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Research Article

Almost Periodic Sequence Solutions of a Discrete Predator-Prey System with Beddington-DeAngelis Functional Response

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This paper considers a discrete predator-prey system with Beddington-DeAngelis functional response. Sufficient conditions are obtained for the existence of the almost periodic solution which is uniformly asymptotically stable by constructing a Lyapunov function.

1. Introduction

In the past decades, the predator-prey competition models have been extensively studied by many authors (see [1–7]). Many excellent works have been done for the predator-prey model with functional response, such as Zhu and Wang [8] who considered a Volterra model with modified Leslie-Gower Holling-type II schemes. Cai et al. [9] studied the positive periodic solution for a multispecies competition-predator system with the Holling III functional and time delays. A well-known model of such systems is the predator-prey model with a Beddington-DeAngelis functional response which was originally proposed by Beddington [10] and DeAngelis et al. [11], independently. The dynamics of this model is described by the following differential equations:

$$\begin{aligned} \dot{x} &= rx(t)\left[1-\frac{x(t)}{K}\right]-\frac{bx(t)\,y(t)}{1+nx(t)+my(t)}, \\ \dot{y} &= y(t)\left[-d+\frac{fbx(t)}{1+nx(t)+my(t)}\right], \end{aligned} \tag{1}$$

where x and y represent prey and predator densities, respectively. Usually, the constant r is called intrinsic growth rate of the prey; K is the carrying capacity of the prey; b and n are positive constants that describe the effects of capture rate and handling time, respectively, on the feeding rate; d is the death rate of the predator; f is the birth rate of the predator; $m \geq 0$ (units: 1/predator) is a constant describing the magnitude of interference among predators. Many excellent works have been done for the predator-prey system with a Beddington-DeAngelis functional response, such as Fan and Kuang [12], Liu et al. [13], and Baek [14].

It has been found that discrete time models governed by difference equations are more appropriate than the continuous ones when the populations have nonoverlapping generations. Discrete time models can also provide efficient computational models of continuous models for numerical simulations. There are some existing results on discrete predator-prey systems [15–17]. For example, in [15], by using the method of upper and lower solutions and the degree theory the authors have studied the existence of periodic positive solutions for a competitive system with two parameters. Zhang and Wang [16] studied the following discrete model:

$$\begin{aligned} x(k+1) &= x(k)\exp\left[a(k)-b(k)\,x(k)-\frac{c(k)\,y(k)}{\alpha(k)+\beta(k)\,x(k)+\gamma(k)\,y(k)}\right], \\ y(k+1) &= y(k)\exp\left[-d(k)+\frac{f(k)\,y(k)}{\alpha(k)+\beta(k)\,x(k)+\gamma(k)\,y(k)}\right]. \end{aligned} \tag{2}$$

The authors used the Mawhin's coincidence degree theory to obtain some sufficient conditions for the existence of positive solutions.

But, nowadays, models with almost periodic coefficient have drawn more attention. Li and Chen [18] considered the almost periodic solutions of the following discrete almost periodic logistic equation:

$$x(n+1) = x(n)\exp\left\{r(n)\left(1-\frac{x(n)}{K(n)}\right)\right\}. \tag{3}$$

Niu and Chen [19] proposed the following model:

$$\begin{aligned}
x_1(n+1) &= x_1(n) \times \exp\left[r_1(n) \times \left(1-\frac{x_1(n)}{K_1(n)} - \mu_2(n)x_2(n) - b_1(n)\mu_1(n)\right)\right],\\
x_2(n+1) &= x_2(n) \times \exp\left[r_2(n) \times \left(1-\frac{x_2(n)}{K_2(n)} - \mu_1(n)x_1(n) - b_2(n)\mu_2(n)\right)\right],\\
\Delta\mu_1(n) &= -\alpha_1(n)\mu_1(n) + \beta_1(n)x_1(n),\\
\Delta\mu_2(n) &= -\alpha_2(n)\mu_2(n) + \beta_2(n)x_2(n).
\end{aligned} \tag{4}$$

By constructing a suitable Lyapunov function, they obtained the existence and uniqueness of the almost periodic solution which is uniformly asymptotically stable.

Li et al. [20] studied the following system:

$$\begin{aligned}
x(k+1) &= x(k) \times \exp\left\{a(k) - b(k)x(k) - p(k, x(k), y(k), x(k-\mu), y(k-\nu))\frac{y(k)}{x(k)}\right\},\\
y(k+1) &= y(k)\exp\left\{c(k) - \frac{d(k)y(k)}{x(k-\mu)}\right\}.
\end{aligned} \tag{5}$$

With the method of the theory of difference inequality and constructing a suitable Lyapunov function, they obtained the permanence and the almost periodic solution of the system.

Inspired by the above papers, in this paper, we consider the following discrete predator-prey system:

$$\begin{aligned}
x_1(n+1) &= x_1(n) \times \exp\left[r_1(n)\left(1-\frac{x_1(n)}{K_1(n)}\right) - \frac{b(n)x_2(n)}{\alpha(n)+\beta(n)x_1(n)+\gamma(n)x_2(n)}\right],\\
x_2(n+1) &= x_2(n) \times \exp\left[-r_2(n) + \frac{c(n)x_1(n)}{\alpha(n)+\beta(n)x_1(n)+\gamma(n)x_2(n)}\right],
\end{aligned} \tag{6}$$

where $x_1(n)$ and $x_2(n)$ represent prey and predator densities at time t.

From the point of view of biology, we assumed that the initial conditions of (6) are of the form

$$x_i(0) > 0, \quad i = 1, 2. \tag{7}$$

Then, it is easy to see that the solutions of (6) with the initial condition (7) are defined and remain positive for all $n \in Z^+$. Throughout this paper, for any bounded sequence $\{a(n)\}$, we denote $a^M = \sup_{n\in N}\{a(n)\}$ and $a^L = \inf_{n\in N}\{a(n)\}$, and we assume that

(H_1) $\{r_1(n)\}$, $\{r_2(n)\}$, $\{K(n)\}$, $\{b(n)\}$, $\{c(n)\}$, $\{\alpha(n)\}$, $\{\beta(n)\}$, and $\{\gamma(n)\}$ are bounded nonnegative almost periodic sequences such that

$$\begin{aligned}
&0 < r_1^L \le r_1(n) \le r_1^M, && 0 < r_2^L \le r_2(n) \le r_2^M,\\
&0 < K^L \le K(n) \le K^M, && 0 < \alpha^L \le \alpha(n) \le \alpha^M,\\
&0 < \beta^L \le \beta(n) \le \beta^M, && 0 < \gamma^L \le \gamma(n) \le \gamma^M,\\
&0 < b^L \le b(n) \le b^M, && 0 < c^L \le c(n) \le c^M,
\end{aligned} \tag{8}$$

(H_2) $r_1^L - (b^M/\gamma^L) > 0$,

(H_3) $(c^L/\beta^M)(1-\alpha^M) - r_2^M > 0$.

The organization of this paper is as follows. In Section 2, we will introduce some definitions and several useful definitions and lemmas. In Section 3, by applying the theory of difference inequality, we get the permanence of system (6). In Section 4, by constructing a suitable Lyapunov function, we obtain the existence of the almost periodic solution for system (6) which is uniformly asymptotically stable. Finally, we give some examples and numerical simulations to verify our results.

2. Preliminaries

In this section, we will introduce some basic definitions and several useful lemmas.

Definition 1. System (6) is said to be permanent if there exist positive constants M_i, m_i, which are independent of the solutions of the system, such that any positive solution $(x_1(n), x_2(n))^{\mathrm{T}}$ of system (6) satisfies

$$m_i \leq \lim_{n\to+\infty} \inf x_i(n) \leq \lim_{n\to+\infty} \sup x_i(n) \leq M_i, \quad i = 1, 2. \tag{9}$$

Lemma 2 (see [21]). *Assume that $\{x(n)\}$ satisfies $x(n) > 0$ and*

$$x(n+1) \leq x(n) \exp\{a(n) - b(n)x(n)\}, \quad n \in N, \tag{10}$$

where $a(n)$ and $b(n)$ are nonnegative sequences bounded above and below by positive constants. Then,

$$\lim_{n\to+\infty} \sup x(n) \leq \frac{1}{b^L} \exp\left\{a^M - 1\right\}. \tag{11}$$

Lemma 3 (see [21]). *Assume that $\{x(n)\}$ satisfies $x(n) > 0$ and*

$$x(n+1) \geq x(n) \exp\{a(n) - b(n)x(n)\}, \quad n \geq N_0, \tag{12}$$

and $\lim_{n\to+\infty} \sup x(n) \leq x^*$, $x(N_0) > 0$, where $a(n)$ and $b(n)$ are nonnegative sequences bounded above and below by positive constants and $N_0 \in N$. Then,

$$\lim_{n\to+\infty} \inf x(n) \geq \min\left\{\frac{a^L}{b^M} \exp\left\{a^M - b^M x^*\right\}, \frac{a^L}{b^M}\right\}. \tag{13}$$

Lemma 4 (see [22]). *Let $x(n)$ and $b(n)$ be nonnegative sequences defined on N, and $c \geq 0$ is a constant. If*

$$x(n) \leq c + \sum_{s=0}^{n-1} b(s)x(s), \quad n \in N, \tag{14}$$

then

$$x(n) \leq c\prod_{s=0}^{n-1}[1 + b(s)], \quad n \in N. \tag{15}$$

Definition 5 (see [23]). A sequence $x : Z \to R$ is called an almost periodic sequence if the ϵ-translation number set of x

$$E\{\epsilon, x\} = \{\tau \in Z : |x(n+\tau) - x(n)| < \epsilon, \forall n \in Z\} \tag{16}$$

is a relatively dense set in Z for all $\epsilon > 0$; that is, for any given $\epsilon > 0$, there exists an integer $l(\epsilon) > 0$ such that each interval of length $l(\epsilon)$ contains an integer $\tau \in E\{\epsilon, x\}$ such that

$$|x(n+\tau) - x(n)| < \epsilon, \quad \forall n \in Z, \tag{17}$$

τ is called the ϵ-translation number or ϵ-almost period.

Definition 6 (see [23]). Let $f : Z \times D \to R$, where D is an open set in R; $f(n, x)$ is said to be almost periodic in n uniformly for $x \in D$, or uniformly almost periodic for short, if, for any $\epsilon > 0$ and any compact set S in D, there exists a positive integer $l(\epsilon, S)$ such that any interval of length $l(\epsilon, S)$ contains a integer τ for which

$$\left|f(n+\tau, x) - f(n, x)\right| < \epsilon, \tag{18}$$

for all $n \in Z$ and $x \in S$. τ is called the ϵ-translation number of $f(n, x)$.

Lemma 7 (see [23]). *$\{x(n)\}$ is an almost periodic sequence if and only if for any sequence $\{m_i\} \subset Z$ there exists a subsequence $\{m_{i_k}\} \subset \{m_i\}$ such that $x(n+m_{i_k})$ converges uniformly on $n \in Z$ as $k \to \infty$. Furthermore, the limit sequence is also an almost periodic sequence.*

In [24], Zhang considered the following almost periodic difference system:

$$x(n+1) = f(n, x(n)), \quad n \in Z^+, \tag{19}$$

where $f : Z \times S_B \to R$, $S_B = \{x \in R : \|x\| < B\}$, and $f(n, x)$ is almost periodic in n uniformly for $x \in S_B$ and is continuous in x. The product system of (19) is the following system:

$$x(n+1) = f(n, x(n)), \qquad y(n+1) = f(n, y(n)), \tag{20}$$

and the following lemma is obtained.

Lemma 8 (see [24]). *Suppose that there exists a Lyapunov functional $V(n, x, y)$ defined for $n \in Z^+$, $\|x\| < B$, $\|y\| < B$ satisfying the following conditions:*

[C_1] *$a(\|x-y\|) \leq V(n, x, y) \leq b(\|x-y\|)$, where $a, b \in \widehat{T}$ with*

$$\widehat{T} = \{a \in C(R^+, R^+) : a(0) = 0 \text{ and } a \text{ is increasing}\}, \tag{21}$$

[C_2] *$|V(n, x_1, y_1) - V(n, x_2, y_2)| \leq L(\|x_1 - x_2\| + \|y_1 - y_2\|)$, where $L > 0$ is a constant,*

[C_3] *$\Delta V_{(20)} \leq -\lambda V(n, x, y)$, where $0 < \lambda < 1$ is a constant and*

$$\Delta V_{(20)} = V(n+1, f(n, x), f(n, y)) - V(n, x, y). \tag{22}$$

Moreover, if there exists a solution $\varphi(n)$ of system (20) such that $\|\varphi(n)\| < B^* < B$ for $n \in Z^+$, then there exists a unique uniformly asymptotically stable almost periodic solution $p(n)$ of system (20) which is bounded by B^*. In particular, if $f(n, x)$ is ω-periodic function, then there exists a unique uniformly asymptotically stable ω-periodic solution of (20).

3. Permanence

Theorem 9. *Assume that (H_1), (H_2), and (H_3) hold, then every solution $(x_1(n), x_2(n))^{\mathrm{T}}$ of system (6) satisfies*

$$x_{i*} \leq \lim_{n\to+\infty} \inf x_i(n) \leq \lim_{n\to+\infty} \sup x_i(n) \leq x_i^*, \quad i = 1, 2. \tag{23}$$

That is, system (6) *is permanent, where*

$$
\begin{aligned}
x_1^* &= \frac{K^M}{r_1^L}\exp\left\{r_1^M - 1\right\},\\
x_{1*} &= \frac{K^L\left(r_1^L\gamma^L - b^M\right)}{\gamma^L r_1^M}\exp\left\{r_1^L - \frac{b^M}{\gamma^L} - \frac{r_1^M}{K^L}x_1^*\right\},\\
x_2^* &= \exp\left\{2\left(\frac{c^M}{\beta^L} - r_2^L\right)\right\},\\
x_{2*} &= \min\left\{T\exp\left\{\frac{c^L\left(1-\alpha^M\right) - r_2^M\beta^M}{\beta^M} - \frac{c^M\gamma^M}{\beta^L}x_2^*\right\}, T\right\}\\
T &= \frac{\beta^L c^L\left(1-\alpha^M\right) - \beta^L\beta^M r_2^M}{c^M\gamma^M\beta^M}.
\end{aligned}
\tag{24}
$$

Proof. Let $(x_1(n), x_2(n))^{\mathbf{T}}$ be any positive solution of system (6); from the first equation of system (6), it follows that

$$
x_1(n+1) \le x_1(n)\exp\left\{r_1(n) - \frac{r_1(n)}{K(n)}x_1(n)\right\}. \tag{25}
$$

Thus, as a direct corollary of Lemma 2, it follows that

$$
\lim_{n\to+\infty}\sup x_1(n) \le \frac{K^M}{r_1^L}\exp\left\{r_1^M - 1\right\} := x_1^*. \tag{26}
$$

On the other hand, from the first equation of system (6), we obtain

$$
\begin{aligned}
x_1(n+1) &\ge x_1(n)\exp\left\{r_1(n) - \frac{r_1(n)}{K(n)}x_1(n) - \frac{b(n)x_2(n)}{\gamma(n)x_2(n)}\right\}\\
&= x_1(n)\exp\left\{r_1(n) - \frac{b(n)}{\gamma(n)} - \frac{r_1(n)}{K(n)}x_1(n)\right\}.
\end{aligned}
\tag{27}
$$

According to Lemma 3 and assumption (H_2), we have

$$
\begin{aligned}
&\lim_{n\to+\infty}\inf x_1(n)\\
&\quad\ge \min\left\{\frac{K^L\left(r_1^L\gamma^L - b^M\right)}{\gamma^L r_1^M}\right.\\
&\qquad\left.\times\exp\left\{r_1^L - \frac{b^M}{\gamma^L} - \frac{r_1^M}{K^L}x_1^*\right\}, \frac{K^L\left(r_1^L\gamma^L - b^M\right)}{\gamma^L r_1^M}\right\}.
\end{aligned}
\tag{28}
$$

Notice that

$$
\begin{aligned}
\frac{r_1^M/K^L}{r_1^L - b^M/\gamma^L}\cdot x_1^* &= \frac{r_1^M/K^L}{r_1^L - b^M/\gamma^L}\cdot\frac{K^M}{r_1^L}\exp\left\{r_1^M - 1\right\}\\
&= \frac{r_1^M}{r_1^L}\cdot\frac{K^M}{K^L}\cdot\frac{\exp\left\{r_1^M - 1\right\}}{r_1^L - b^M/\gamma^L}\\
&> \frac{r_1^M}{r_1^L}\cdot\frac{K^M}{K^L}\cdot\frac{r_1^M}{r_1^L}\\
&> 1.
\end{aligned}
\tag{29}
$$

In the above inequality, using the Bernoulli inequality $e^x > 1 + x$, for $x > -1$, one has

$$
\begin{aligned}
&\lim_{n\to+\infty}\inf x_1(n)\\
&\quad\ge \frac{K^L\left(r_1^L\gamma^L - b^M\right)}{\gamma^L r_1^M}\exp\left\{r_1^L - \frac{b^M}{\gamma^L} - \frac{r_1^M}{K^L}x_1^*\right\} := x_{1*}.
\end{aligned}
\tag{30}
$$

From the second equation of system (6), we can obtain

$$
x_2(n+1) \le x_2(n)\exp\left\{-r_2(n) + \frac{c(n)}{\beta(n)}\right\}. \tag{31}
$$

Let $x_2(n) = \exp\{y(n)\}$; then

$$
\begin{aligned}
y(n+1) &\le y(n) + \frac{c(n)}{\beta(n)} - r_2(n)\\
&\le \sum_{s=0}^{n} f(s)\,y(s) + \frac{c^M}{\beta^L} - r_2^L,
\end{aligned}
\tag{32}
$$

where

$$
f(s) = \begin{cases} 0, & 0 \le s \le n-1,\\ 1, & s = n. \end{cases} \tag{33}
$$

Since

$$
\frac{c^M}{\beta^L} - r_2^L \ge \frac{c^L}{\beta^M}\left(1-\alpha^M\right) - r_2^M > 0, \tag{34}
$$

by applying Lemma 4, we have

$$
y(n+1) \le 2\left(\frac{c^M}{\beta^L} - r_2^L\right). \tag{35}
$$

Thus,

$$
\lim_{n\to+\infty}\sup x_2(n) \le \exp\left\{2\left(\frac{c^M}{\beta^L} - r_2^L\right)\right\} := x_2^*. \tag{36}
$$

By the second equation of system (6), we can get that

$$\begin{aligned}
&x_2(n+1) \\
&= x_2(n)\exp\Bigg[-r_2(n) \\
&\qquad + \frac{c(n)x_1(n)}{\alpha(n)+\beta(n)x_1(n)+\gamma(n)x_2(n)}\Bigg] \\
&= x_2(n)\exp\Bigg\{-r_2(n)+c(n) \\
&\qquad \times\Bigg[\frac{x_1(n)}{\alpha(n)+\beta(n)x_1(n)+\gamma(n)x_2(n)} \\
&\qquad -\frac{1}{\beta(n)}+\frac{1}{\beta(n)}\Bigg]\Bigg\} \\
&= x_2(n)\exp\Bigg\{\frac{c(n)}{\beta(n)}-r_2(n)-\frac{c(n)}{\beta(n)} \\
&\qquad \times\frac{\alpha(n)+\gamma(n)x_2(n)}{\alpha(n)+\beta(n)x_1(n)+\gamma(n)x_2(n)}\Bigg\} \\
&= x_2(n) \\
&\quad \times\exp\Bigg\{\frac{c(n)}{\beta(n)} \\
&\qquad \times\left(1-\frac{\alpha(n)}{\alpha(n)+\beta(n)x_1(n)+\gamma(n)x_2(n)}\right) \\
&\qquad -r_2(n)-\frac{c(n)}{\beta(n)} \\
&\qquad \times\frac{\gamma(n)}{\alpha(n)+\beta(n)x_1(n)+\gamma(n)x_2(n)}x_2(n)\Bigg\} \\
&\geq x_2(n)\exp\left\{\frac{c^L}{\beta^M}\left(1-\alpha^M\right)-r_2^M-\frac{c^M\gamma^M}{\beta^L}x_2(n)\right\}.
\end{aligned} \tag{37}$$

According to (H_3), applying Lemma 3 to inequality (37), one has

$$\begin{aligned}
&\lim_{n\to+\infty}\inf x_2(n) \\
&\geq \min\left\{T\exp\left\{\frac{c^L\left(1-\alpha^M\right)}{\beta^M}-r_2^M-\frac{c^M\gamma^M}{\beta^L}x_2^*\right\},T\right\} \\
&:= x_{2*}.
\end{aligned} \tag{38}$$

The proof of the theorem is completed. □

4. Existence of Globally Attractive Almost Periodic Solutions

In this section, we study the existence of a globally attractive almost periodic sequence solution of system (6).

First, we denote by Ω the set of all solutions $(x_1(n), x_2(n))$ of system (6) satisfying $x_{i*} \leq x_i(n) \leq x_i^*$, $i = 1, 2$ for all $n \in Z^+$.

In order to apply Lemma 8, we should prove that there exists a bounded solution of system (6) and then construct an adaptive Lyapunov functional for system (6).

Theorem 10. *Assume that (H_1), (H_2), and (H_3) hold. Then* $\Omega \neq \emptyset$.

Proof. By the almost periodicity of $\{r_1(n)\}$, $\{r_2(n)\}$, $\{K(n)\}$, $\{b(n)\}$, $\{c(n)\}$, $\{\alpha(n)\}$, $\{\beta(n)\}$, and $\{\gamma(n)\}$, there exists an integer valued sequence τ_p with $\tau_p \to \infty$ as $p \to \infty$ such that

$$\begin{aligned}
&r_1(n+\tau_p)\longrightarrow r_1(n), \qquad r_2(n+\tau_p)\longrightarrow r_2(n), \\
&K(n+\tau_p)\longrightarrow K(n), \qquad b(n+\tau_p)\longrightarrow b(n), \\
&c(n+\tau_p)\longrightarrow c(n), \qquad \alpha(n+\tau_p)\longrightarrow \alpha(n), \\
&\beta(n+\tau_p)\longrightarrow \beta(n), \qquad \gamma(n+\tau_p)\longrightarrow \gamma(n)
\end{aligned} \tag{39}$$

as $p \to \infty$ uniformly on Z^+.

Let ϵ be an arbitrary small positive number. It follows from Theorem 9 that there exists a positive integer N_0 such that

$$x_{i*}-\epsilon \leq x_i(n) \leq x_i^*+\epsilon, \quad i=1,2,\ \forall n > N_0. \tag{40}$$

Write $x_{ip}(n) = x_i(n+\tau_p)$, $i = 1, 2$, for $n \geq N_0 - \tau_p$, $p = 1, 2, \ldots$. For any positive integer q, it is easy to see that there exist sequence $\{x_{ip} : p \geq q, i = 1, 2\}$ such that the sequence $\{x_{ip}\}$ have subsequences, denoted by x_{ip} again, converging on any finite interval of Z as $p \to \infty$, respectively. Thus we have sequence $y_i(n)$ such that

$$x_{ip}\longrightarrow y_i(n), \quad i=1,2 \text{ for } n\in Z^+ \text{ as } p\longrightarrow\infty. \tag{41}$$

Combined with

$$\begin{aligned}
&x_{1p}(n+1) \\
&\quad = x_{1p}(n)\exp\Bigg[r_1(n+\tau_p)\left(1-\frac{x_{1p}(n)}{K(n+\tau_p)}\right) \\
&\qquad - b(n+\tau_p)x_{2p}(n) \\
&\qquad \times\Big(\alpha(n+\tau_p)+\beta(n+\tau_p)x_{1p}(n) \\
&\qquad +\gamma(n+\tau_p)x_{2p}(n)\Big)^{-1}\Bigg],
\end{aligned}$$

$$\begin{aligned} x_{2p}(n+1) &= x_{2p}(n)\exp\Big[-r_2(n+\tau_p) \\ &\quad + c(n+\tau_p)\,x_{1p}(n) \\ &\quad \times\big(\alpha(n+\tau_p)+\beta(n+\tau_p)\,x_{1p}(n) \\ &\quad +\gamma(n+\tau_p)\,x_{2p}(n)\big)^{-1}\Big], \end{aligned} \tag{42}$$

it gives

$$\begin{aligned} y_1(n+1) &= y_1(n)\exp\left[r_1(n)\left(1-\frac{y_1(n)}{K(n)}\right) - \frac{b(n)\,y_2(n)}{\alpha(n)+\beta(n)\,y_1(n)+\gamma(n)\,y_2(n)}\right], \\ y_2(n+1) &= y_2(n)\exp\left[-r_2(n) + \frac{c(n)\,y_1(n)}{\alpha(n)+\beta(n)\,y_1(n)+\gamma(n)\,y_2(n)}\right]. \end{aligned} \tag{43}$$

We can easily see that $(y_1(n), y_2(n))$ is a solution of system (6); furthermore,

$$x_{i*}-\epsilon \le y_i(n) \le x_i^* + \epsilon, \quad i=1,2, \text{ for } n\in Z^+. \tag{44}$$

Since ϵ is an arbitrary small positive number, it follows that

$$x_{i*} \le y_i(n) \le x_i^*, \quad i=1,2, \text{ for } n\in Z^+. \tag{45}$$

The proof of theorem is completed. □

In the following, we denote

$$\begin{aligned} &\forall x\in[x_{1*}, x_1^*], \quad \forall y\in[x_{2*}, x_2^*], \\ &g(n,x,y)=\alpha(n)+\beta(n)\,x+\gamma(n)\,y, \\ &H=\max\{g(n,x,y)\}, \quad h=\min\{g(n,x,y)\}, \\ \eta_1 &= \frac{2r_1^L x_{1*}}{K^M}+\frac{2r_1^L b^L\beta^L x_{2*}x_{1*}^2}{K^M H^2} - \frac{r_1^M b^M\alpha^M x_1^* x_2^{*2}+r_1^M b^M\beta^M x_1^{*2}x_2^{*2}}{K^L h^2} \\ &\quad -\Big(\Big(\big(b^{M^2}\beta^{M^2}+c^{M^2}\gamma^{M^2}\big)x_1^{*2}x_2^{*2} +2c^M\alpha^M\gamma^M x_1^{*2}x_2^{*}+c^{M^2}\alpha^M x_1^{*2}\Big)\times\big(h^4\big)^{-1}\Big) \\ &\quad - \frac{c^M\alpha^M x_1^* + \big(c^M\gamma^M+2b^M\beta^M\big)x_1^*x_2^*}{h^2}, \\ \eta_2 &= \frac{2c^L\gamma^L x_{1*}x_{2*}}{H^2} - \frac{r_1^M b^M\alpha^M x_1^* x_2^{*2}+r_1^M b^M\beta^M x_1^{*2}x_2^{*2}}{K^L h^2} \\ &\quad -\Big(\Big(\alpha^{M^2}b^{M^2}x_2^{*4}+2b^{M^2}\alpha^M\beta^M x_1^* x_2^{*4} +b^{M^2}\beta^{M^2}x_1^{*2}x_2^{*4}+c^M\gamma^M x_1^{*2}x_2^{*2}\Big)\times\big(h^4\big)^{-1}\Big) \\ &\quad - \frac{c^M\alpha^M x_1^* + c^M\gamma^M x_1^* x_2^*}{h^2}, \\ \eta &= \min\{\eta_1,\eta_2\}. \end{aligned} \tag{46}$$

Theorem 11. *Suppose the conditions* (H_1), (H_2), *and* (H_3) *are satisfied, and let* $0<\eta<1$; *then there exists a uniqueness uniformly asymptotically stable almost periodic solution* $X=(x_1(n), x_2(n))$ *of system* (6) *which is bounded by* Ω *for all* $n\in Z^+$.

Proof. Let $x_1(n)=\exp\{p_1(n)\}$, $x_2(n)=\exp\{p_2(n)\}$. From system (6),we have

$$\begin{aligned} p_1(n+1) &= p_1(n) + r_1(n)\left(1-\frac{e^{p_1(n)}}{K(n)}\right) - \frac{b(n)\,e^{p_2(n)}}{g\big(n,e^{p_1(n)},e^{p_2(n)}\big)}, \\ p_2(n+1) &= p_2(n)-r_2(n)+\frac{c(n)\,e^{p_1(n)}}{g\big(n,e^{p_1(n)},e^{p_2(n)}\big)}. \end{aligned} \tag{47}$$

From Theorem 11, we know that the system (47) have bounded solution $Y(n)=(p_1(n),p_2(n))$ satisfying

$$\ln x_{i*} \le p_i(n) \le \ln x_i^*, \quad i=1,2,\ n\in Z^+. \tag{48}$$

Hence, $|p_i(n)| \le A_i$, where $A_i=\max\{|\ln x_{i*}|, |\ln x_i^*|\}$, $i=1,2$. For $(p_1(n),p_2(n))\in R^2$, we define the norm $\|(p_1(n),p_2(n))\|=|p_1(n)|+|p_2(n)|$.

Consider the product system of system (47)

$$\begin{aligned} p_1(n+1) &= p_1(n)+r_1(n)\left(1-\frac{e^{p_1(n)}}{K(n)}\right) - \frac{b(n)\,e^{p_2(n)}}{g\big(n,e^{p_1(n)},e^{p_2(n)}\big)}, \\ p_2(n+1) &= p_2(n)-r_2(n)+\frac{c(n)\,e^{p_1(n)}}{g\big(n,e^{p_1(n)},e^{p_2(n)}\big)}, \\ q_1(n+1) &= q_1(n)+r_1(n)\left(1-\frac{e^{q_1(n)}}{K(n)}\right) - \frac{b(n)\,e^{q_2(n)}}{g\big(n,e^{q_1(n)},e^{q_2(n)}\big)}, \\ q_2(n+1) &= q_2(n)-r_2(n)+\frac{c(n)\,e^{q_1(n)}}{g\big(n,e^{q_1(n)},e^{q_2(n)}\big)}. \end{aligned} \tag{49}$$

Suppose $X = (p_1(n), p_2(n))$, $Y = (q_1(n), q_2(n))$ are any two solutions of system (49) defined on $Z^+ \times S^* \times S^*$; then $\|X\| \le A$, $\|Y\| \le A$, where $A = A_1 + A_2$,

$$S^* = \{(p_1(n), p_2(n)) \mid \ln x_{i*} \le p_i(n) \le \ln x_i^*, \quad i = 1, 2, n \in Z^+\}. \tag{50}$$

Consider a Lyapunov function defined on $Z^+ \times S^* \times S^*$ as follows:

$$V(n, X, Y) = \sum_{i=1}^{2} (p_i(n) - q_i(n))^2. \tag{51}$$

It is easy to see that the norm $\|X - Y\| = \sum_{i=1}^{2} |(p_i(n) - q_i(n))|$ and the norm $\|X - Y\|_* = \sqrt{\sum_{i=1}^{2} (p_i(n) - q_i(n))^2}$ are equivalent; that is, there exist two constants $C_1 > 0$, $C_2 > 0$, such that

$$C_1 \|X - Y\| \le \|X - Y\|_* \le C_2 \|X - Y\|; \tag{52}$$

then

$$(C_1 \|X - Y\|)^2 \le V(n, X, Y) \le (C_2 \|X - Y\|)^2. \tag{53}$$

Let $a \in C(R^+, R^+)$, $a(x) = C_1^2 x^2$, $b \in C(R^+, R^+)$, and $b(x) = C_2^2 x^2$; thus the condition [C_1] in Lemma 8 is satisfied.

In addition,

$$\begin{aligned}
&\left|V(n, X, Y) - V(n, \widetilde{X}, \widetilde{Y})\right| \\
&= \left|\sum_{i=1}^{2} (p_i(n) - q_i(n))^2 - \sum_{i=1}^{2} (\widetilde{p}_i(n) - \widetilde{q}_i(n))^2\right| \\
&\le \left|(p_1(n) - q_1(n))^2 - (\widetilde{p}_1(n) - \widetilde{q}_1(n))^2\right| \\
&\quad + \left|(p_2(n) - q_2(n))^2 - (\widetilde{p}_2(n) - \widetilde{q}_2(n))^2\right| \\
&= \left|p_1(n) - q_1(n) + \widetilde{p}_1(n) - \widetilde{q}_1(n)\right| \\
&\quad \times \left|p_1(n) - q_1(n) - \widetilde{p}_1(n) + \widetilde{q}_1(n)\right| \\
&\quad + \left|p_2(n) - q_2(n) + \widetilde{p}_2(n) - \widetilde{q}_2(n)\right| \\
&\quad \times \left|p_2(n) - q_2(n) - \widetilde{p}_2(n) + \widetilde{q}_2(n)\right| \\
&\le \left(|p_1(n)| + |q_1(n)| + |\widetilde{p}_1(n)| + |\widetilde{q}_1(n)|\right) \\
&\quad \times \left(|p_1(n) - \widetilde{p}_1(n)| + |q_1(n) - \widetilde{q}_1(n)|\right) \\
&\quad + \left(|p_2(n)| + |q_2(n)| + |\widetilde{p}_2(n)| + |\widetilde{q}_2(n)|\right) \\
&\quad \times \left(|p_2(n) - \widetilde{p}_2(n)| + |q_2(n) - \widetilde{q}_2(n)|\right) \\
&\le L\{|p_1(n) - \widetilde{p}_1(n)| + |q_1(n) - \widetilde{q}_1(n)| \\
&\qquad + |p_2(n) - \widetilde{p}_2(n)| + |q_2(n) - \widetilde{q}_2(n)|\} \\
&= L\left\{\left\|X - \widetilde{X}\right\| + \left\|Y - \widetilde{Y}\right\|\right\},
\end{aligned} \tag{54}$$

where $L = 4\max\{A_1, A_2\}$. Hence, the condition [C_2] of Lemma 8 is satisfied.

Finally, calculate the ΔV of $V(n)$ along the solutions of (49); we can obtain

$$\begin{aligned}
\Delta V_{(49)}(n, X, Y) &= V(n+1, X, Y) - V(n, X, Y) \\
&= \sum_{i=1}^{2} (p_i(n+1) - q_i(n+1))^2 \\
&\quad - \sum_{i=1}^{2} (p_i(n) - q_i(n))^2 \\
&= (p_1(n+1) - q_1(n+1))^2 \\
&\quad - (p_1(n) - q_1(n))^2 \\
&\quad + (p_2(n+1) - q_2(n+1))^2 \\
&\quad - (p_2(n) - q_2(n))^2.
\end{aligned} \tag{55}$$

In the view of (49), we get

$$\begin{aligned}
&(p_1(n+1) - q_1(n+1))^2 - (p_1(n) - q_1(n))^2 \\
&= \Bigg[(p_1(n) - q_1(n)) - \frac{r_1(n)}{K(n)}\left(e^{p_1(n)} - e^{q_1(n)}\right) \\
&\qquad - b(n)\left(\frac{e^{p_2(n)}}{g(n, e^{p_1(n)}, e^{p_2(n)})} - \frac{e^{q_2(n)}}{g(n, e^{q_1(n)}, e^{q_2(n)})}\right)\Bigg]^2 \\
&\quad - (p_1(n) - q_1(n))^2 \\
&= \Bigg[(p_1(n) - q_1(n)) - \frac{r_1(n)}{K(n)}\left(e^{p_1(n)} - e^{q_1(n)}\right) \\
&\qquad - b(n)\Bigg(\frac{e^{p_2(n)}}{g(n, e^{p_1(n)}, e^{p_2(n)})} - \frac{e^{p_2(n)}}{g(n, e^{q_1(n)}, e^{q_2(n)})} \\
&\qquad + \frac{e^{p_2(n)}}{g(n, e^{q_1(n)}, e^{q_2(n)})} - \frac{e^{q_2(n)}}{g(n, e^{q_1(n)}, e^{q_2(n)})}\Bigg)\Bigg]^2 \\
&\quad - (p_1(n) - q_1(n))^2 \\
&= \Bigg[(p_1(n) - q_1(n)) \\
&\qquad - \frac{b(n)e^{p_2(n)}\left(\alpha(n) + \beta(n)e^{p_1(n)}\right)}{g(n, e^{q_1(n)}, e^{q_2(n)})\, g(n, e^{p_1(n)}, e^{p_2(n)})} \\
&\qquad \times \left(e^{p_2(n)} - e^{q_2(n)}\right) \\
&\qquad - \left(\frac{r_1(n)}{K(n)} - \frac{b(n)e^{p_2(n)}\beta(n)}{g(n, e^{q_1(n)}, e^{q_2(n)})\, g(n, e^{p_1(n)}, e^{p_2(n)})}\right) \\
&\qquad \times \left(e^{p_1(n)} - e^{q_1(n)}\right)\Bigg]^2 - (p_1(n) - q_1(n))^2
\end{aligned}$$

$$
\begin{aligned}
&= \frac{b^2(n)\, e^{2p_2(n)}\left(\alpha(n) + \beta(n)\, e^{p_1(n)}\right)^2}{g^2\left(n, e^{q_1(n)}, e^{q_2(n)}\right) g^2\left(n, e^{p_1(n)}, e^{p_2(n)}\right)} \\
&\quad \times \left(e^{p_2(n)} - e^{q_2(n)}\right)^2 \\
&\quad + \left(\frac{r_1(n)}{K(n)} - \frac{b(n)\, e^{p_2(n)} \beta(n)}{g\left(n, e^{q_1(n)}, e^{q_2(n)}\right) g\left(n, e^{p_1(n)}, e^{p_2(n)}\right)}\right)^2 \\
&\quad \times \left(e^{p_1(n)} - e^{q_1(n)}\right)^2 \\
&\quad - 2 \frac{b(n)\, e^{p_2(n)} \left(\alpha(n) + \beta(n)\, e^{p_1(n)}\right)}{g\left(n, e^{q_1(n)}, e^{q_2(n)}\right) g\left(n, e^{p_1(n)}, e^{p_2(n)}\right)} \\
&\quad \times \left(e^{p_2(n)} - e^{q_2(n)}\right) \left(p_1(n) - q_1(n)\right) \\
&\quad - 2\left(\frac{r_1(n)}{K(n)} - \frac{b(n)\, e^{p_2(n)} \beta(n)}{g\left(n, e^{q_1(n)}, e^{q_2(n)}\right) g\left(n, e^{p_1(n)}, e^{p_2(n)}\right)}\right) \\
&\quad \times \left(e^{p_1(n)} - e^{q_1(n)}\right) \left(p_1(n) - q_1(n)\right) \\
&\quad + 2\left(\frac{r_1(n)}{K(n)} - \frac{b(n)\, e^{p_2(n)} \beta(n)}{g\left(n, e^{q_1(n)}, e^{q_2(n)}\right) g\left(n, e^{p_1(n)}, e^{p_2(n)}\right)}\right) \\
&\quad \times \frac{b(n)\, e^{p_2(n)} \left(\alpha(n) + \beta(n)\, e^{p_1(n)}\right)}{g\left(n, e^{q_1(n)}, e^{q_2(n)}\right) g\left(n, e^{p_1(n)}, e^{p_2(n)}\right)} \\
&\quad \times \left(e^{p_1(n)} - e^{q_1(n)}\right) \left(e^{p_2(n)} - e^{q_2(n)}\right),
\end{aligned}
$$

$$
\begin{aligned}
&\left(p_2(n+1) - q_2(n+1)\right)^2 - \left(p_2(n) - q_2(n)\right)^2 \\
&= \Bigg[\left(p_2(n) - q_2(n)\right) + c(n) \\
&\quad \times \left(\frac{e^{p_1(n)}}{g\left(n, e^{p_1(n)}, e^{p_2(n)}\right)} - \frac{e^{q_1(n)}}{g\left(n, e^{q_1(n)}, e^{q_2(n)}\right)}\right)\Bigg]^2 \\
&\quad - \left(p_2(n) - q_2(n)\right)^2 \\
&= \Bigg[\left(p_2(n) - q_2(n)\right) \\
&\quad + c(n)\Bigg(\frac{e^{p_1(n)}}{g\left(n, e^{p_1(n)}, e^{p_2(n)}\right)} - \frac{e^{p_1(n)}}{g\left(n, e^{q_1(n)}, e^{q_2(n)}\right)} \\
&\qquad + \frac{e^{p_1(n)}}{g\left(n, e^{q_1(n)}, e^{q_2(n)}\right)} - \frac{e^{q_1(n)}}{g\left(n, e^{q_1(n)}, e^{q_2(n)}\right)}\Bigg)\Bigg]^2 \\
&\quad - \left(p_2(n) - q_2(n)\right)^2 \\
&= \Bigg[\left(p_2(n) - q_2(n)\right) \\
&\quad + \frac{c(n)\,\alpha(n) + c(n)\,\gamma(n)\, e^{p_2(n)}}{g\left(n, e^{q_1(n)}, e^{q_2(n)}\right) g\left(n, e^{p_1(n)}, e^{p_2(n)}\right)} \\
&\quad \times \left(e^{p_1(n)} - e^{q_1(n)}\right) \\
&\quad - \frac{c(n)\,\gamma(n)\, e^{p_1(n)}}{g\left(n, e^{q_1(n)}, e^{q_2(n)}\right) g\left(n, e^{p_1(n)}, e^{p_2(n)}\right)} \\
&\quad \times \left(e^{p_2(n)} - e^{q_2(n)}\right)\Bigg]^2 - \left(p_2(n) - q_2(n)\right)^2 \\
&= \frac{\left(c(n)\,\alpha(n) + c(n)\,\gamma(n)\, e^{p_2(n)}\right)^2}{g^2\left(n, e^{q_1(n)}, e^{q_2(n)}\right) g^2\left(n, e^{p_1(n)}, e^{p_2(n)}\right)} \\
&\quad \times \left(e^{p_1(n)} - e^{q_1(n)}\right)^2 \\
&\quad + \frac{c^2(n)\,\gamma^2(n)\, e^{2p_1(n)}}{g^2\left(n, e^{q_1(n)}, e^{q_2(n)}\right) g^2\left(n, e^{p_1(n)}, e^{p_2(n)}\right)} \\
&\quad \times \left(e^{p_2(n)} - e^{q_2(n)}\right)^2 \\
&\quad + 2 \frac{c(n)\,\alpha(n) + c(n)\,\gamma(n)\, e^{p_2(n)}}{g\left(n, e^{q_1(n)}, e^{q_2(n)}\right) g\left(n, e^{p_1(n)}, e^{p_2(n)}\right)} \\
&\quad \times \left(e^{p_1(n)} - e^{q_1(n)}\right) \left(p_2(n) - q_2(n)\right) \\
&\quad - 2 \frac{c(n)\,\gamma(n)\, e^{p_1(n)}}{g\left(n, e^{q_1(n)}, e^{q_2(n)}\right) g\left(n, e^{p_1(n)}, e^{p_2(n)}\right)} \\
&\quad \times \left(e^{p_2(n)} - e^{q_2(n)}\right) \left(p_2(n) - q_2(n)\right) \\
&\quad - 2 \frac{\left(c(n)\,\alpha(n) + c(n)\,\gamma(n)\, e^{p_2(n)}\right) c(n)\,\gamma(n)\, e^{p_1(n)}}{g^2\left(n, e^{q_1(n)}, e^{q_2(n)}\right) g^2\left(n, e^{p_1(n)}, e^{p_2(n)}\right)} \\
&\quad \times \left(e^{p_1(n)} - e^{q_1(n)}\right) \left(e^{p_2(n)} - e^{q_2(n)}\right).
\end{aligned}
\tag{56}
$$

Substituting (56) into (55), we have

$$
\begin{aligned}
&\Delta V_{(49)}(n, X, Y) \\
&\quad = V(n+1, X, Y) - V(n, X, Y) \\
&\quad = \Bigg[\left(\frac{r_1(n)}{K(n)} - \frac{b(n)\, e^{p_2(n)} \beta(n)}{g\left(n, e^{q_1(n)}, e^{q_2(n)}\right) g\left(n, e^{p_1(n)}, e^{p_2(n)}\right)}\right)^2 \\
&\qquad + \frac{\left(c(n)\,\alpha(n) + c(n)\,\gamma(n)\, e^{p_2(n)}\right)^2}{g^2\left(n, e^{q_1(n)}, e^{q_2(n)}\right) g^2\left(n, e^{p_1(n)}, e^{p_2(n)}\right)}\Bigg] \\
&\qquad \times \left(e^{p_1(n)} - e^{q_1(n)}\right)^2 \\
&\qquad + \Bigg(\Big(b^2(n)\, e^{2p_2(n)}\left(\alpha(n) + \beta(n)\, e^{p_1(n)}\right)^2 \\
&\qquad\qquad + c^2(n)\,\gamma^2(n)\, e^{2p_1(n)}\Big) \\
&\qquad\quad \times \left(g^2\left(n, e^{q_1(n)}, e^{q_2(n)}\right) g^2\left(n, e^{p_1(n)}, e^{p_2(n)}\right)\right)^{-1}\Bigg) \\
&\qquad \times \left(e^{p_2(n)} - e^{q_2(n)}\right)^2
\end{aligned}
$$

$$
\begin{aligned}
&+2\left[\left(\frac{r_1(n)}{K(n)}-\frac{b(n)e^{p_2(n)}\beta(n)}{g(n,e^{q_1(n)},e^{q_2(n)})\,g(n,e^{p_1(n)},e^{p_2(n)})}\right)\right.\\
&\quad\times\frac{b(n)e^{p_2(n)}\left(\alpha(n)+\beta(n)e^{p_1(n)}\right)}{g(n,e^{q_1(n)},e^{q_2(n)})\,g(n,e^{p_1(n)},e^{p_2(n)})}\\
&\quad\left.-\frac{\left(c(n)\alpha(n)+c(n)\gamma(n)e^{p_2(n)}\right)\left(c(n)\gamma(n)e^{p_1(n)}\right)}{g^2(n,e^{q_1(n)},e^{q_2(n)})\,g^2(n,e^{p_1(n)},e^{p_2(n)})}\right]\\
&\times\left(e^{p_1(n)}-e^{q_1(n)}\right)\left(e^{p_2(n)}-e^{q_2(n)}\right)\\
&-2\frac{b(n)e^{p_2(n)}\left(\alpha(n)+\beta(n)e^{p_1(n)}\right)}{g(n,e^{q_1(n)},e^{q_2(n)})\,g(n,e^{p_1(n)},e^{p_2(n)})}\\
&\times\left(e^{p_2(n)}-e^{q_2(n)}\right)\left(p_1(n)-q_1(n)\right)\\
&-2\left(\frac{r_1(n)}{K(n)}-\frac{b(n)e^{p_2(n)}\beta(n)}{g(n,e^{q_1(n)},e^{q_2(n)})\,g(n,e^{p_1(n)},e^{p_2(n)})}\right)\\
&\times\left(e^{p_1(n)}-e^{q_1(n)}\right)\left(p_1(n)-q_1(n)\right)\\
&-2\frac{c(n)\gamma(n)e^{p_1(n)}}{g(n,e^{q_1(n)},e^{q_2(n)})\,g(n,e^{p_1(n)},e^{p_2(n)})}\\
&\times\left(e^{p_2(n)}-e^{q_2(n)}\right)\left(p_2(n)-q_2(n)\right)\\
&+2\frac{c(n)\alpha(n)+c(n)\gamma(n)e^{p_2(n)}}{g(n,e^{q_1(n)},e^{q_2(n)})\,g(n,e^{p_1(n)},e^{p_2(n)})}\\
&\times\left(e^{p_1(n)}-e^{q_1(n)}\right)\left(p_2(n)-q_2(n)\right).
\end{aligned}
\tag{57}
$$

Using the mean value theorem, we get

$$
e^{p_i(n)}-e^{q_i(n)}=\xi_i(n)\left(p_i(n)-q_i(n)\right),\quad i=1,2,
\tag{58}
$$

where $\xi_i(n)$ lies between $e^{p_i(n)}$ and $e^{q_i(n)}$, $i=1,2$. From (57) and (58), we have

$$
\begin{aligned}
&\Delta V_{(49)}(n,X,Y)\\
&\quad=V(n+1,X,Y)-V(n,X,Y)\\
&\quad=\left[\left(\frac{r_1(n)}{K(n)}-\frac{b(n)e^{p_2(n)}\beta(n)}{g(n,e^{q_1(n)},e^{q_2(n)})\,g(n,e^{p_1(n)},e^{p_2(n)})}\right)^2\xi_1^2(n)\right.\\
&\qquad+\frac{\left(c(n)\alpha(n)+c(n)\gamma(n)e^{p_2(n)}\right)^2}{g^2(n,e^{q_1(n)},e^{q_2(n)})\,g^2(n,e^{p_1(n)},e^{p_2(n)})}\xi_1^2(n)-2\\
&\qquad\times\left(\frac{r_1(n)}{K(n)}-\frac{b(n)e^{p_2(n)}\beta(n)}{g(n,e^{q_1(n)},e^{q_2(n)})\,g(n,e^{p_1(n)},e^{p_2(n)})}\right)\\
&\qquad\left.\times\xi_1(n)\right]\left(p_1(n)-q_1(n)\right)^2\\
&\qquad+\left[\left(\left(b^2(n)e^{2p_2(n)}\left(\alpha(n)+\beta(n)e^{p_1(n)}\right)^2\right.\right.\right.\\
&\qquad\qquad\left.+c^2(n)\gamma^2(n)e^{2p_1(n)}\right)\\
&\qquad\qquad\left.\times\left(g^2(n,e^{q_1(n)},e^{q_2(n)})\,g^2(n,e^{p_1(n)},e^{p_2(n)})\right)^{-1}\right)\xi_2^2(n)\\
&\qquad\qquad\left.-2\frac{c(n)\gamma(n)e^{p_1(n)}}{g(n,e^{q_1(n)},e^{q_2(n)})\,g(n,e^{p_1(n)},e^{p_2(n)})}\xi_2\right]\\
&\qquad\times\left(p_2(n)-q_2(n)\right)^2\\
&\qquad+2\left[\left(\frac{r_1(n)}{K(n)}-\frac{b(n)e^{p_2(n)}\beta(n)}{g(n,e^{q_1(n)},e^{q_2(n)})\,g(n,e^{p_1(n)},e^{p_2(n)})}\right)\right.\\
&\qquad\qquad\times\frac{b(n)e^{p_2(n)}\left(\alpha(n)+\beta(n)e^{p_1(n)}\right)}{g(n,e^{q_1(n)},e^{q_2(n)})\,g(n,e^{p_1(n)},e^{p_2(n)})}\xi_1(n)\xi_2(n)\\
&\qquad\qquad-\frac{\left(c(n)\alpha(n)+c(n)\gamma(n)e^{p_2(n)}\right)\left(c(n)\gamma(n)e^{p_1(n)}\right)}{g^2(n,e^{q_1(n)},e^{q_2(n)})\,g^2(n,e^{p_1(n)},e^{p_2(n)})}\\
&\qquad\qquad\times\xi_1(n)\xi_2(n)\\
&\qquad\qquad-\frac{b(n)e^{p_2(n)}\left(\alpha(n)+\beta(n)e^{p_1(n)}\right)}{g(n,e^{q_1(n)},e^{q_2(n)})\,g(n,e^{p_1(n)},e^{p_2(n)})}\xi_2(n)\\
&\qquad\qquad\left.+\frac{c(n)\alpha(n)+c(n)\gamma(n)e^{p_2(n)}}{g(n,e^{q_1(n)},e^{q_2(n)})\,g(n,e^{p_1(n)},e^{p_2(n)})}\xi_1(n)\right]\\
&\qquad\times\left(p_1(n)-q_1(n)\right)\left(p_2(n)-q_2(n)\right),
\end{aligned}
\tag{59}
$$

and we get

$$
\Delta V_{(49)}=V_1+V_2+V_3,
\tag{60}
$$

where

$$
\begin{aligned}
&V_1(n)\\
&\quad=\left[\left(\frac{r_1(n)}{K(n)}-\frac{b(n)e^{p_2(n)}\beta(n)}{g(n,e^{q_1(n)},e^{q_2(n)})\,g(n,e^{p_1(n)},e^{p_2(n)})}\right)^2\xi_1^2(n)\right.\\
&\qquad+\frac{\left(c(n)\alpha(n)+c(n)\gamma(n)e^{p_2(n)}\right)^2}{g^2(n,e^{q_1(n)},e^{q_2(n)})\,g^2(n,e^{p_1(n)},e^{p_2(n)})}\xi_1^2(n)\\
&\qquad-2\left(\frac{r_1(n)}{K(n)}-\frac{b(n)e^{p_2(n)}\beta(n)}{g(n,e^{q_1(n)},e^{q_2(n)})\,g(n,e^{p_1(n)},e^{p_2(n)})}\right)\\
&\qquad\left.\times\xi_1(n)\right]\left(p_1(n)-q_1(n)\right)^2
\end{aligned}
$$

$$\leq \Bigg[\left(\left(\left(b^{M^2}\beta^{M^2} + c^{M^2}\gamma^{M^2} \right) x_1^{*2} x_2^{*2} + c^{M^2}\alpha^M x_1^{*2} + 2c^M\alpha^M\gamma^M x_2^* x_1^{*2} \right) \times \left(h^4\right)^{-1} \right) + \frac{2b^M\beta^M x_2^* x_1^*}{h^2} - \frac{2r_1^L b^L \beta^L x_{2*} x_{1*}^2}{K^M H^2} + \frac{r_1^{M^2} x_1^{*2}}{K^{L^2}} - \frac{2r_1^L x_{1*}}{K^M} \Bigg] \times \left(p_1(n) - q_1(n)\right)^2,$$

$$\begin{aligned}
V_2(n) &= \Bigg[\Bigg(\left(b^2(n) e^{2p_2(n)} \left(\alpha(n) + \beta(n) e^{p_1(n)}\right)^2 + c^2(n)\gamma^2(n) e^{2p_1(n)} \right) \\
&\quad \times \left(g^2\left(n, e^{q_1(n)}, e^{q_2(n)}\right) g^2\left(n, e^{p_1(n)}, e^{p_2(n)}\right) \right)^{-1} \Bigg) \xi_2^2(n) \\
&\quad - 2\frac{c(n)\gamma(n) e^{p_1(n)}}{g\left(n, e^{q_1(n)}, e^{q_2(n)}\right) g\left(n, e^{p_1(n)}, e^{p_2(n)}\right)} \xi_2 \Bigg] \times \left(p_2(n) - q_2(n)\right)^2 \\
&\leq \Bigg[\left(\left(\alpha^{M^2} b^{M^2} x_2^{*4} + 2b^{M^2}\alpha^M\beta^M x_1^* x_2^{*4} + b^{M^2}\beta^{M^2} x_1^{*2} x_2^{*4} + c^M\gamma^M x_1^{*2} x_2^{*2} \right) \times \left(h^4\right)^{-1} \right) - \frac{2c^L\gamma^L x_{1*} x_{2*}}{H^2} \Bigg] \\
&\quad \times \left(p_2(n) - q_2(n)\right)^2,
\end{aligned}$$

$$\begin{aligned}
V_3(n) &= 2\Bigg[\left(\frac{r_1(n)}{K(n)} - \frac{b(n) e^{p_2(n)} \beta(n)}{g\left(n, e^{q_1(n)}, e^{q_2(n)}\right) g\left(n, e^{p_1(n)}, e^{p_2(n)}\right)} \right) \\
&\quad \times \frac{b(n) e^{p_2(n)} \left(\alpha(n) + \beta(n) e^{p_1(n)}\right)}{g\left(n, e^{q_1(n)}, e^{q_2(n)}\right) g\left(n, e^{p_1(n)}, e^{p_2(n)}\right)} \xi_1(n) \xi_2(n) \\
&\quad - \frac{\left(c(n)\alpha(n) + c(n)\gamma(n) e^{p_2(n)}\right)\left(c(n)\gamma(n) e^{p_1(n)}\right)}{g^2\left(n, e^{q_1(n)}, e^{q_2(n)}\right) g^2\left(n, e^{p_1(n)}, e^{p_2(n)}\right)} \times \xi_1(n) \xi_2(n) \\
&\quad - \frac{b(n) e^{p_2(n)} \left(\alpha(n) + \beta(n) e^{p_1(n)}\right)}{g\left(n, e^{q_1(n)}, e^{q_2(n)}\right) g\left(n, e^{p_1(n)}, e^{p_2(n)}\right)} \xi_2(n) \\
&\quad + \frac{c(n)\alpha(n) + c(n)\gamma(n) e^{p_2(n)}}{g\left(n, e^{q_1(n)}, e^{q_2(n)}\right) g\left(n, e^{p_1(n)}, e^{p_2(n)}\right)} \xi_1(n) \Bigg] \\
&\quad \times \left(p_1(n) - q_1(n)\right)\left(p_2(n) - q_2(n)\right) \\
&\leq 2\Bigg[\frac{c^M\alpha^M x_1^* + c^M\gamma^M x_1^* x_2^*}{h^2} + \frac{r_1^M b^M \alpha^M x_1^* x_2^{*2} + r_1^M b^M \beta^M x_1^{*2} x_2^{*2}}{K^L h^2} \Bigg] \\
&\quad \times \left[\left(p_1(n) - q_1(n)\right)^2 + \left(p_2(n) - q_2(n)\right)^2 \right].
\end{aligned} \tag{61}$$

Hence,

$$\begin{aligned}
\Delta V_{(49)} &\leq \Bigg[\left(\left(\left(b^{M^2}\beta^{M^2} + c^{M^2}\gamma^{M^2} \right) x_1^{*2} x_2^{*2} + c^{M^2}\alpha^M x_1^{*2} + 2c^M\alpha^M\gamma^M x_2^* x_1^{*2} \right) \times \left(h^4\right)^{-1} \right) \\
&\quad + \frac{2b^M\beta^M x_2^* x_1^*}{h^2} - \frac{2r_1^M b^M \beta^M x_2^* x_1^{*2}}{K^L h^2} + \frac{r_1^{M^2} x_1^{*2}}{K^{L^2}} - \frac{2r_1^L x_{1*}}{K^M} \Bigg] \left(p_1(n) - q_1(n)\right)^2 \\
&\quad + \Bigg[\left(\left(\alpha^{M^2} b^{M^2} x_2^{*3} + 2b^{M^2}\alpha^M\beta^M x_1^* x_2^{*3} + b^{M^2}\beta^{M^2} x_1^{*2} x_2^{*4} + c^M\gamma^M x_1^* x_2^{*2} \right) \times \left(h^4\right)^{-1} \right) - \frac{2c^L\gamma^L x_{1*} x_{2*}}{H^2} \Bigg] \\
&\quad \times \left(p_2(n) - q_2(n)\right)^2 \\
&\quad + \Bigg[\frac{c^M\alpha^M x_1^* + c^M\gamma^M x_1^* x_2^*}{h^2} + \frac{r_1^M b^M \alpha^M x_1^* x_2^{*2} + r_1^M b^M \beta^M x_1^{*2} x_2^{*2}}{K^L h^2} \Bigg] \\
&\quad \times \left[\left(p_1(n) - q_1(n)\right)^2 + \left(p_2(n) - q_2(n)\right)^2 \right] \\
&\leq -\Bigg[\frac{2r_1^L x_{1*}}{K^M} + \frac{2r_1^L b^L \beta^L x_{2*} x_{1*}^2}{K^M H^2} - \frac{r_1^M b^M \alpha^M x_1^* x_2^{*2} + r_1^M b^M \beta^M x_1^{*2} x_2^{*2}}{K^L h^2} \\
&\quad - \left(\left(\left(b^{M^2}\beta^{M^2} + c^{M^2}\gamma^{M^2} \right) x_1^{*2} x_2^{*2} + 2c^M\alpha^M\gamma^M x_1^{*2} x_2^* + c^{M^2}\alpha^M x_1^{*2} \right) \times \left(h^4\right)^{-1} \right) \\
&\quad - \frac{c^M\alpha^M x_1^* + \left(c^M\gamma^M + 2b^M\beta^M\right) x_1^* x_2^*}{h^2} \Bigg] \\
&\quad \times \left(p_1(n) - q_1(n)\right)^2
\end{aligned}$$

$$
\begin{aligned}
&- \Bigg[\frac{2c^L\gamma^L x_{1*}x_{2*}}{H^2} - \frac{r_1^M b^M \alpha^M x_1^* x_2^{*2} + r_1^M b^M \beta^M x_1^{*2} x_2^{*2}}{K^L h^2} \\
&\quad - \Big(\Big(\alpha^{M^2} b^{M^2} x_2^{*4} + 2b^{M^2} \alpha^M \beta^M x_1^* x_2^{*4} \\
&\qquad + b^{M^2} \beta^{M^2} x_1^{*2} x_2^{*4} + c^M \gamma^M x_1^{*2} x_2^{*2} \Big) \times \left(h^4\right)^{-1} \Big) \\
&\quad - \frac{c^M \alpha^M x_1^* + c^M \gamma^M x_1^* x_2^*}{h^2} \Bigg] \left(p_2(n) - q_2(n)\right)^2 \\
&\leq -\eta_1 \left(p_1(n) - q_1(n)\right)^2 - \eta_2 \left(p_2(n) - q_2(n)\right)^2 \\
&\leq -\eta \left[\left(p_1(n) - q_1(n)\right)^2 + \left(p_2(n) - q_2(n)\right)^2 \right] \\
&= -\eta V(n, X, Y),
\end{aligned} \tag{62}
$$

where $\eta = \min\{\eta_1, \eta_2\}$. That is, there exists a positive constant $0 < \eta < 1$ such that

$$\Delta V_{(49)} \leq -\eta V(n, X, Y). \tag{63}$$

From $0 < \eta < 1$, the condition $[C_3]$ of Lemma 8 is satisfied. So, from Lemma 8, there exists a uniqueness uniformly asymptotically stable almost periodic solution $X(n) = (p_1(n), p_2(n))$ of (47) which is bounded by S^* for all $n \in Z^+$. Which means that there exists a uniqueness uniformly asymptotically stable almost periodic solution $X(n) = (x_1(n), x_2(n))$ of (6) which is bounded by Ω for all $n \in Z^+$. The proof is completed. □

5. Application

Example 12. Consider the following discrete model:

$$
\begin{aligned}
x_1(n+1) &= x_1(n) \\
&\quad \times \exp \Big[(1.1 + 0.4\sin(n))(1 - x_1(n)) \\
&\qquad - (0.3 - 0.2\cos(n)) x_2(n) \\
&\qquad \times \big(0.3 - 0.1\sin(n) \\
&\qquad\quad + (0.6 + 0.1\cos(n)) x_1(n) + x_2(n)\big)^{-1} \Big], \\
x_2(n+1) &= x_2(n) \exp \Big[-(0.8 + 0.2\sin(n)) \\
&\qquad + (1.5 - 0.3\sin(n)) x_1(n) \\
&\qquad \times \big(0.3 - 0.1\sin(n) \\
&\qquad\quad + (0.6 + 0.1\cos(n)) x_1(n) + x_2(n)\big)^{-1} \Big],
\end{aligned} \tag{64}
$$

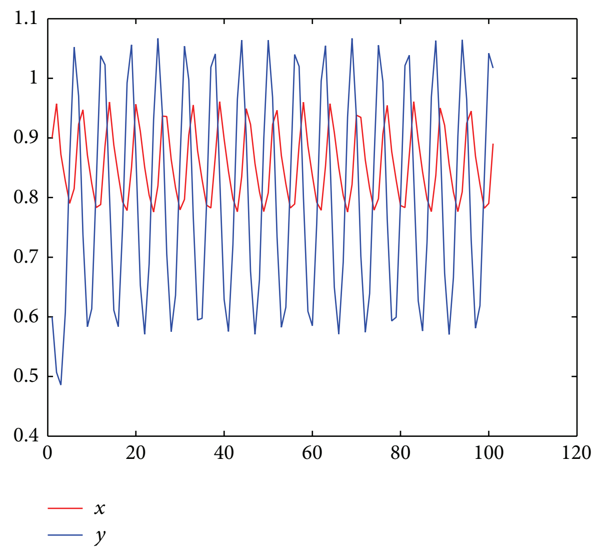

Figure 1: The orbit of predator (x_2)-prey (x_1).

with initial conditions (0.9, 0.6); then system (64) is persistent and has a globally attractive almost periodic solution.

By simple computation, we derive

$$r_1^L - \frac{b^M}{\gamma^L} = 0.2 > 0, \qquad \frac{c^L}{\beta^M}\left(1 - \alpha^M\right) - r_2^M \approx 0.2857 > 0. \tag{65}$$

Thus, the system (64) is persistent. Its integral curves and orbits are shown in Figure 1, respectively.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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