parameter.

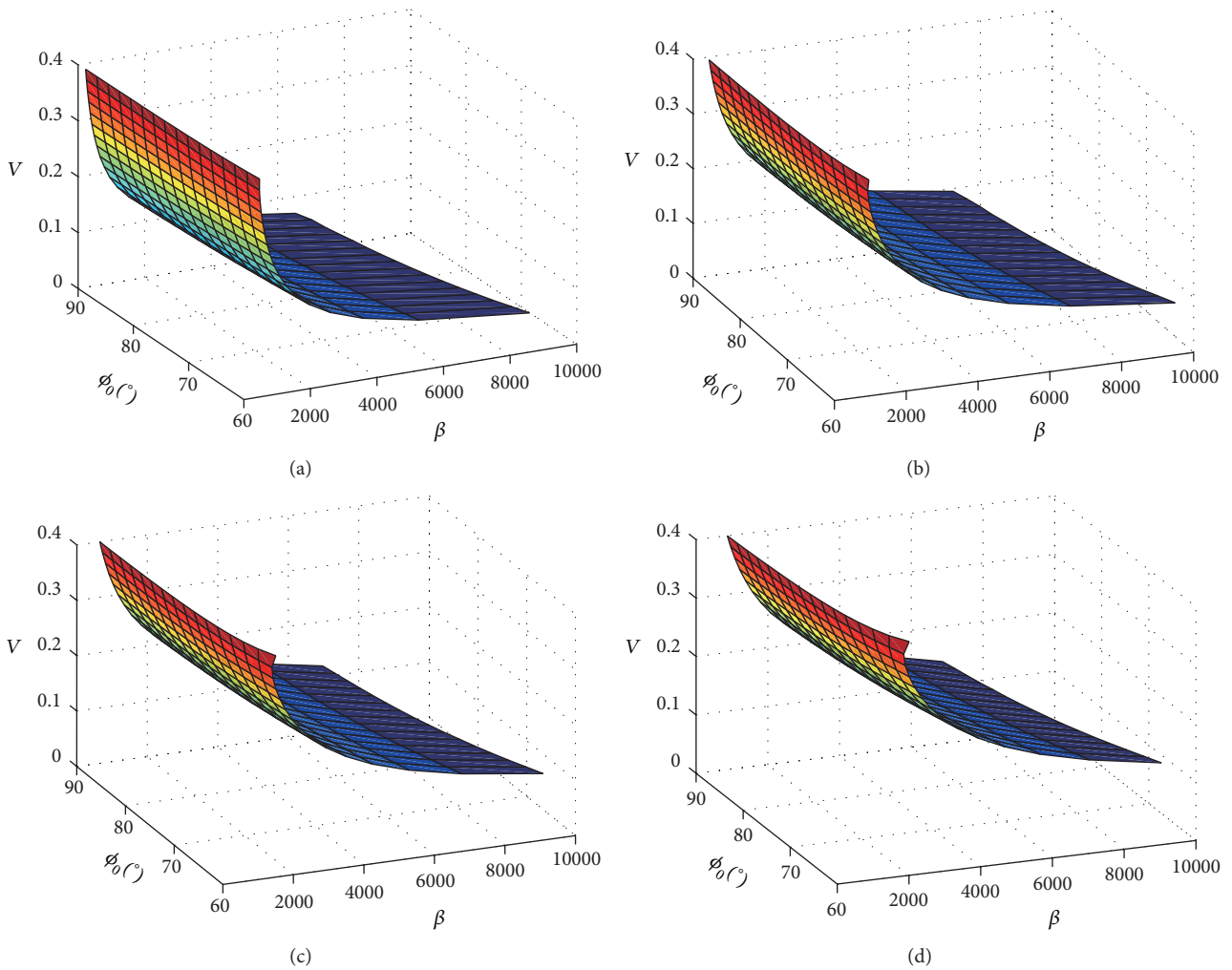


FIGURE 4: Dropping damage boundary surfaces of the system when the product fragility (a)  $A_c = 5$ , (b)  $A_c = 10$ , (c)  $A_c = 15$ , and (d)  $A_c = 20$ .  $V$ : the dimensionless dropping shock velocity;  $\beta$ : the system parameter;  $\phi_0$ : the suspension angle.

By selecting the suspension angle as the third evaluation quantity, set product fragility  $A_c = 5, 10, 15,$  and  $20,$  respectively, and then dropping damage boundary surfaces can be obtained by applying the VIM, just like Figure 4 indicated. The safe area is under the dropping damage boundary surfaces, and the product is safe when the coordinate point  $(\beta, \phi_0, V)$  enters the area.

According to Figures 3 and 4, it is shown that decreasing the suspension angle  $\phi_0,$  the security performance of the system will improve, and increasing the product fragility  $A_c,$  the damage boundary curves and surface will move up obviously. So the characteristics of suspension geometry nonlinear ( $\phi_0 < 90^\circ$ ) to protect products are superior to the linear system ( $\phi_0 = 90^\circ$ ).

## 5. Conclusion

A dynamic model with nonlinear cubic-quintic Duffing oscillators is proposed for the suspension spring packaging system, and the first-order approximate solution of the equation is obtained by He's variable iteration method. Based on the results, the damage evaluation equation for the packaging system is derived, revealing the main controlling parameters of the damage potential of dropping shock to packaged products. Finally, the damage boundary curves and surfaces for the system are discussed. It is found that decreasing the suspension angle can help to protect the packaged product.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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