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Research Article

Resilient Robust Finite-Time L_2-L_∞ Controller Design for Uncertain Neutral System with Mixed Time-Varying Delays

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The delay-dependent resilient robust finite-time L_2-L_∞ control problem of uncertain neutral time-delayed system is studied. The disturbance input is assumed to be energy bounded and the time delays are time-varying. Based on the Lyapunov function approach and linear matrix inequalities (LMIs) techniques, a state feedback controller is designed to guarantee that the resulted closed-loop system is finite-time bounded for all uncertainties and to satisfy a given L_2-L_∞ constraint condition. Simulation results illustrate the validity of the proposed approach.

1. Introduction

Dynamical systems with time delays and uncertain parameters have been of considerable interest over the past decades. In fact, time delays are always the important source of system instability and poor performance [1–4]. As a special class of time-delay systems, the neutral type time-delayed system has also received some attention in recent years. This time-delayed system contains time delays both in its state and in the derivative of its states. Moreover, neutral time-delayed systems are frequently encountered in many dynamics, such as automatic control, distributed network system containing lossless transmission line, heat exchangers, and population ecology. Various analysis approaches have been utilized to find stability criteria and control design conditions for asymptotic stability of neutral time delays [5–10].

It is now worth pointing out that the control performances mentioned above concern the desired behavior of control dynamics over an infinite-time interval and it always deals with the asymptotic property of system trajectories. For controlling a dynamical system, it can meet the requirements of asymptotic stability, but it will not reflect the transient characteristics. Asymptotic stability is unable to satisfy the transient requirements of industrial production if there exists large amount of overshoot, oscillation change, and nonlinear disturbance within a finite-time interval. To deal with this transient performance of control dynamics, Dorato gave the concept of finite-time stability [11] (or short-time stability) in the early 1960s. Then, the relevant concepts of finite-time bounded (FTB) [12], finite-time stabilization [13], finite-time H_∞ control [14], and finite-time L_2-L_∞ [15] control have been revisited in form of linear matrix inequalities (LMIs) techniques. And this transient performance is widely applied to time-delay systems, uncertain systems, nonlinear systems, stochastic systems, and so forth. However, to the best of our knowledge, very few results in the literature consider the related control problems of neutral time-varying delays in the finite-time interval.

On the other hand, the L_2-L_∞ performance has attracted considerable attention as an important performance evaluation index when it was first proposed in 1989 [16]. In engineering practice, although the study of the impact of noise and delay on the system performance is important, the extremum problem of the controlled output cannot be ignored, because the controlled output should be controlled within a certain range. In control theory and engineering application, the L_2-L_∞ control has very important significance that lies in its performance index which can control the output value minimization. Unfortunately, up to now, the theme of L_2-L_∞ control design of uncertain neutral systems with time-varying delays has received little attention.

Motivated by the above discussion, this paper focuses on the problem of finite-time L_2-L_∞ controller design for a class of neutral systems with mixed time-varying delays and uncertainties. By constructing a suitable Lyapunov function, the sufficient conditions are derived that closed-loop controlled system is FTB and satisfies the given finite-time interval induced L_2-L_∞ norm of the operator from the unknown disturbance to the output. We also show that the L_2-L_∞ controller designing problem can be dealt with by solving a set of coupled LMIs. Finally, a numerical example illustrates the effectiveness of the developed techniques.

2. Problem Statement

Consider the following neutral time-delayed system with uncertainties:

$$\Sigma_0 : \begin{cases} \dot{\mathbf{x}}(t) - (\mathbf{C} + \Delta\mathbf{C}(t))\dot{\mathbf{x}}(t-\tau(t)) = (\mathbf{A} + \Delta\mathbf{A}(t))\mathbf{x}(t) \\ \qquad + (\mathbf{A}_d + \Delta\mathbf{A}_d(t))\mathbf{x}(t-h(t)) + \mathbf{Bu}(t) \\ \qquad + (\mathbf{D} + \Delta\mathbf{D}(t))\mathbf{w}(t) \\ \mathbf{y}(t) = (\mathbf{F} + \Delta\mathbf{F}(t))\mathbf{x}(t) + \mathbf{Gu}(t) \\ \mathbf{x}(t_0+\theta) = \boldsymbol{\phi}(\theta), \quad \theta \in [-\max\{h,\tau\}, 0], t_0 = 0, \end{cases} \tag{1}$$

where $\mathbf{x}(t) \in \mathbf{R}^n$ is the state, $\mathbf{u}(t) \in \mathbf{R}^m$ is the controlled input, $\mathbf{y}(t) \in \mathbf{R}^q$ is the controlled output, and $\mathbf{w}(t) \in \mathbf{R}^p$ is the disturbance input that belongs to $L_2[0,+\infty)$ and for a given positive number δ and constant time T, the following form is satisfied:

$$\int_0^T \mathbf{w}^T(t)\mathbf{w}(t)\,dt \le \delta, \quad \delta \ge 0. \tag{2}$$

$h(t)$ and $\tau(t)$ are time-varying delays and satisfy

$$\begin{gathered} 0 \le h(t) \le h, \qquad \dot{h}(t) \le h_d, \\ 0 \le \tau(t) \le \tau, \qquad \dot{\tau}(t) \le \tau_d < 1, \end{gathered} \tag{3}$$

where h, τ, h_d, and τ_d are constant scalars. $\boldsymbol{\phi}(\theta) \in L_2[-\max\{h,\tau\},0]$ is the continuous initial function. $\mathbf{A}$, $\mathbf{A}_d$, $\mathbf{C}$, $\mathbf{D}$, and $\mathbf{F} \in \mathbf{R}^{n\times n}$ are known constant matrices, and $\Delta\mathbf{A}(t)$, $\Delta\mathbf{A}_d(t)$, $\Delta\mathbf{C}(t)$, $\Delta\mathbf{D}(t)$, and $\Delta\mathbf{F}(t)$ are unknown time-variant matrices representing the norm-bounded parameter uncertainties and satisfy the following form:

$$\begin{gathered} [\Delta\mathbf{A}(t) \;\; \Delta\mathbf{A}_d(t) \;\; \Delta\mathbf{C}(t) \;\; \Delta\mathbf{D}(t)] \\ = \mathbf{M}_1\boldsymbol{\sigma}(t)[\mathbf{H}_1 \;\; \mathbf{H}_2 \;\; \mathbf{H}_3 \;\; \mathbf{H}_4], \end{gathered} \tag{4}$$

$$\Delta\mathbf{F}(t) = \mathbf{M}_2\boldsymbol{\sigma}(t)\mathbf{H}_1, \tag{5}$$

where $\mathbf{M}_1$, $\mathbf{M}_2$, $\mathbf{H}_1$, $\mathbf{H}_2$, $\mathbf{H}_3$, and $\mathbf{H}_4$ are known real matrices with suitable dimension and $\boldsymbol{\sigma}(t)$ is an unknown real and possibly time-varying matrix with Lebesgue measurable elements satisfying

$$\boldsymbol{\sigma}^T(t)\boldsymbol{\sigma}(t) \le \mathbf{I}. \tag{6}$$

In this paper, we consider the state feedback controller as follows:

$$\mathbf{u}(t) = (\mathbf{K} + \Delta\mathbf{K}(t))\mathbf{x}(t), \tag{7}$$

where $\mathbf{K}$ is the unknown controller gain and $\Delta\mathbf{K}(t)$ is the time-varying controller gain which satisfies

$$\Delta\mathbf{K}(t) = \mathbf{N}\boldsymbol{\eta}(t)\mathbf{S}, \qquad \boldsymbol{\eta}^T(t)\boldsymbol{\eta}(t) \le \mathbf{I}. \tag{8}$$

Then, we can get the following closed-loop control system:

$$\Sigma: \begin{cases} \dot{\mathbf{x}}(t) - \overline{\mathbf{C}}\dot{\mathbf{x}}(t-\tau(t)) = \widehat{\mathbf{A}}\mathbf{x}(t) \\ \qquad + \overline{\mathbf{A}}_d\mathbf{x}(t-h(t)) + \overline{\mathbf{D}}\mathbf{w}(t) \\ \mathbf{y}(t) = \widehat{\mathbf{F}}\mathbf{x}(t) \\ \mathbf{x}(t_0+\theta) = \boldsymbol{\phi}(\theta), \quad \theta \in [-\max\{h,\tau\},0], t_0 = 0, \end{cases} \tag{9}$$

where $\widehat{\mathbf{A}} = \overline{\mathbf{A}} + \Delta\overline{\mathbf{A}}(t)$, $\overline{\mathbf{A}} = \mathbf{A} + \mathbf{BK}$, $\Delta\overline{\mathbf{A}}(t) = \Delta\mathbf{A}(t) + \mathbf{B}\Delta\mathbf{K}(t)$, $\overline{\mathbf{A}}_d = \mathbf{A}_d + \Delta\mathbf{A}_d(t)$, $\overline{\mathbf{C}} = \mathbf{C} + \Delta\mathbf{C}(t)$, $\overline{\mathbf{D}} = \mathbf{D} + \Delta\mathbf{D}(t)$, $\widehat{\mathbf{F}} = \overline{\mathbf{F}} + \Delta\overline{\mathbf{F}}(t)$, $\overline{\mathbf{F}} = \mathbf{F} + \mathbf{GK}$, and $\Delta\overline{\mathbf{F}}(t) = \Delta\mathbf{F}(t) + \mathbf{G}\Delta\mathbf{K}(t)$.

The main purpose of this paper is to design an appropriate resilient state feedback controller (7), such that the closed-loop control system Σ is finite-time bounded and satisfies the given performance index constraints.

Before proceeding with the study, we give the relevant definitions and lemmas first.

Definition 1. For given positive scalars c_1, δ, and T and a symmetrical positive determined matrix $\mathbf{R}$, the closed-loop system Σ is robust finite-time bounded (FTB) with respect to $(c_1, c_2, \delta, \mathbf{R}, T)$, if there exists a positive constant c_2 with $c_2 > c_1$, such that, for all the external disturbances $\mathbf{w}(t)$ satisfying condition (2), the following formula is satisfied:

$$\boldsymbol{\phi}^T(\theta)\mathbf{R}\boldsymbol{\phi}(\theta) \le c_1 \Longrightarrow \mathbf{x}^T(t)\mathbf{Rx}(t) < c_2, \quad \forall t \in [0,T]. \tag{10}$$

Remark 2. If the disturbance input is not present in the closed-loop system, that is, $\mathbf{w}(t) = 0$, the concept of FTB will reduce into finite-time stability (FTS). It is worth mentioning that Lyapunov stability and finite-time stability are two different concepts. The former is largely known to the control characteristic in infinite-time interval, but the latter concerns the boundedness analysis of the controlled states within a finite-time interval. Obviously, a finite-time stable system may not be Lyapunov stochastically stable and vice versa.

Definition 3. The state feedback controller in the form of (7) is considered as a robust finite-time L_2-L_∞ controller for the closed-loop system Σ, if the system Σ is FTB with respect to $(c_1, c_2, \delta, \mathbf{R}, T)$ and under the zero initial condition, there exist two positive scalars γ and T for all disturbance which satisfy condition (2), such that

$$\|\mathbf{y}(t)\|_\infty^2 \le \gamma^2\|\mathbf{w}(t)\|_2^2, \tag{11}$$

where $\|\mathbf{y}(t)\|_\infty^2 = \sup_{t\in[0,T]}|\mathbf{y}^T(t)\mathbf{y}(t)|$, $\|\mathbf{w}(t)\|_2^2 = \int_0^T \mathbf{w}^T(t)\mathbf{w}(t)dt$.

Lemma 4 (see [17]). *For any real positive scalars α, β (where $\alpha > \beta$) and a positive definite symmetric matrix* $\mathbf{S}$*, then the following inequality holds for a vector function $\boldsymbol{\omega} : [\beta, \alpha] \rightarrow \mathbf{R}^n$ which can let the integrals converge:*

$$\left(\int_{\beta}^{\alpha} \boldsymbol{\omega}(\boldsymbol{\sigma})\, d\sigma\right)^{T} \mathbf{S} \left(\int_{\beta}^{\alpha} \boldsymbol{\omega}(\boldsymbol{\sigma})\, d\sigma\right) \le (\alpha - \beta)\left(\int_{\beta}^{\alpha} \boldsymbol{\omega}^{T}(\boldsymbol{\sigma})\, \mathbf{S}\boldsymbol{\omega}(\boldsymbol{\sigma})\, d\sigma\right). \quad (12)$$

Lemma 5 (see [17]). *For any positive scalar h and positive definite symmetric matrix* $\mathbf{S}$*, the following inequality is satisfied:*

$$\frac{2}{h^2}\left(\int_{-h}^{0}\int_{t+\theta}^{t} \boldsymbol{\omega}(\boldsymbol{\sigma})\, d\sigma\, d\theta\right)^{T} \mathbf{S} \left(\int_{-h}^{0}\int_{t+\theta}^{t} \boldsymbol{\omega}(\boldsymbol{\sigma})\, d\sigma\, d\theta\right) \le \int_{-h}^{0}\int_{t+\theta}^{t} \boldsymbol{\omega}^{T}(\boldsymbol{\sigma})\, \mathbf{S}\boldsymbol{\omega}(\sigma)\, d\sigma\, d\theta. \quad (13)$$

Lemma 6 (see [15]). *For any given appropriate dimension matrix* $\mathbf{H}$ *and* $\mathbf{E}$*, if there exists a matrix $\mathbf{W}(t)$ which satisfied $\mathbf{W}^T(t)\mathbf{W}(t) \le \mathbf{I}$ and a scalar $\varepsilon > 0$, then*

$$\mathbf{H}\mathbf{W}(t)\,\mathbf{E} + \mathbf{E}^{T}\mathbf{W}^{\mathbf{T}}(t)\,\mathbf{H}^{T} \le \varepsilon^{-1}\mathbf{H}\mathbf{H}^{T} + \varepsilon\mathbf{E}^{T}\mathbf{E}. \quad (14)$$

3. Main Results

In this section, our main purpose is to solve the design problem of a resilient robust finite-time L_2-L_∞ controller for a class of uncertain neutral systems with mixed time-varying delays.

Theorem 7. *Given positive scalars c_1, δ, T, and α, positive definite symmetric matrix* $\mathbf{R}$*, and time-delay parameters $h > 0$, $h_d > 0$, $\tau > 0$, and $\tau_d > 0$, the closed-loop system Σ is FTB with respect to $(c_1, c_2, \delta, \mathbf{R}, T)$, if there exist positive scalars $\lambda_i, i = 1, 2, \ldots, 6$, c_2, and symmetric positive definite matrices $\mathbf{P}_i, i = 1, 2, \ldots, 6$, $\mathbf{Q}_i, i = 1, 2, \ldots, 4$, and $\mathbf{W}_i, i = 1, 2, \ldots, 6$, such that*

$$\boldsymbol{\Pi} = \begin{bmatrix} \boldsymbol{\Pi}_1 & \boldsymbol{\Pi}_2 & \boldsymbol{\Pi}_3 \\ * & \boldsymbol{\Pi}_4 & \boldsymbol{\Pi}_5 \\ * & * & \boldsymbol{\Pi}_6 \end{bmatrix} < 0, \quad (15)$$

$$c_1\left[\lambda_2 + h\lambda_3 + h\lambda_4 + \tau\lambda_5 + \tau\lambda_6\right] + \delta\left(1 - e^{-\alpha T}\right) < \lambda_1 c_2 e^{-\alpha T}, \quad (16)$$

where

$$\boldsymbol{\Pi}_1 = \left[\boldsymbol{\Pi}_{ij}\right]_{7\times 7},$$

$$\begin{aligned}\boldsymbol{\Pi}_{11} = {} & \widehat{\mathbf{A}}^{T}\mathbf{P}_1 + \mathbf{P}_1\widehat{\mathbf{A}} + \mathbf{P}_2 + \mathbf{P}_3 + \mathbf{P}_4 + \mathbf{P}_5 \\ & + \mathbf{W}_1 + \mathbf{W}_3 + \mathbf{W}_4 + \mathbf{W}_6 + \mathbf{W}_1^T + \mathbf{W}_3^T + \mathbf{W}_4^T + \mathbf{W}_6^T \\ & - \alpha\mathbf{P}_1 - \frac{\alpha}{\tau}\mathbf{P}_6 - 2\alpha\mathbf{Q}_1 - 2\alpha\mathbf{Q}_2 - 2\alpha\mathbf{Q}_3 - 2\alpha\mathbf{Q}_4,\end{aligned}$$

$$\boldsymbol{\Pi}_{12} = \mathbf{P}_1\overline{\mathbf{A}}_d - \mathbf{W}_1 + \mathbf{W}_2 + \mathbf{W}_1^T + \mathbf{W}_3^T + \mathbf{W}_4^T + \mathbf{W}_6^T,$$

$$\boldsymbol{\Pi}_{13} = \mathbf{P}_1\overline{\mathbf{C}} + \mathbf{W}_1^T + \mathbf{W}_3^T + \mathbf{W}_4^T + \mathbf{W}_6^T,$$

$$\boldsymbol{\Pi}_{14} = -\mathbf{W}_4 + \mathbf{W}_5 + \mathbf{W}_1^T + \mathbf{W}_3^T + \mathbf{W}_4^T + \mathbf{W}_6^T + \frac{\alpha}{\tau}\mathbf{P}_6,$$

$$\boldsymbol{\Pi}_{15} = -\mathbf{W}_2 - \mathbf{W}_3 + \mathbf{W}_1^T + \mathbf{W}_3^T + \mathbf{W}_4^T + \mathbf{W}_6^T,$$

$$\boldsymbol{\Pi}_{16} = -\mathbf{W}_5 - \mathbf{W}_6 + \mathbf{W}_1^T + \mathbf{W}_3^T + \mathbf{W}_4^T + \mathbf{W}_6^T,$$

$$\boldsymbol{\Pi}_{17} = \mathbf{P}_1\overline{\mathbf{D}},$$

$$\boldsymbol{\Pi}_{22} = -\left(1 - h_d\right)\mathbf{P}_2 - \mathbf{W}_1 + \mathbf{W}_2 - \mathbf{W}_1^T + \mathbf{W}_2^T,$$

$$\boldsymbol{\Pi}_{23} = -\mathbf{W}_1^T + \mathbf{W}_2^T,$$

$$\boldsymbol{\Pi}_{24} = -\mathbf{W}_4 + \mathbf{W}_5 - \mathbf{W}_1^T + \mathbf{W}_2^T,$$

$$\boldsymbol{\Pi}_{25} = -\mathbf{W}_2 - \mathbf{W}_3 - \mathbf{W}_1^T + \mathbf{W}_2^T,$$

$$\boldsymbol{\Pi}_{26} = -\mathbf{W}_5 - \mathbf{W}_6 - \mathbf{W}_1^T + \mathbf{W}_2^T,$$

$$\boldsymbol{\Pi}_{27} = 0,$$

$$\boldsymbol{\Pi}_{33} = -\left(1 - \tau_d\right)\mathbf{P}_6,$$

$$\boldsymbol{\Pi}_{34} = -\mathbf{W}_4 + \mathbf{W}_5,$$

$$\boldsymbol{\Pi}_{35} = -\mathbf{W}_2 - \mathbf{W}_3,$$

$$\boldsymbol{\Pi}_{36} = -\mathbf{W}_5 - \mathbf{W}_6,$$

$$\boldsymbol{\Pi}_{37} = 0,$$

$$\boldsymbol{\Pi}_{44} = -\left(1 - \tau_d\right)\mathbf{P}_4 - \mathbf{W}_4 + \mathbf{W}_5 - \mathbf{W}_4^T + \mathbf{W}_5^T - \frac{\alpha}{\tau}\mathbf{P}_6,$$

$$\boldsymbol{\Pi}_{45} = -\mathbf{W}_2 - \mathbf{W}_3 - \mathbf{W}_4^T + \mathbf{W}_5^T,$$

$$\boldsymbol{\Pi}_{46} = -\mathbf{W}_5 - \mathbf{W}_6 - \mathbf{W}_4^T + \mathbf{W}_5^T,$$

$$\boldsymbol{\Pi}_{47} = 0,$$

$$\boldsymbol{\Pi}_{55} = -\mathbf{P}_3 - \mathbf{W}_2 - \mathbf{W}_3 - \mathbf{W}_2^T - \mathbf{W}_3^T,$$

$$\boldsymbol{\Pi}_{56} = -\mathbf{W}_5 - \mathbf{W}_6 - \mathbf{W}_2^T - \mathbf{W}_3^T,$$

$$\boldsymbol{\Pi}_{57} = 0,$$

$$\boldsymbol{\Pi}_{66} = -\mathbf{P}_5 - \mathbf{W}_5 - \mathbf{W}_6 - \mathbf{W}_5^T - \mathbf{W}_6^T,$$

$$\boldsymbol{\Pi}_{67} = 0,$$

$$\boldsymbol{\Pi}_{77} = -\alpha\mathbf{I},$$

$$\boldsymbol{\Pi}_2 = \begin{bmatrix} h\mathbf{W}_1 & h\mathbf{W}_2 & h\mathbf{W}_3 & \tau\mathbf{W}_4 & \tau\mathbf{W}_5 & \tau\mathbf{W}_6 \\ h\mathbf{W}_1 & h\mathbf{W}_2 & h\mathbf{W}_3 & \tau\mathbf{W}_4 & \tau\mathbf{W}_5 & \tau\mathbf{W}_6 \\ h\mathbf{W}_1 & h\mathbf{W}_2 & h\mathbf{W}_3 & \tau\mathbf{W}_4 & \tau\mathbf{W}_5 & \tau\mathbf{W}_6 \\ h\mathbf{W}_1 & h\mathbf{W}_2 & h\mathbf{W}_3 & \tau\mathbf{W}_4 & \tau\mathbf{W}_5 & \tau\mathbf{W}_6 \\ h\mathbf{W}_1 & h\mathbf{W}_2 & h\mathbf{W}_3 & \tau\mathbf{W}_4 & \tau\mathbf{W}_5 & \tau\mathbf{W}_6 \\ h\mathbf{W}_1 & h\mathbf{W}_2 & h\mathbf{W}_3 & \tau\mathbf{W}_4 & \tau\mathbf{W}_5 & \tau\mathbf{W}_6 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{\Pi}_3 = \begin{bmatrix} \widehat{\mathbf{A}}^T\mathbf{P}_6 & h\widehat{\mathbf{A}}^T\mathbf{Q}_1 & h\widehat{\mathbf{A}}^T\mathbf{Q}_2 & \tau\widehat{\mathbf{A}}^T\mathbf{Q}_3 & \tau\widehat{\mathbf{A}}^T\mathbf{Q}_4 \\ \overline{\mathbf{A}}_d^T\mathbf{P}_6 & h\overline{\mathbf{A}}_d^T\mathbf{Q}_1 & h\overline{\mathbf{A}}_d^T\mathbf{Q}_2 & \tau\overline{\mathbf{A}}_d^T\mathbf{Q}_3 & \tau\overline{\mathbf{A}}_d^T\mathbf{Q}_4 \\ \overline{\mathbf{C}}^T\mathbf{P}_6 & h\overline{\mathbf{C}}^T\mathbf{Q}_1 & h\overline{\mathbf{C}}^T\mathbf{Q}_2 & \tau\overline{\mathbf{C}}^T\mathbf{Q}_3 & \tau\overline{\mathbf{C}}^T\mathbf{Q}_4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \overline{\mathbf{D}}^T\mathbf{P}_6 & h\overline{\mathbf{D}}^T\mathbf{Q}_1 & h\overline{\mathbf{D}}^T\mathbf{Q}_2 & \tau\overline{\mathbf{D}}^T\mathbf{Q}_3 & \tau\overline{\mathbf{D}}^T\mathbf{Q}_4 \end{bmatrix},$$

$$\mathbf{\Pi}_4 = \operatorname{diag}\{-h\mathbf{Q}_1 \;\; -h\mathbf{Q}_1 \;\; -h\mathbf{Q}_2 \;\; -\tau\mathbf{Q}_3 \;\; -\tau\mathbf{Q}_3 \;\; -\tau\mathbf{Q}_4\},$$

$$\mathbf{\Pi}_5 = [0]_{6\times 5},$$

$$\mathbf{\Pi}_6 = \operatorname{diag}\{-\mathbf{P}_6 \;\; -h\mathbf{Q}_1 \;\; -h\mathbf{Q}_2 \;\; -\tau\mathbf{Q}_3 \;\; -\tau\mathbf{Q}_3\}. \tag{17}$$

Proof. Construct a positive definite Lyapunov function as follows:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t), \tag{18}$$

where

$$\begin{aligned} V_1(t) &= \mathbf{x}^T(t)\,\mathbf{P}_1\mathbf{x}(t), \\ V_2(t) &= \int_{t-h(t)}^{t} \mathbf{x}^T(s)\,\mathbf{P}_2\mathbf{x}(s)\,ds + \int_{t-h}^{t} \mathbf{x}^T(s)\,\mathbf{P}_3\mathbf{x}(s)\,ds, \\ V_3(t) &= \int_{t-\tau(t)}^{t} \mathbf{x}^T(s)\,\mathbf{P}_4\mathbf{x}(s)\,ds + \int_{t-\tau}^{t} \mathbf{x}^T(s)\,\mathbf{P}_5\mathbf{x}(s)\,ds, \\ V_4(t) &= \int_{t-\tau(t)}^{t} \dot{\mathbf{x}}^T(s)\,\mathbf{P}_6\dot{\mathbf{x}}(s)\,ds, \\ V_5(t) &= \int_{-h}^{0}\int_{t+\theta}^{t} \dot{\mathbf{x}}^T(s)\left(\mathbf{Q}_1 + \mathbf{Q}_2\right)\dot{\mathbf{x}}(s)\,ds\,d\theta \\ &\quad + \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{\mathbf{x}}^T(s)\left(\mathbf{Q}_3 + \mathbf{Q}_4\right)\dot{\mathbf{x}}(s)\,ds\,d\theta. \end{aligned} \tag{19}$$

We take the time derivative of $V(t)$ along the trajectory of system Σ and it yields the following:

$$\begin{aligned} \dot{V}_1(t) &= \mathbf{x}^T(t)\left(\mathbf{P}_1\widehat{\mathbf{A}} + \widehat{\mathbf{A}}^T\mathbf{P}_1\right)\mathbf{x}(t) \\ &\quad + \mathbf{x}^T(t)\,\mathbf{P}_1\overline{\mathbf{A}}_d\mathbf{x}(t-h(t)) \\ &\quad + \mathbf{x}^T(t)\,\mathbf{P}_1\overline{\mathbf{C}}\dot{\mathbf{x}}(t-\tau(t)) \\ &\quad + \mathbf{x}^T(t)\,\mathbf{P}_1\overline{\mathbf{D}}\mathbf{w}(t) + \mathbf{x}^T(t-h(t))\,\overline{\mathbf{A}}_d^T\mathbf{P}_1\mathbf{x}(t) \\ &\quad + \dot{\mathbf{x}}^T(t-\tau(t))\,\overline{\mathbf{C}}^T\mathbf{P}_1\mathbf{x}(t) + \boldsymbol{\omega}^T(t)\,\overline{\mathbf{D}}^T\mathbf{P}_1\mathbf{x}(t), \\ \dot{V}_2(t) &\le \mathbf{x}^T(t)\left(\mathbf{P}_2 + \mathbf{P}_3\right)\mathbf{x}(t) \\ &\quad - \left(1 - h_d\right)\mathbf{x}^T(t-h(t))\,\mathbf{P}_2\mathbf{x}(t-h(t)) \\ &\quad - \mathbf{x}^T(t-h)\,\mathbf{P}_3\mathbf{x}(t-h), \\ \dot{V}_3(t) &\le \mathbf{x}^T(t)\left(\mathbf{P}_4 + \mathbf{P}_5\right)\mathbf{x}(t) \\ &\quad - \left(1 - \tau_d\right)\mathbf{x}^T(t-\tau(t))\,\mathbf{P}_4\mathbf{x}(t-\tau(t)) \\ &\quad - \mathbf{x}^T(t-\tau)\,\mathbf{P}_5\mathbf{x}(t-\tau), \\ \dot{V}_4(t) &\le \dot{\mathbf{x}}^T(t)\,\mathbf{P}_6\dot{\mathbf{x}}(t) \\ &\quad - \left(1 - \tau_d\right)\dot{\mathbf{x}}^T(t-\tau(t))\,\mathbf{P}_6\dot{\mathbf{x}}(t-\tau(t)), \\ \dot{V}_5&(t) \\ &= \dot{\mathbf{x}}^T(t)\left(h\left(\mathbf{Q}_1 + \mathbf{Q}_2\right) + \tau\left(\mathbf{Q}_3 + \mathbf{Q}_4\right)\right)\dot{\mathbf{x}}(t) \\ &\quad - \int_{t-h(t)}^{t} \dot{\mathbf{x}}^T(s)\,\mathbf{Q}_1\dot{\mathbf{x}}(s)\,ds - \int_{t-h}^{t-h(t)} \dot{\mathbf{x}}^T(s)\,\mathbf{Q}_1\dot{\mathbf{x}}(s)\,ds \\ &\quad - \int_{t-h}^{t} \dot{\mathbf{x}}^T(s)\,\mathbf{Q}_2\dot{\mathbf{x}}(s)\,ds - \int_{t-\tau(t)}^{t} \dot{\mathbf{x}}^T(s)\,\mathbf{Q}_3\dot{\mathbf{x}}(s)\,ds \\ &\quad - \int_{t-\tau}^{t-\tau(t)} \dot{\mathbf{x}}^T(s)\,\mathbf{Q}_3\dot{\mathbf{x}}(s)\,ds - \int_{t-\tau}^{t} \dot{\mathbf{x}}^T(s)\,\mathbf{Q}_4\dot{\mathbf{x}}(s)\,ds. \end{aligned} \tag{20}$$

For any symmetric positive definite matrices $\mathbf{W}_i, i = 1, 2, \ldots, 6$, the following equations are satisfied according to Leibniz-Newton lemma:

$$\begin{aligned} 2\boldsymbol{\zeta}^T(t)\,\mathbf{W}_1\left[\mathbf{x}(t) - \mathbf{x}(t-h(t)) - \int_{t-h(t)}^{t}\dot{\mathbf{x}}(s)\,ds\right] &= 0, \\ 2\boldsymbol{\zeta}^T(t)\,\mathbf{W}_2\left[\mathbf{x}(t-h(t)) - \mathbf{x}(t-h) - \int_{t-h}^{t-h(t)}\dot{\mathbf{x}}(s)\,ds\right] &= 0, \\ 2\boldsymbol{\zeta}^T(t)\,\mathbf{W}_3\left[\mathbf{x}(t) - \mathbf{x}(t-h) - \int_{t-h}^{t}\dot{\mathbf{x}}(s)\,ds\right] &= 0, \\ 2\boldsymbol{\zeta}^T(t)\,\mathbf{W}_4\left[\mathbf{x}(t) - \mathbf{x}(t-\tau(t)) - \int_{t-\tau(t)}^{t}\dot{\mathbf{x}}(s)\,ds\right] &= 0, \\ 2\boldsymbol{\zeta}^T(t)\,\mathbf{W}_5\left[\mathbf{x}(t-\tau(t)) - \mathbf{x}(t-\tau) - \int_{t-\tau}^{t-\tau(t)}\dot{\mathbf{x}}(s)\,ds\right] &= 0, \\ 2\boldsymbol{\zeta}^T(t)\,\mathbf{W}_6\left[\mathbf{x}(t) - \mathbf{x}(t-\tau) - \int_{t-\tau}^{t}\dot{\mathbf{x}}(s)\,ds\right] &= 0, \end{aligned} \tag{21}$$

where

$$\begin{aligned} \boldsymbol{\zeta}(t) &= \left[\mathbf{x}^T(t) \;\; \mathbf{x}^T(t-h(t)) \;\; \dot{\mathbf{x}}^T(t-\tau(t)) \;\; \mathbf{x}^T(t-\tau(t)) \;\; \mathbf{x}^T(t-h) \;\; \mathbf{x}^T(t-\tau)\right]^T, \\ \boldsymbol{\xi}(t) &= \left[\boldsymbol{\zeta}^T(t) \;\; \mathbf{w}^T(t)\right]^T. \end{aligned} \tag{22}$$

According to (20)-(21), we can obtain

$$
\begin{aligned}
\dot{V}(t) &= \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t) + \dot{V}_5(t) \\
&\le \boldsymbol{\xi}^T(t)\,\mathbf{\Omega}_1\boldsymbol{\xi}(t) - \int_{t-h(t)}^{t} \dot{\mathbf{x}}^T(s)\,\mathbf{Q}_1\dot{\mathbf{x}}(s)\,ds \\
&\quad - \int_{t-h}^{t-h(t)} \dot{\mathbf{x}}^T(s)\,\mathbf{Q}_1\dot{\mathbf{x}}(s)\,ds - \int_{t-h}^{t} \dot{\mathbf{x}}^T(s)\,\mathbf{Q}_2\dot{\mathbf{x}}(s)\,ds \\
&\quad - \int_{t-\tau(t)}^{t} \dot{\mathbf{x}}^T(s)\,\mathbf{Q}_3\dot{\mathbf{x}}(s)\,ds - \int_{t-\tau}^{t-\tau(t)} \dot{\mathbf{x}}^T(s)\,\mathbf{Q}_3\dot{\mathbf{x}}(s)\,ds \\
&\quad - \int_{t-\tau}^{t} \dot{\mathbf{x}}^T(s)\,\mathbf{Q}_4\dot{\mathbf{x}}(s)\,ds \\
&\quad + 2\boldsymbol{\zeta}^T(t)\,\mathbf{W}_1\left[\mathbf{x}(t) - \mathbf{x}(t-h(t)) - \int_{t-h(t)}^{t} \dot{\mathbf{x}}(s)\,ds\right] \\
&\quad + 2\boldsymbol{\zeta}^T(t)\,\mathbf{W}_2\Bigg[\mathbf{x}(t-h(t)) - \mathbf{x}(t-h) \\
&\qquad\qquad - \int_{t-h}^{t-h(t)} \dot{\mathbf{x}}(s)\,ds\Bigg] \\
&\quad + 2\boldsymbol{\zeta}^T(t)\,\mathbf{W}_3\left[\mathbf{x}(t) - \mathbf{x}(t-h) - \int_{t-h}^{t} \dot{\mathbf{x}}(s)\,ds\right] \\
&\quad + 2\boldsymbol{\zeta}^T(t)\,\mathbf{W}_4\left[\mathbf{x}(t) - \mathbf{x}(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{\mathbf{x}}(s)\,ds\right] \\
&\quad + 2\boldsymbol{\zeta}^T(t)\,\mathbf{W}_5\Bigg[\mathbf{x}(t-\tau(t)) - \mathbf{x}(t-\tau) \\
&\qquad\qquad - \int_{t-\tau}^{t-\tau(t)} \dot{\mathbf{x}}(s)\,ds\Bigg] \\
&\quad + 2\boldsymbol{\zeta}^T(t)\,\mathbf{W}_6\left[\mathbf{x}(t) - \mathbf{x}(t-\tau) - \int_{t-\tau}^{t} \dot{\mathbf{x}}(s)\,ds\right] \\
&= \boldsymbol{\xi}^T(t)\,\mathbf{\Omega}_1\boldsymbol{\xi}(t) + \boldsymbol{\zeta}^T(t)\,\mathbf{\Omega}_2\boldsymbol{\zeta}(t) \\
&\quad - \int_{t-h(t)}^{t} \left(\boldsymbol{\zeta}^T(t)\,\mathbf{W}_1 + \dot{\mathbf{x}}^T(s)\,\mathbf{Q}_1\right) \\
&\qquad\qquad \times \mathbf{Q}_1^{-1}\left(\mathbf{W}_1^T\boldsymbol{\zeta}(t) + \mathbf{Q}_1\dot{\mathbf{x}}(s)\right) ds \\
&\quad - \int_{t-h}^{t-h(t)} \left(\boldsymbol{\zeta}^T(t)\,\mathbf{W}_2 + \dot{\mathbf{x}}^T(s)\,\mathbf{Q}_1\right) \\
&\qquad\qquad \times \mathbf{Q}_1^{-1}\left(\mathbf{W}_2^T\boldsymbol{\zeta}(t) + \mathbf{Q}_1\dot{\mathbf{x}}(s)\right) ds \\
&\quad - \int_{t-h}^{t} \left(\boldsymbol{\zeta}^T(t)\,\mathbf{W}_3 + \dot{\mathbf{x}}^T(s)\,\mathbf{Q}_2\right) \\
&\qquad\qquad \times \mathbf{Q}_2^{-1}\left(\mathbf{W}_3^T\boldsymbol{\zeta}(t) + \mathbf{Q}_2\dot{\mathbf{x}}(s)\right) ds \\
&\quad - \int_{t-\tau(t)}^{t} \left(\boldsymbol{\zeta}^T(t)\,\mathbf{W}_4 + \dot{\mathbf{x}}^T(s)\,\mathbf{Q}_3\right) \\
&\qquad\qquad \times \mathbf{Q}_3^{-1}\left(\mathbf{W}_4^T\boldsymbol{\zeta}(t) + \mathbf{Q}_3\dot{\mathbf{x}}(s)\right) ds \\
&\quad - \int_{t-\tau}^{t-\tau(t)} \left(\boldsymbol{\zeta}^T(t)\,\mathbf{W}_5 + \dot{\mathbf{x}}^T(s)\,\mathbf{Q}_3\right) \\
&\qquad\qquad \times \mathbf{Q}_3^{-1}\left(\mathbf{W}_5^T\boldsymbol{\zeta}(t) + \mathbf{Q}_3\dot{\mathbf{x}}(s)\right) ds \\
&\quad - \int_{t-\tau}^{t} \left(\boldsymbol{\zeta}^T(t)\,\mathbf{W}_6 + \dot{\mathbf{x}}^T(s)\,\mathbf{Q}_4\right) \\
&\qquad\qquad \times \mathbf{Q}_4^{-1}\left(\mathbf{W}_6^T\boldsymbol{\zeta}(t) + \mathbf{Q}_4\dot{\mathbf{x}}(s)\right) ds.
\end{aligned}
\tag{23}
$$

Since $\mathbf{Q}_1$, $\mathbf{Q}_2$, $\mathbf{Q}_3$, and $\mathbf{Q}_4$ are positive definite symmetric matrices, we have

$$\dot{V}(t) \le \boldsymbol{\xi}^T(t)\,\mathbf{\Omega}_1\boldsymbol{\xi}(t) + \boldsymbol{\zeta}^T(t)\,\mathbf{\Omega}_2\boldsymbol{\zeta}(t), \tag{24}$$

where

$$
\begin{aligned}
\mathbf{\Omega}_1 &= \left[\mathbf{\Omega}_{ij}\right]_{7\times 7}, \\
\mathbf{\Omega}_{11} &= \widehat{\mathbf{A}}^T\mathbf{P}_1 + \mathbf{P}_1\widehat{\mathbf{A}} + \mathbf{P}_2 + \mathbf{P}_3 + \mathbf{P}_4 + \mathbf{P}_5 \\
&\quad + \widehat{\mathbf{A}}^T\left(\mathbf{P}_6 + h\left(\mathbf{Q}_1 + \mathbf{Q}_2\right) + \tau\left(\mathbf{Q}_3 + \mathbf{Q}_4\right)\right)\widehat{\mathbf{A}} \\
&\quad + \mathbf{W}_1 + \mathbf{W}_3 + \mathbf{W}_4 + \mathbf{W}_6 + \mathbf{W}_1^T + \mathbf{W}_3^T + \mathbf{W}_4^T + \mathbf{W}_6^T, \\
\mathbf{\Omega}_{12} &= \mathbf{P}_1\overline{\mathbf{A}}_d + \widehat{\mathbf{A}}^T\left(\mathbf{P}_6 + h\left(\mathbf{Q}_1 + \mathbf{Q}_2\right) + \tau\left(\mathbf{Q}_3 + \mathbf{Q}_4\right)\right)\overline{\mathbf{A}}_d \\
&\quad - \mathbf{W}_1 + \mathbf{W}_2 + \mathbf{W}_1^T + \mathbf{W}_3^T + \mathbf{W}_4^T + \mathbf{W}_6^T, \\
\mathbf{\Omega}_{13} &= \mathbf{P}_1\overline{\mathbf{C}} + \widehat{\mathbf{A}}^T\left(\mathbf{P}_6 + h\left(\mathbf{Q}_1 + \mathbf{Q}_2\right) + \tau\left(\mathbf{Q}_3 + \mathbf{Q}_4\right)\right)\overline{\mathbf{C}} \\
&\quad + \mathbf{W}_1^T + \mathbf{W}_3^T + \mathbf{W}_4^T + \mathbf{W}_6^T, \\
\mathbf{\Omega}_{14} &= -\mathbf{W}_4 + \mathbf{W}_5 + \mathbf{W}_1^T + \mathbf{W}_3^T + \mathbf{W}_4^T + \mathbf{W}_6^T, \\
\mathbf{\Omega}_{15} &= -\mathbf{W}_2 - \mathbf{W}_3 + \mathbf{W}_1^T + \mathbf{W}_3^T + \mathbf{W}_4^T + \mathbf{W}_6^T, \\
\mathbf{\Omega}_{16} &= -\mathbf{W}_5 - \mathbf{W}_6 + \mathbf{W}_1^T + \mathbf{W}_3^T + \mathbf{W}_4^T + \mathbf{W}_6^T, \\
\mathbf{\Omega}_{17} &= \mathbf{P}_1\overline{\mathbf{D}} + \widehat{\mathbf{A}}^T\left(\mathbf{P}_6 + h\left(\mathbf{Q}_1 + \mathbf{Q}_2\right) + \tau\left(\mathbf{Q}_3 + \mathbf{Q}_4\right)\right)\overline{\mathbf{D}}, \\
\mathbf{\Omega}_{22} &= -\left(1 - h_d\right)\mathbf{P}_2 \\
&\quad + \overline{\mathbf{A}}_d^T\left(\mathbf{P}_6 + h\left(\mathbf{Q}_1 + \mathbf{Q}_2\right) + \tau\left(\mathbf{Q}_3 + \mathbf{Q}_4\right)\right)\overline{\mathbf{A}}_d \\
&\quad - \mathbf{W}_1 + \mathbf{W}_2 - \mathbf{W}_1^T + \mathbf{W}_2^T, \\
\mathbf{\Omega}_{23} &= \overline{\mathbf{A}}_d^T\left(\mathbf{P}_6 + h\left(\mathbf{Q}_1 + \mathbf{Q}_2\right) + \tau\left(\mathbf{Q}_3 + \mathbf{Q}_4\right)\right)\overline{\mathbf{C}} \\
&\quad - \mathbf{W}_1^T + \mathbf{W}_2^T, \\
\mathbf{\Omega}_{24} &= -\mathbf{W}_4 + \mathbf{W}_5 - \mathbf{W}_1^T + \mathbf{W}_2^T, \\
\mathbf{\Omega}_{25} &= -\mathbf{W}_2 - \mathbf{W}_3 - \mathbf{W}_1^T + \mathbf{W}_2^T, \\
\mathbf{\Omega}_{26} &= -\mathbf{W}_5 - \mathbf{W}_6 - \mathbf{W}_1^T + \mathbf{W}_2^T,
\end{aligned}
$$

$$\begin{aligned}
\mathbf{\Omega}_{27} &= \overline{\mathbf{A}}_d^T\left(\mathbf{P}_6 + h\left(\mathbf{Q}_1 + \mathbf{Q}_2\right) + \tau\left(\mathbf{Q}_3 + \mathbf{Q}_4\right)\right)\overline{\mathbf{D}},\\
\mathbf{\Omega}_{33} &= -\left(1 - \tau_d\right)\mathbf{P}_6\\
&\quad + \overline{\mathbf{C}}^T\left(\mathbf{P}_6 + h\left(\mathbf{Q}_1 + \mathbf{Q}_2\right) + \tau\left(\mathbf{Q}_3 + \mathbf{Q}_4\right)\right)\overline{\mathbf{C}},\\
\mathbf{\Omega}_{34} &= -\mathbf{W}_4 + \mathbf{W}_5,\\
\mathbf{\Omega}_{35} &= -\mathbf{W}_2 - \mathbf{W}_3,\\
\mathbf{\Omega}_{36} &= -\mathbf{W}_5 - \mathbf{W}_6,\\
\mathbf{\Omega}_{37} &= \overline{\mathbf{C}}^T\left(\mathbf{P}_6 + h\left(\mathbf{Q}_1 + \mathbf{Q}_2\right) + \tau\left(\mathbf{Q}_3 + \mathbf{Q}_4\right)\right)\overline{\mathbf{D}},\\
\mathbf{\Omega}_{44} &= -\left(1 - \tau_d\right)\mathbf{P}_4 - \mathbf{W}_4 + \mathbf{W}_5 - \mathbf{W}_4^T + \mathbf{W}_5^T,\\
\mathbf{\Omega}_{45} &= -\mathbf{W}_2 - \mathbf{W}_3 - \mathbf{W}_4^T + \mathbf{W}_5^T,\\
\mathbf{\Omega}_{46} &= -\mathbf{W}_5 - \mathbf{W}_6 - \mathbf{W}_4^T + \mathbf{W}_5^T,\\
\mathbf{\Omega}_{47} &= 0,\\
\mathbf{\Omega}_{55} &= -\mathbf{P}_3 - \mathbf{W}_2 - \mathbf{W}_3 - \mathbf{W}_2^T - \mathbf{W}_3^T,\\
\mathbf{\Omega}_{56} &= -\mathbf{W}_5 - \mathbf{W}_6 - \mathbf{W}_2^T - \mathbf{W}_3^T,\\
\mathbf{\Omega}_{57} &= 0,\\
\mathbf{\Omega}_{66} &= -\mathbf{P}_5 - \mathbf{W}_5 - \mathbf{W}_6 - \mathbf{W}_5^T - \mathbf{W}_6^T,\\
\mathbf{\Omega}_{67} &= 0,\\
\mathbf{\Omega}_{77} &= \overline{\mathbf{D}}^T\left(\mathbf{P}_6 + h\left(\mathbf{Q}_1 + \mathbf{Q}_2\right) + \tau\left(\mathbf{Q}_3 + \mathbf{Q}_4\right)\right)\overline{\mathbf{D}},\\
\mathbf{\Omega}_2 &= h\mathbf{W}_1\mathbf{Q}_1^{-1}\mathbf{W}_1^T + h\mathbf{W}_2\mathbf{Q}_1^{-1}\mathbf{W}_3^T + h\mathbf{W}_3\mathbf{Q}_2^{-1}\mathbf{W}_3^T\\
&\quad + \tau\mathbf{W}_4\mathbf{Q}_3^{-1}\mathbf{W}_4^T + \tau\mathbf{W}_5\mathbf{Q}_3^{-1}\mathbf{W}_5^T + \tau\mathbf{W}_6\mathbf{Q}_4^{-1}\mathbf{W}_6^T.
\end{aligned} \tag{25}$$

Recalling formula (24) and Lemmas 4 and 5 and using Schur complement, we can get

$$\dot{V}(t) - \alpha V(t) - \alpha\mathbf{w}^T(t)\mathbf{w}(t) \le \boldsymbol{\xi}^T(t)\mathbf{\Pi}\boldsymbol{\xi}(t) < 0; \tag{26}$$

that is,

$$\dot{V}(t) < \alpha V(t) + \alpha\mathbf{w}^T(t)\mathbf{w}(t). \tag{27}$$

Pre- and postmultiplying (27) by $e^{-\alpha t}$, we have

$$\frac{d}{dt}\left(e^{-\alpha t}V(t)\right) < \alpha e^{-\alpha t}\mathbf{w}^T(t)\mathbf{w}(t). \tag{28}$$

Then integrating the aforementioned inequality from 0 to t, where $t \in [0, T]$, it yields

$$e^{-\alpha t}V(t) - V(0) < \alpha\int_0^t e^{-\alpha\tau}\mathbf{w}^T(\tau)\mathbf{w}(\tau)\,d\tau. \tag{29}$$

Considering condition (2), (29) can be simplified as

$$\begin{aligned}
V(t) &< e^{\alpha t}\left[V(0) + \alpha\int_0^t e^{-\alpha\tau}\mathbf{w}^T(\tau)\mathbf{w}(\tau)\,d\tau\right]\\
&< e^{\alpha T}\left[V(0) + \delta\left(1 - e^{-\alpha T}\right)\right].
\end{aligned} \tag{30}$$

On the other hand,

$$\begin{aligned}
V(t) &\ge V_1(t) = \mathbf{x}^T(t)\mathbf{P}_1\mathbf{x}(t) \ge \lambda_{\min}\left(\widetilde{\mathbf{P}}_1\right)\mathbf{x}^T(t)\mathbf{R}\mathbf{x}(t),\\
V(0) &\le \boldsymbol{\phi}^T(\theta)\mathbf{P}_1\boldsymbol{\phi}(\theta) + h\boldsymbol{\phi}^T(\theta)\mathbf{P}_2\boldsymbol{\phi}(\theta)\\
&\quad + h\boldsymbol{\phi}^T(\theta)\mathbf{P}_3\boldsymbol{\phi}(\theta) + \tau\boldsymbol{\phi}^T(\theta)\mathbf{P}_4\boldsymbol{\phi}(\theta)\\
&\quad + \tau\boldsymbol{\phi}^T(\theta)\mathbf{P}_5\boldsymbol{\phi}(\theta)\\
&\le \lambda_{\max}\left(\widetilde{\mathbf{P}}_1\right)\boldsymbol{\phi}^T(\theta)\mathbf{R}\boldsymbol{\phi}(\theta) + h\lambda_{\max}\left(\widetilde{\mathbf{P}}_2\right)\boldsymbol{\phi}^T(\theta)\mathbf{R}\boldsymbol{\phi}(\theta)\\
&\quad + h\lambda_{\max}\left(\widetilde{\mathbf{P}}_3\right)\boldsymbol{\phi}^T(\theta)\mathbf{R}\boldsymbol{\phi}(\theta)\\
&\quad + \tau\lambda_{\max}\left(\widetilde{\mathbf{P}}_4\right)\boldsymbol{\phi}^T(\theta)\mathbf{R}\boldsymbol{\phi}(\theta)\\
&\quad + \tau\lambda_{\max}\left(\widetilde{\mathbf{P}}_5\right)\boldsymbol{\phi}^T(\theta)\mathbf{R}\boldsymbol{\phi}(\theta)\\
&\le \lambda_{\max}\left(\widetilde{\mathbf{P}}_1\right)c_1 + h\lambda_{\max}\left(\widetilde{\mathbf{P}}_2\right)c_1 + h\lambda_{\max}\left(\widetilde{\mathbf{P}}_3\right)c_1\\
&\quad + \tau\lambda_{\max}\left(\widetilde{\mathbf{P}}_4\right)c_1 + \tau\lambda_{\max}\left(\widetilde{\mathbf{P}}_5\right)c_1.
\end{aligned} \tag{31}$$

Then, formula (27) can be written as

$$\begin{aligned}
&\mathbf{x}^T(t)\mathbf{R}\mathbf{x}(t)\\
&\quad \le \frac{c_1\left[\lambda_2 + h\lambda_3 + h\lambda_4 + \tau\lambda_5 + \tau\lambda_6\right] + \delta\left(1 - e^{-\alpha T}\right)}{\lambda_1 e^{-\alpha T}},
\end{aligned} \tag{32}$$

which can be guaranteed by condition (16). This completes the proof. □

According to Theorem 7, we will obtain the resilient robust finite-time L_2-L_∞ controller for a class of uncertain neutral system with mixed time-varying delays.

Theorem 8. *Given positive scalars* c_1, T, δ, *and* α, *positive definite symmetric matrix* $\mathbf{R}$, *and time-delay parameters* $h > 0$, $h_d > 0$, $\tau > 0$, *and* $\tau_d > 0$, *the closed-loop neutral system* Σ *is FTB with respect to* $(c_1, c_2, \delta, \mathbf{R}, T)$ *and satisfies the cost function* (11) *for all admissible disturbance* $\mathbf{w}(t)$, *if there exist positive scalars* c_2 *and* β *and symmetric positive definite matrices* $\mathbf{P}_i, i = 1, 2, \ldots, 6$, $\mathbf{Q}_i, i = 1, 2, \ldots, 4$, $\mathbf{W}_i, i = 1, 2, \ldots, 6$, *such that conditions* (15) *and* (16) *and the following LMI hold:*

$$\mathbf{\Psi} = \begin{bmatrix} -\mathbf{P}_1 & \widehat{\mathbf{F}}^T \\ * & -\beta\mathbf{I} \end{bmatrix} < 0. \tag{33}$$

Proof. Similar to the proof of Theorem 7, (29) can be rewritten as

$$e^{-\alpha t}V(t) < \alpha\int_0^t e^{-\alpha\tau}\mathbf{w}^T(\tau)\mathbf{w}(\tau)\,d\tau. \tag{34}$$

Then, we have

$$\mathbf{x}^T(t)\,\mathbf{P}_1\mathbf{x}(t) \le V(t) < \alpha e^{\alpha T}\int_0^t \mathbf{w}^T(\tau)\,\mathbf{w}(\tau)\,d\tau. \tag{35}$$

From (33), we can obviously get

$$\widehat{\mathbf{F}}^T\widehat{\mathbf{F}} < \beta\mathbf{P}_1. \tag{36}$$

Considering system Σ, we have

$$\mathbf{y}^T(t)\,\mathbf{y}(t) = \left[\widehat{\mathbf{F}}\mathbf{x}(t)\right]^T\left[\widehat{\mathbf{F}}\mathbf{x}(t)\right] = \mathbf{x}^T(t)\,\widehat{\mathbf{F}}^T\widehat{\mathbf{F}}\mathbf{x}(t). \tag{37}$$

Combining (35)–(37), we can obtain

$$\mathbf{x}^T(t)\,\widehat{\mathbf{F}}^T\widehat{\mathbf{F}}\mathbf{x}(t) \le \beta\mathbf{x}^T(t)\,\mathbf{P}_1\mathbf{x}(t) \le \beta\alpha e^{\alpha T}\int_0^t \mathbf{w}^T(\tau)\,\mathbf{w}(\tau)\,d\tau; \tag{38}$$

that is,

$$\mathbf{y}^T(t)\,\mathbf{y}(t) \le \beta\alpha e^{\alpha T}\int_0^t \mathbf{w}^T(\tau)\,\mathbf{w}(\tau)\,d\tau. \tag{39}$$

Letting $\gamma^2 = \beta\alpha e^{\alpha T}$, we have $\|\mathbf{y}(t)\|_\infty^2 < \gamma^2\|\mathbf{w}(t)\|_2^2$. This completes the proof. □

Theorem 9. *Given positive scalars* c_1, T, δ, *and* α, *positive definite symmetric matrix* $\mathbf{R}$, *and time-delay parameters* $h > 0$, $h_d > 0$, $\tau > 0$, *and* $\tau_d > 0$, *the closed-loop neutral system* Σ *is FTB with respect to* $(c_1, c_2, \delta, \mathbf{R}, T)$, *satisfies the cost function* (11) *for all admissible disturbance* $\mathbf{w}(t)$, *and exists as a state feedback controller in the form of* (7) *with* $\mathbf{K} = \mathbf{U}\mathbf{P}_1^{-1}$, *if there exist positive scalars* c_2, β, ε_i, $i = 1, 2, \ldots, 4$, *and* μ_i, $i = 1, 2, \ldots, 5$, *and symmetric positive definite matrices* L_i, $i = 1, 2, \ldots, 10$, T_i, $i = 1, 2, \ldots, 6$, $\overline{\mathbf{Q}}_i$, $i = 1, 2, \ldots, 4$, $\mathbf{P}_i$, $i = 2, 3, \ldots, 5$, $\overline{\mathbf{P}}_6$, *and* $\mathbf{U}$, *such that the following LMIs are feasible:*

$$\widetilde{\mathbf{\Pi}} = \begin{bmatrix} \widetilde{\mathbf{\Pi}}_1 & \widetilde{\mathbf{\Pi}}_2 & \widetilde{\mathbf{\Pi}}_3 & \widetilde{\mathbf{\Pi}}_7 \\ * & \widetilde{\mathbf{\Pi}}_4 & \widetilde{\mathbf{\Pi}}_5 & \widetilde{\mathbf{\Pi}}_8 \\ * & * & \widetilde{\mathbf{\Pi}}_6 & \widetilde{\mathbf{\Pi}}_9 \\ * & * & * & \widetilde{\mathbf{\Pi}}_{10} \end{bmatrix} < 0, \tag{40}$$

$$\widetilde{\mathbf{\Psi}} = \begin{bmatrix} -\mathbf{L}_1 & \mathbf{L}_1\mathbf{F}^T + \mathbf{U}^T\mathbf{G}^T & \mathbf{L}_1\mathbf{H}_1^T & \mathbf{L}_1\mathbf{S}^T \\ * & \widetilde{\mathbf{\Psi}}_{22} & 0 & 0 \\ * & * & -\varepsilon_3\mathbf{I} & 0 \\ * & * & * & -\varepsilon_4\mathbf{I} \end{bmatrix} < 0, \tag{41}$$

$$\mu_1\mathbf{R}^{-1} < \mathbf{L}_1 < \mathbf{R}^{-1}, \tag{42}$$

$$0 < \mathbf{P}_2 < \mu_2\mathbf{R}, \tag{43}$$

$$0 < \mathbf{P}_3 < \mu_3\mathbf{R}, \tag{44}$$

$$0 < \mathbf{P}_4 < \mu_4\mathbf{R}, \tag{45}$$

$$0 < \mathbf{P}_5 < \mu_5\mathbf{R}, \tag{46}$$

$$\begin{bmatrix} c_1\left[h\left(\mu_2 + \mu_3\right) + \tau\left(\mu_4 + \mu_5\right)\right] + \delta\left(1 - e^{-\alpha T}\right) - c_2 e^{-\alpha T} & \sqrt{c_1} \\ * & -\mu_1 \end{bmatrix} < 0, \tag{47}$$

where

$$\widetilde{\mathbf{\Pi}}_1 = \left[\widetilde{\mathbf{\Pi}}_{ij}\right]_{7\times 7}, \qquad \widetilde{\mathbf{\Pi}}_{11} = \mathbf{L}_1\mathbf{A}^T + \mathbf{A}\mathbf{L}_1 + \mathbf{U}^T\mathbf{B}^T + \mathbf{B}\mathbf{U} + \mathbf{L}_2 + \mathbf{L}_3 + \mathbf{L}_4 + \mathbf{L}_5 + \mathbf{T}_1 + \mathbf{T}_3 + \mathbf{T}_4 + \mathbf{T}_6$$
$$+ \mathbf{T}_1^T + \mathbf{T}_3^T + \mathbf{T}_4^T + \mathbf{T}_6^T - \alpha\mathbf{L}_1 - \frac{\alpha}{\tau}\mathbf{L}_6 - 2\alpha\mathbf{L}_7 - 2\alpha\mathbf{L}_8 - 2\alpha\mathbf{L}_9 - 2\alpha\mathbf{L}_{10},$$

$$\widetilde{\mathbf{\Pi}}_{12} = \mathbf{A}_d\mathbf{L}_1 - \mathbf{T}_1 + \mathbf{T}_2 + \mathbf{T}_1^T + \mathbf{T}_3^T + \mathbf{T}_4^T + \mathbf{T}_6^T, \qquad \widetilde{\mathbf{\Pi}}_{13} = \mathbf{C}\mathbf{L}_1 + \mathbf{T}_1^T + \mathbf{T}_3^T + \mathbf{T}_4^T + \mathbf{T}_6^T,$$

$$\widetilde{\mathbf{\Pi}}_{14} = -\mathbf{T}_4 + \mathbf{T}_5 + \mathbf{T}_1^T + \mathbf{T}_3^T + \mathbf{T}_4^T + \mathbf{T}_6^T + \frac{\alpha}{\tau}\mathbf{L}_6, \qquad \widetilde{\mathbf{\Pi}}_{15} = -\mathbf{T}_2 - \mathbf{T}_3 + \mathbf{T}_1^T + \mathbf{T}_3^T + \mathbf{T}_4^T + \mathbf{T}_6^T,$$

$$\widetilde{\mathbf{\Pi}}_{16} = -\mathbf{T}_5 - \mathbf{T}_6 + \mathbf{T}_1^T + \mathbf{T}_3^T + \mathbf{T}_4^T + \mathbf{T}_6^T, \qquad \widetilde{\mathbf{\Pi}}_{17} = \mathbf{D}, \qquad \widetilde{\mathbf{\Pi}}_{22} = -\left(1 - h_d\right)\mathbf{L}_2 - \mathbf{T}_1 + \mathbf{T}_2 - \mathbf{T}_1^T + \mathbf{T}_2^T,$$

$$\widetilde{\mathbf{\Pi}}_{23} = -\mathbf{T}_1^T + \mathbf{T}_2^T, \qquad \widetilde{\mathbf{\Pi}}_{24} = -\mathbf{T}_4 + \mathbf{T}_5 - \mathbf{T}_1^T + \mathbf{T}_2^T, \qquad \widetilde{\mathbf{\Pi}}_{25} = -\mathbf{T}_2 - \mathbf{T}_3 - \mathbf{T}_1^T + \mathbf{T}_2^T,$$

$$\widetilde{\mathbf{\Pi}}_{26} = -\mathbf{T}_5 - \mathbf{T}_6 - \mathbf{T}_1^T + \mathbf{T}_2^T, \qquad \widetilde{\mathbf{\Pi}}_{27} = 0, \qquad \widetilde{\mathbf{\Pi}}_{33} = -\left(1 - \tau_d\right)\mathbf{L}_6, \qquad \widetilde{\mathbf{\Pi}}_{34} = -\mathbf{T}_4 + \mathbf{T}_5,$$

$$\widetilde{\mathbf{\Pi}}_{35} = -\mathbf{T}_2 - \mathbf{T}_3, \qquad \widetilde{\mathbf{\Pi}}_{36} = -\mathbf{T}_5 - \mathbf{T}_6, \qquad \widetilde{\mathbf{\Pi}}_{37} = 0, \qquad \widetilde{\mathbf{\Pi}}_{44} = -\left(1 - \tau_d\right)\mathbf{L}_4 - \mathbf{T}_4 + \mathbf{T}_5 - \mathbf{T}_4^T + \mathbf{T}_5^T - \frac{\alpha}{\tau}\mathbf{L}_6,$$

$$\widetilde{\mathbf{\Pi}}_{45} = -\mathbf{T}_2 - \mathbf{T}_3 - \mathbf{T}_4^T + \mathbf{T}_5^T, \qquad \widetilde{\mathbf{\Pi}}_{46} = -\mathbf{T}_5 - \mathbf{T}_6 - \mathbf{T}_4^T + \mathbf{T}_5^T, \qquad \widetilde{\mathbf{\Pi}}_{47} = 0,$$

$$\widetilde{\mathbf{\Pi}}_{55} = -\mathbf{L}_3 - \mathbf{T}_2 - \mathbf{T}_3 - \mathbf{T}_2^T - \mathbf{T}_3^T, \qquad \widetilde{\mathbf{\Pi}}_{56} = -\mathbf{T}_5 - \mathbf{T}_6 - \mathbf{T}_2^T - \mathbf{T}_3^T, \qquad \widetilde{\mathbf{\Pi}}_{57} = 0,$$

$$\widetilde{\mathbf{\Pi}}_{66} = -\mathbf{L}_5 - \mathbf{T}_5 - \mathbf{T}_6 - \mathbf{T}_5^T - \mathbf{T}_6^T, \qquad \widetilde{\mathbf{\Pi}}_{67} = 0, \qquad \widetilde{\mathbf{\Pi}}_{77} = -\alpha\mathbf{I},$$

$$\widetilde{\mathbf{\Pi}}_2 = \begin{bmatrix} h\mathbf{T}_1 & h\mathbf{T}_2 & h\mathbf{T}_3 & \tau\mathbf{T}_4 & \tau\mathbf{T}_5 & \tau\mathbf{T}_6 \\ h\mathbf{T}_1 & h\mathbf{T}_2 & h\mathbf{T}_3 & \tau\mathbf{T}_4 & \tau\mathbf{T}_5 & \tau\mathbf{T}_6 \\ h\mathbf{T}_1 & h\mathbf{T}_2 & h\mathbf{T}_3 & \tau\mathbf{T}_4 & \tau\mathbf{T}_5 & \tau\mathbf{T}_6 \\ h\mathbf{T}_1 & h\mathbf{T}_2 & h\mathbf{T}_3 & \tau\mathbf{T}_4 & \tau\mathbf{T}_5 & \tau\mathbf{T}_6 \\ h\mathbf{T}_1 & h\mathbf{T}_2 & h\mathbf{T}_3 & \tau\mathbf{T}_4 & \tau\mathbf{T}_5 & \tau\mathbf{T}_6 \\ h\mathbf{T}_1 & h\mathbf{T}_2 & h\mathbf{T}_3 & \tau\mathbf{T}_4 & \tau\mathbf{T}_5 & \tau\mathbf{T}_6 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\widetilde{\mathbf{\Pi}}_3 = \begin{bmatrix} \mathbf{L}_1\mathbf{A}^T + \mathbf{U}^T\mathbf{B}^T & h\mathbf{L}_1\mathbf{A}^T + h\mathbf{U}^T\mathbf{B}^T & h\mathbf{L}_1\mathbf{A}^T + h\mathbf{U}^T\mathbf{B}^T & \tau\mathbf{L}_1\mathbf{A}^T + \tau\mathbf{U}^T\mathbf{B}^T & \tau\mathbf{L}_1\mathbf{A}^T + \tau\mathbf{U}^T\mathbf{B}^T \\ \mathbf{L}_1\mathbf{A}_d^T & h\mathbf{L}_1\mathbf{A}_d^T & h\mathbf{L}_1\mathbf{A}_d^T & \tau\mathbf{L}_1\mathbf{A}_d^T & \tau\mathbf{L}_1\mathbf{A}_d^T \\ \mathbf{L}_1\mathbf{C}^T & h\mathbf{C}^T\mathbf{Q}_1 & h\mathbf{C}^T\mathbf{Q}_2 & \tau\mathbf{C}^T\mathbf{Q}_3 & \tau\mathbf{C}^T\mathbf{Q}_4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \mathbf{D}^T & h\mathbf{D}^T & h\mathbf{D}^T & \tau\mathbf{D}^T & \tau\mathbf{D}^T \end{bmatrix},$$

$$\widetilde{\mathbf{\Pi}}_4 = \operatorname{diag}\left\{-h\mathbf{L}_7 \quad -h\mathbf{L}_7 \quad -h\mathbf{L}_8 \quad -\tau\mathbf{L}_9 \quad -\tau\mathbf{L}_9 \quad -\tau\mathbf{L}_{10}\right\}, \qquad \widetilde{\mathbf{\Pi}}_5 = [0]_{6\times 5},$$

$$\widetilde{\mathbf{\Pi}}_6 = \operatorname{diag}\left\{-\overline{\mathbf{P}}_6 \quad -h\overline{\mathbf{Q}}_1 \quad -h\overline{\mathbf{Q}}_2 \quad -\tau\overline{\mathbf{Q}}_3 \quad -\tau\overline{\mathbf{Q}}_3\right\}, \qquad \widetilde{\mathbf{\Pi}}_7 = \begin{bmatrix} \mathbf{L}_1\mathbf{H}_1^T & \mathbf{L}_1\mathbf{S}^T & \varepsilon_1\mathbf{M}_1 & \varepsilon_2\mathbf{B}\mathbf{N} \\ \mathbf{L}_1\mathbf{H}_2^T & 0 & 0 & 0 \\ \mathbf{L}_1\mathbf{H}_3^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \mathbf{L}_1\mathbf{H}_4^T & 0 & 0 & 0 \end{bmatrix},$$

$$\widetilde{\mathbf{\Pi}}_8 = [0]_{6\times 4}, \qquad \widetilde{\mathbf{\Pi}}_{10} = \operatorname{diag}\left\{-\varepsilon_1\mathbf{I} \quad -\varepsilon_1\mathbf{I} \quad -\varepsilon_2\mathbf{I} \quad -\varepsilon_2\mathbf{I}\right\}, \qquad \widetilde{\mathbf{\Psi}}_{22} = -\beta\mathbf{I} + \varepsilon_3\mathbf{M}^T\mathbf{M}_2^T + \varepsilon_4\mathbf{G}\mathbf{N}^T\mathbf{N}\mathbf{G}^T. \tag{48}$$

Proof. Replacing $\widehat{\mathbf{A}}$, $\overline{\mathbf{A}}_d$, $\overline{\mathbf{C}}$, and $\overline{\mathbf{D}}$ in (15) with $\widehat{\mathbf{A}} = \overline{\mathbf{A}} + \Delta\overline{\mathbf{A}}(t)$, $\overline{\mathbf{A}} = \mathbf{A} + \mathbf{B}\mathbf{K}$, $\Delta\overline{\mathbf{A}}(t) = \Delta\mathbf{A}(t) + \mathbf{B}\Delta\mathbf{K}(t)$, $\overline{\mathbf{A}}_d = \mathbf{A}_d + \Delta\mathbf{A}_d(t)$, $\overline{\mathbf{C}} = \mathbf{C} + \Delta\mathbf{C}(t)$, and $\overline{\mathbf{D}} = \mathbf{D} + \Delta\mathbf{D}(t)$, respectively, we have

$$\mathbf{\Pi} = \overline{\mathbf{\Pi}} + \Delta\overline{\mathbf{\Pi}} < 0, \tag{49}$$

where

$$\overline{\mathbf{\Pi}} = \begin{bmatrix} \overline{\mathbf{\Pi}}_1 & \mathbf{\Pi}_2 & \overline{\mathbf{\Pi}}_3 \\ * & \mathbf{\Pi}_4 & \mathbf{\Pi}_5 \\ * & * & \mathbf{\Pi}_6 \end{bmatrix} < 0, \qquad \overline{\mathbf{\Pi}}_1 = \left[\overline{\mathbf{\Pi}}_{ij}\right]_{7\times 7}, \qquad \overline{\mathbf{\Pi}}_1 = \begin{bmatrix} \overline{\mathbf{\Pi}}_{11} & \overline{\mathbf{\Pi}}_{12} & \overline{\mathbf{\Pi}}_{13} & \mathbf{\Pi}_{14} & \mathbf{\Pi}_{15} & \mathbf{\Pi}_{16} & \mathbf{P}_1\mathbf{D} \\ * & \mathbf{\Pi}_{22} & \mathbf{\Pi}_{23} & \mathbf{\Pi}_{24} & \mathbf{\Pi}_{25} & \mathbf{\Pi}_{26} & \mathbf{\Pi}_{27} \\ * & * & \mathbf{\Pi}_{33} & \mathbf{\Pi}_{34} & \mathbf{\Pi}_{35} & \mathbf{\Pi}_{36} & \mathbf{\Pi}_{37} \\ * & * & * & \mathbf{\Pi}_{44} & \mathbf{\Pi}_{45} & \mathbf{\Pi}_{46} & \mathbf{\Pi}_{47} \\ * & * & * & * & \mathbf{\Pi}_{55} & \mathbf{\Pi}_{56} & \mathbf{\Pi}_{57} \\ * & * & * & * & * & \mathbf{\Pi}_{66} & \mathbf{\Pi}_{67} \\ * & * & * & * & * & * & \mathbf{\Pi}_{77} \end{bmatrix},$$

$$\begin{aligned} \overline{\mathbf{\Pi}}_{11} = {} & \overline{\mathbf{A}}^T\mathbf{P}_1 + \mathbf{P}_1\overline{\mathbf{A}} + \mathbf{P}_2 + \mathbf{P}_3 + \mathbf{P}_4 + \mathbf{P}_5 + \mathbf{W}_1 + \mathbf{W}_3 + \mathbf{W}_4 + \mathbf{W}_6 + \mathbf{W}_1^T + \mathbf{W}_3^T + \mathbf{W}_4^T + \mathbf{W}_6^T \\ & - \alpha\mathbf{P}_1 - \frac{\alpha}{\tau}\mathbf{P}_6 - 2\alpha\mathbf{Q}_1 - 2\alpha\mathbf{Q}_2 - 2\alpha\mathbf{Q}_3 - 2\alpha\mathbf{Q}_4, \\ \overline{\mathbf{\Pi}}_{12} = {} & \mathbf{P}_1\mathbf{A}_d - \mathbf{W}_1 + \mathbf{W}_2 + \mathbf{W}_1^T + \mathbf{W}_3^T + \mathbf{W}_4^T + \mathbf{W}_6^T, \qquad \overline{\mathbf{\Pi}}_{13} = \mathbf{P}_1\mathbf{C} + \mathbf{W}_1^T + \mathbf{W}_3^T + \mathbf{W}_4^T + \mathbf{W}_6^T, \end{aligned}$$

$$\overline{\boldsymbol{\Pi}}_3=\begin{bmatrix}\overline{\mathbf{A}}^T\mathbf{P}_6 & h\overline{\mathbf{A}}^T\mathbf{Q}_1 & h\overline{\mathbf{A}}^T\mathbf{Q}_2 & \tau\overline{\mathbf{A}}^T\mathbf{Q}_3 & \tau\overline{\mathbf{A}}^T\mathbf{Q}_4\\ \mathbf{A}_d^T\mathbf{P}_6 & h\mathbf{A}_d^T\mathbf{Q}_1 & h\mathbf{A}_d^T\mathbf{Q}_2 & \tau\mathbf{A}_d^T\mathbf{Q}_3 & \tau\mathbf{A}_d^T\mathbf{Q}_4\\ \mathbf{C}^T\mathbf{P}_6 & h\mathbf{C}^T\mathbf{Q}_1 & h\mathbf{C}^T\mathbf{Q}_2 & \tau\mathbf{C}^T\mathbf{Q}_3 & \tau\mathbf{C}^T\mathbf{Q}_4\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ \mathbf{D}^T\mathbf{P}_6 & h\mathbf{D}^T\mathbf{Q}_1 & h\mathbf{D}^T\mathbf{Q}_2 & \tau\mathbf{D}^T\mathbf{Q}_3 & \tau\mathbf{D}^T\mathbf{Q}_4\end{bmatrix},\qquad \Delta\overline{\boldsymbol{\Pi}}=\begin{bmatrix}\Delta\overline{\boldsymbol{\Pi}}_1 & 0 & \Delta\overline{\boldsymbol{\Pi}}_3\\ * & 0 & 0\\ * & * & 0\end{bmatrix}<0,$$

$$\Delta\overline{\boldsymbol{\Pi}}_1=\begin{bmatrix}\Delta\overline{\mathbf{A}}^T(t)\mathbf{P}_1+\mathbf{P}_1\Delta\overline{\mathbf{A}}(t) & \mathbf{P}_1\Delta\overline{\mathbf{A}}_d(t) & \mathbf{P}_1\Delta\overline{\mathbf{C}}(t) & 0 & 0 & 0 & \mathbf{P}_1\Delta\overline{\mathbf{D}}(t)\\ * & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & 0 & 0 & 0 & 0 & 0\\ * & * & * & 0 & 0 & 0 & 0\\ * & * & * & * & 0 & 0 & 0\\ * & * & * & * & * & 0 & 0\\ * & * & * & * & * & * & 0\end{bmatrix},$$

$$\Delta\overline{\boldsymbol{\Pi}}_3=\begin{bmatrix}\Delta\overline{\mathbf{A}}^T(t)\mathbf{P}_6 & h\Delta\overline{\mathbf{A}}^T(t)\mathbf{Q}_1 & h\Delta\overline{\mathbf{A}}^T(t)\mathbf{Q}_2 & \tau\Delta\overline{\mathbf{A}}^T(t)\mathbf{Q}_3 & \tau\Delta\overline{\mathbf{A}}^T(t)\mathbf{Q}_4\\ \Delta\mathbf{A}_d^T(t)\mathbf{P}_6 & h\Delta\mathbf{A}_d^T(t)\mathbf{Q}_1 & h\Delta\mathbf{A}_d^T(t)\mathbf{Q}_2 & \tau\Delta\mathbf{A}_d^T(t)\mathbf{Q}_3 & \tau\Delta\mathbf{A}_d^T(t)\mathbf{Q}_4\\ \Delta\mathbf{C}^T(t)\mathbf{P}_6 & h\Delta\mathbf{C}^T(t)\mathbf{Q}_1 & h\Delta\mathbf{C}^T(t)\mathbf{Q}_2 & \tau\Delta\mathbf{C}^T(t)\mathbf{Q}_3 & \tau\Delta\mathbf{C}^T(t)\mathbf{Q}_4\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ \Delta\mathbf{D}^T(t)\mathbf{P}_6 & h\Delta\mathbf{D}^T(t)\mathbf{Q}_1 & h\Delta\mathbf{D}^T(t)\mathbf{Q}_2 & \tau\Delta\mathbf{D}^T(t)\mathbf{Q}_3 & \tau\Delta\mathbf{D}^T(t)\mathbf{Q}_4\end{bmatrix},\qquad(50)$$

bring formulas (4) and (8) into $\Delta\overline{\boldsymbol{\Pi}}$, and according to Lemma 6, we have

$$\boldsymbol{\Gamma}_{d1}\boldsymbol{\sigma}(t)\boldsymbol{\Gamma}_{e1}+\left(\boldsymbol{\Gamma}_{d1}\boldsymbol{\sigma}(t)\boldsymbol{\Gamma}_{e1}\right)^T\le\varepsilon_1\boldsymbol{\Gamma}_{d1}\boldsymbol{\Gamma}_{d1}^T+\varepsilon_1^{-1}\boldsymbol{\Gamma}_{e1}^T\boldsymbol{\Gamma}_{e1},$$

$$\boldsymbol{\Gamma}_{d2}\boldsymbol{\eta}(t)\boldsymbol{\Gamma}_{e2}+\left(\boldsymbol{\Gamma}_{d2}\boldsymbol{\eta}(t)\boldsymbol{\Gamma}_{e2}\right)^T\le\varepsilon_2\boldsymbol{\Gamma}_{d2}\boldsymbol{\Gamma}_{d2}^T+\varepsilon_2^{-1}\boldsymbol{\Gamma}_{e2}^T\boldsymbol{\Gamma}_{e2},\qquad(51)$$

where

$$\begin{gathered}\boldsymbol{\Gamma}_{d1}=\left[\mathbf{M}_1^T\mathbf{P}_1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ \mathbf{M}_1^T\mathbf{P}_6\ \ h\mathbf{M}_1^T\mathbf{Q}_1\ \ h\mathbf{M}_1^T\mathbf{Q}_2\ \ \tau\mathbf{M}_1^T\mathbf{Q}_3\ \ \tau\mathbf{M}_1^T\mathbf{Q}_4\right]^T,\\ \boldsymbol{\Gamma}_{e1}=\left[\mathbf{H}_1\ \ \mathbf{H}_2\ \ \mathbf{H}_3\ 0\ 0\ 0\ \mathbf{H}_4\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\right],\\ \boldsymbol{\Gamma}_{d2}=\left[\mathbf{N}^T\mathbf{B}^T\mathbf{P}_1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ \mathbf{N}^T\mathbf{B}^T\mathbf{P}_6\ \ h\mathbf{N}^T\mathbf{B}^T\mathbf{Q}_1\ \ h\mathbf{N}^T\mathbf{B}^T\mathbf{Q}_2\ \ \tau\mathbf{N}^T\mathbf{B}^T\mathbf{Q}_3\ \ \tau\mathbf{N}^T\mathbf{B}^T\mathbf{Q}_4\right]^T,\\ \boldsymbol{\Gamma}_{e2}=\left[\mathbf{S}\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\right].\end{gathered}\qquad(52)$$

Considering

$$\begin{aligned}\Delta\overline{\boldsymbol{\Pi}}&=\boldsymbol{\Gamma}_{d1}\boldsymbol{\sigma}(t)\boldsymbol{\Gamma}_{e1}+\left(\boldsymbol{\Gamma}_{d1}\boldsymbol{\sigma}(t)\boldsymbol{\Gamma}_{e1}\right)^T\\&\quad+\boldsymbol{\Gamma}_{d2}\boldsymbol{\eta}(t)\boldsymbol{\Gamma}_{e2}+\left(\boldsymbol{\Gamma}_{d2}\boldsymbol{\eta}(t)\boldsymbol{\Gamma}_{e2}\right)^T\\&\le\varepsilon_1\boldsymbol{\Gamma}_{d1}\boldsymbol{\Gamma}_{d1}^T+\varepsilon_1^{-1}\boldsymbol{\Gamma}_{e1}^T\boldsymbol{\Gamma}_{e1}+\varepsilon_2\boldsymbol{\Gamma}_{d2}\boldsymbol{\Gamma}_{d2}^T+\varepsilon_2^{-1}\boldsymbol{\Gamma}_{e2}^T\boldsymbol{\Gamma}_{e2},\end{aligned}\qquad(53)$$

(49) can be guaranteed by

$$\overline{\boldsymbol{\Pi}}+\varepsilon_1\boldsymbol{\Gamma}_{d1}\boldsymbol{\Gamma}_{d1}^T+\varepsilon_1^{-1}\boldsymbol{\Gamma}_{e1}^T\boldsymbol{\Gamma}_{e1}+\varepsilon_2\boldsymbol{\Gamma}_{d2}\boldsymbol{\Gamma}_{d2}^T+\varepsilon_2^{-1}\boldsymbol{\Gamma}_{e2}^T\boldsymbol{\Gamma}_{e2}<0.\qquad(54)$$

Using Schur complement, equality (54) can be rewritten as

$$\widehat{\boldsymbol{\Pi}}=\begin{bmatrix}\overline{\boldsymbol{\Pi}}_1 & \boldsymbol{\Pi}_2 & \overline{\boldsymbol{\Pi}}_3 & \widehat{\boldsymbol{\Pi}}_7\\ * & \boldsymbol{\Pi}_4 & \overline{\boldsymbol{\Pi}}_5 & \widehat{\boldsymbol{\Pi}}_8\\ * & * & \overline{\boldsymbol{\Pi}}_6 & \widehat{\boldsymbol{\Pi}}_9\\ * & * & * & \widehat{\boldsymbol{\Pi}}_{10}\end{bmatrix}<0,\qquad(55)$$

where

$$\widehat{\boldsymbol{\Pi}}_7 = \begin{bmatrix} \mathbf{S}_1^T & \mathbf{N}^T & \varepsilon_1 \mathbf{P}_1 \mathbf{M}_1 & \varepsilon_2 \mathbf{P}_1 \mathbf{B}\mathbf{N} \\ \mathbf{S}_2^T & 0 & 0 & 0 \\ \mathbf{S}_3^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \mathbf{S}_4^T & 0 & 0 & 0 \end{bmatrix},$$

$$\widehat{\boldsymbol{\Pi}}_9 = \begin{bmatrix} 0 & 0 & \varepsilon_1 \mathbf{P}_6 \mathbf{M}_1 & \varepsilon_2 \mathbf{P}_6 \mathbf{B}\mathbf{N} \\ 0 & 0 & h\varepsilon_1 \mathbf{Q}_1 \mathbf{M}_1 & h\varepsilon_2 \mathbf{Q}_1 \mathbf{B}\mathbf{N} \\ 0 & 0 & h\varepsilon_1 \mathbf{Q}_2 \mathbf{M}_1 & h\varepsilon_2 \mathbf{Q}_2 \mathbf{B}\mathbf{N} \\ 0 & 0 & \tau\varepsilon_1 \mathbf{Q}_3 \mathbf{M}_1 & \tau\varepsilon_2 \mathbf{Q}_3 \mathbf{B}\mathbf{N} \\ 0 & 0 & \tau\varepsilon_1 \mathbf{Q}_4 \mathbf{M}_1 & \tau\varepsilon_2 \mathbf{Q}_4 \mathbf{B}\mathbf{N} \end{bmatrix},$$

$$\widehat{\boldsymbol{\Pi}}_8 = [0]_{6\times 4},$$
$$\widehat{\boldsymbol{\Pi}}_{10} = \operatorname{diag}\{-\varepsilon_1 \mathbf{I} \quad -\varepsilon_2 \mathbf{I} \quad -\varepsilon_1 \mathbf{I} \quad -\varepsilon_2 \mathbf{I}\}. \tag{56}$$

Letting $\mathbf{L}_1 = \mathbf{P}_1^{-1}$, $\mathbf{L}_2 = \mathbf{L}_1\mathbf{P}_2\mathbf{L}_1$, $\mathbf{L}_3 = \mathbf{L}_1\mathbf{P}_3\mathbf{L}_1$, $\mathbf{L}_4 = \mathbf{L}_1\mathbf{P}_4\mathbf{L}_1$, $\mathbf{L}_5 = \mathbf{L}_1\mathbf{P}_5\mathbf{L}_1$, $\mathbf{L}_6 = \mathbf{L}_1\mathbf{P}_6\mathbf{L}_1$, $\mathbf{L}_7 = \mathbf{L}_1\mathbf{Q}_1\mathbf{L}_1$, $\mathbf{L}_8 = \mathbf{L}_1\mathbf{Q}_2\mathbf{L}_1$, $\mathbf{L}_9 = \mathbf{L}_1\mathbf{Q}_3\mathbf{L}_1$, $\mathbf{L}_{10} = \mathbf{L}_1\mathbf{Q}_4\mathbf{L}_1$, $\mathbf{T}_1 = \mathbf{L}_1\mathbf{W}_1\mathbf{L}_1$, $\mathbf{T}_2 = \mathbf{L}_1\mathbf{W}_2\mathbf{L}_1$, $\mathbf{T}_3 = \mathbf{L}_1\mathbf{W}_3\mathbf{L}_1$, $\mathbf{T}_3 = \mathbf{L}_1\mathbf{W}_3\mathbf{L}_1$, $\mathbf{T}_4 = \mathbf{L}_1\mathbf{W}_4\mathbf{L}_1$, $\mathbf{T}_5 = \mathbf{L}_1\mathbf{W}_5\mathbf{L}_1$, $\mathbf{T}_6 = \mathbf{L}_1\mathbf{W}_6\mathbf{L}_1$, $\overline{\mathbf{P}}_6 = \mathbf{P}_6^{-1}$, $\overline{\mathbf{Q}}_1 = \mathbf{Q}_1^{-1}$, $\overline{\mathbf{Q}}_2 = \mathbf{Q}_2^{-1}$, $\overline{\mathbf{Q}}_3 = \mathbf{Q}_{.3}^{-1}$, $\overline{\mathbf{Q}}_4 = \mathbf{Q}_4^{-1}$, and $\mathbf{U} = \mathbf{K}\mathbf{L}_1$, we can obtain condition (40) by pre- and postmultiplying inequality (55) by block-diagonal matrix

$$\operatorname{diag}\{\mathbf{P}_1^{-1} \; \mathbf{P}_1^{-1} \; \mathbf{P}_1^{-1} \; \mathbf{P}_1^{-1} \; \mathbf{P}_1^{-1} \; \mathbf{P}_1^{-1} \; \mathbf{I} \; \mathbf{P}_1^{-1} \; \mathbf{P}_1^{-1} \; \mathbf{P}_1^{-1} \; \mathbf{P}_1^{-1} \; \mathbf{P}_1^{-1} \; \mathbf{P}_1^{-1} \; \mathbf{P}_6^{-1} \; \mathbf{Q}_1^{-1} \; \mathbf{Q}_2^{-1} \; \mathbf{Q}_3^{-1} \; \mathbf{Q}_4^{-1} \; \mathbf{I} \; \mathbf{I} \; \mathbf{I} \; \mathbf{I}\}. \tag{57}$$

Next, we will prove that condition (33) is equivalent to (41).

Considering

$$\boldsymbol{\Psi} = \overline{\boldsymbol{\Psi}} + \Delta\overline{\boldsymbol{\Psi}} < 0, \tag{58}$$

where

$$\overline{\boldsymbol{\Psi}} = \begin{bmatrix} -\mathbf{P}_1 & \overline{\mathbf{F}}^T \\ * & -\beta\mathbf{I} \end{bmatrix}, \qquad \Delta\overline{\boldsymbol{\Psi}} = \begin{bmatrix} 0 & \Delta\overline{\mathbf{F}}^T(t) \\ * & 0 \end{bmatrix}, \tag{59}$$

combining with formulas (5) and (8), and using Schur complement, we have

$$\begin{aligned} \Delta\overline{\boldsymbol{\Psi}} &= \boldsymbol{\Gamma}_{d3}\boldsymbol{\sigma}(t)\boldsymbol{\Gamma}_{e3} + \left(\boldsymbol{\Gamma}_{d3}\boldsymbol{\sigma}(t)\boldsymbol{\Gamma}_{e3}\right)^T \\ &\quad + \boldsymbol{\Gamma}_{d4}\boldsymbol{\eta}(t)\boldsymbol{\Gamma}_{e4} + \left(\boldsymbol{\Gamma}_{d4}\boldsymbol{\eta}(t)\boldsymbol{\Gamma}_{e4}\right)^T \\ &\leq \varepsilon_3\boldsymbol{\Gamma}_{d3}\boldsymbol{\Gamma}_{d3}^T + \varepsilon_3^{-1}\boldsymbol{\Gamma}_{e3}^T\boldsymbol{\Gamma}_{e3} + \varepsilon_4\boldsymbol{\Gamma}_{d4}\boldsymbol{\Gamma}_{d4}^T + \varepsilon_4^{-1}\boldsymbol{\Gamma}_{e4}^T\boldsymbol{\Gamma}_{e4}, \end{aligned} \tag{60}$$

where $\boldsymbol{\Gamma}_{d3} = \begin{bmatrix} 0 & \mathbf{M}_2^T \end{bmatrix}^T$, $\boldsymbol{\Gamma}_{e3} = [\mathbf{H}_1 \;\; 0]$, $\boldsymbol{\Gamma}_{d4} = \begin{bmatrix} 0 & \mathbf{N}^T\mathbf{G}^T \end{bmatrix}^T$, and $\boldsymbol{\Gamma}_{e4} = [\mathbf{S} \;\; 0]$.

Then, we can get the following inequality which ensures (58):

$$\overline{\boldsymbol{\Psi}} + \varepsilon_3\boldsymbol{\Gamma}_{d3}\boldsymbol{\Gamma}_{d3}^T + \varepsilon_3^{-1}\boldsymbol{\Gamma}_{e3}^T\boldsymbol{\Gamma}_{e3} + \varepsilon_4\boldsymbol{\Gamma}_{d4}\boldsymbol{\Gamma}_{d4}^T + \varepsilon_4^{-1}\boldsymbol{\Gamma}_{e4}^T\boldsymbol{\Gamma}_{e4} < 0, \tag{61}$$

Using the Schur complement, equality (61) can be rewritten as

$$\widehat{\boldsymbol{\Psi}} = \begin{bmatrix} -\mathbf{P}_1 & \mathbf{F}^T + \mathbf{K}^T\mathbf{G}^T & \mathbf{H}_1^T & \mathbf{S}^T \\ * & \widetilde{\boldsymbol{\Psi}}_{22} & 0 & 0 \\ * & * & -\varepsilon_3\mathbf{I} & 0 \\ * & * & * & -\varepsilon_4\mathbf{I} \end{bmatrix} < 0. \tag{62}$$

Then, we can obtain condition (40) by pre- and post-multiplying inequality (62) by block-diagonal matrix $\operatorname{diag}\{\mathbf{P}_1^{-1} \;\; \mathbf{I} \;\; \mathbf{I} \;\; \mathbf{I}\}$.

Denoting $\widetilde{\mathbf{L}}_1 = \mathbf{R}^{1/2}\mathbf{L}_1\mathbf{R}^{1/2}$, $\widetilde{\mathbf{P}}_2 = \mathbf{R}^{-1/2}\mathbf{P}_2\mathbf{R}^{-1/2}$, $\widetilde{\mathbf{P}}_3 = \mathbf{R}^{-1/2}\mathbf{P}_3\mathbf{R}^{-1/2}$, $\widetilde{\mathbf{P}}_4 = \mathbf{R}^{-1/2}\mathbf{P}_4\mathbf{R}^{-1/2}$, and $\widetilde{\mathbf{P}}_5 = \mathbf{R}^{-1/2}\mathbf{P}_5\mathbf{R}^{-1/2}$, we know that condition (16) is equivalent to (47) according to conditions (42)–(46). This completes the proof. □

4. Simulation Example

In this part, we consider a class of neutral time-varying delayed systems with parameters described as

$$\mathbf{A} = \begin{bmatrix} 1.5 & 0.2 \\ 2.1 & 0.9 \end{bmatrix}, \qquad \mathbf{A}_d = \begin{bmatrix} -1.1 & -0.2 \\ -0.1 & -1.1 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 1.0 \\ 0.8 \end{bmatrix},$$
$$\mathbf{C} = \begin{bmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{bmatrix}, \qquad \mathbf{R} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$\mathbf{D} = \begin{bmatrix} 0.1 & 0.2 \\ -0.2 & 0.1 \end{bmatrix}, \qquad \mathbf{F} = \begin{bmatrix} 1.5 & 1.7 \\ 0.2 & 0.9 \end{bmatrix},$$
$$\mathbf{G} = \begin{bmatrix} 2 \\ -1.5 \end{bmatrix}, \qquad \mathbf{M}_1 = \begin{bmatrix} 1.1 \\ -0.7 \end{bmatrix}, \qquad \mathbf{M}_2 = \begin{bmatrix} 0.8 \\ -0.4 \end{bmatrix},$$
$$\mathbf{H}_1 = [1.4 \;\; 0.8], \qquad \mathbf{H}_2 = [0.4 \;\; 1.1],$$
$$\mathbf{H}_3 = [0.7 \;\; 0.2], \qquad \mathbf{H}_4 = [0.5 \;\; 1.3],$$
$$\mathbf{N} = [0.2], \qquad \mathbf{S} = [0.2 \;\; 0.6]. \tag{63}$$

In this note, we choose the initial values for $c_1 = 1$, $T = 5$, $\alpha = 0.3$, and $\delta = 1.0$ and the upper bounds on the delays are $\tau = 0.8$, $h = 0.5$, $h_d = 0.9$, and $\tau_d = 0.9$. By using the LMI toolbox in MATLAB to solve LMIs (40)–(47), we can get the finite-time L_2-L_∞ controller gain as follows:

$$\mathbf{L}_1 = \begin{bmatrix} 0.6515 & -0.1789 \\ -0.1789 & 0.3827 \end{bmatrix}, \qquad \mathbf{U} = [-0.3115 \;\; -0.0343],$$
$$\mathbf{K} = \mathbf{U}\mathbf{L}_1^{-1} = [-0.5768 \;\; -0.3593], \tag{64}$$

with constraint conditions $\beta = 14.7085$, $\gamma = 0.9923$, and $c_2 = 124.6975$.

Selecting $h(t) = 0.9/(1+t^2)$, $\tau(t) = 0.11/(3+t^2)$, $\boldsymbol{\sigma}(t) = (0.9/(1+t^2))\mathbf{I}$, $\boldsymbol{\eta}(t) = (1.5/(1+t^2))\mathbf{I}$, and $h(t) = 0.9/(1+t^2)$,

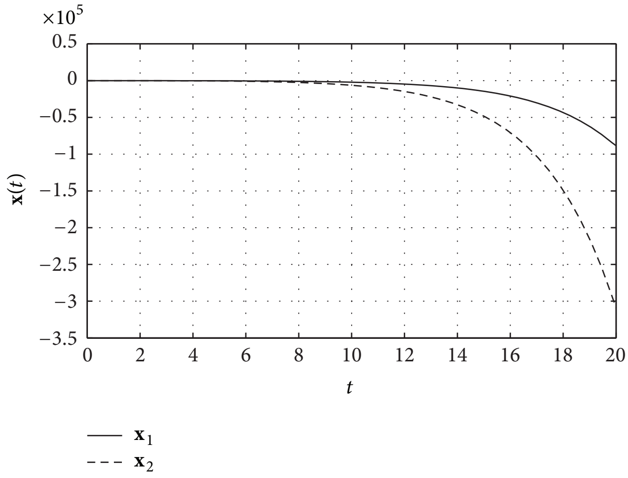

FIGURE 1: The trajectories of open-loop controlled system state $\mathbf{x}(t)$.

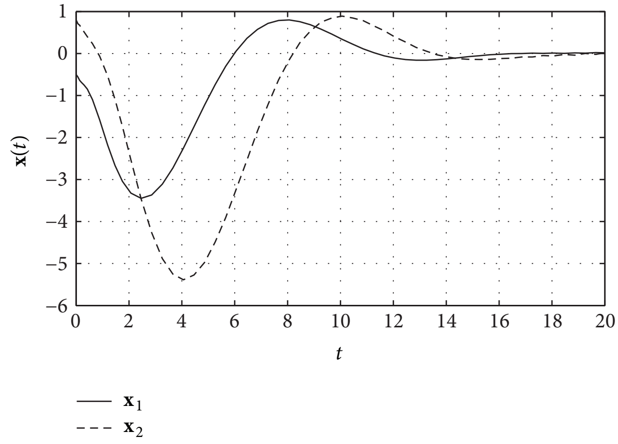

FIGURE 2: The trajectories of closed-loop controlled system state $\mathbf{x}(t)$.

$t \in [0, 20]$, and setting the initial states $\mathbf{x}_0 = \begin{bmatrix} -0.5 & 0.8 \end{bmatrix}^T$ and $\mathbf{w}_0 = \begin{bmatrix} 0.04 & 0.08 \end{bmatrix}^T$, we have the open-loop controlled system state simulation graph and the trajectories of closed-loop controlled system state and output as shown in Figures 1, 2, and 3, respectively. Figure 4 shows the evolution of function $\mathbf{x}^T(t)\mathbf{R}\mathbf{x}(t)$ $(t \in [0, 20])$ of the uncertain neutral time-delayed system Σ_0. Based on comparison between result in Figure 1 and result in Figure 2, we noted that the design finite-time L_2-L_∞ controller can make the closed-loop controlled system achieve FTB.

5. Conclusions

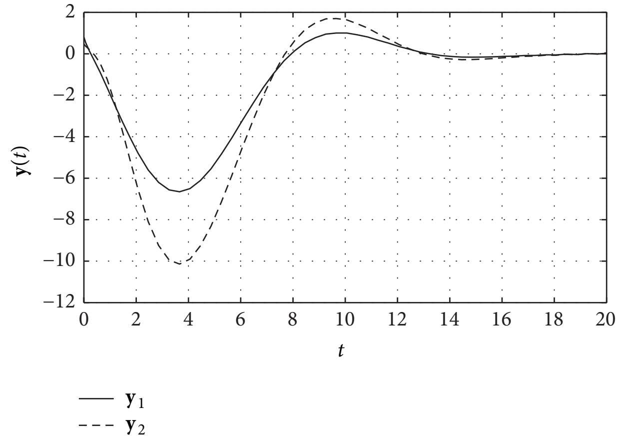

FIGURE 3: The trajectories of closed-loop controlled system output $\mathbf{y}(t)$.

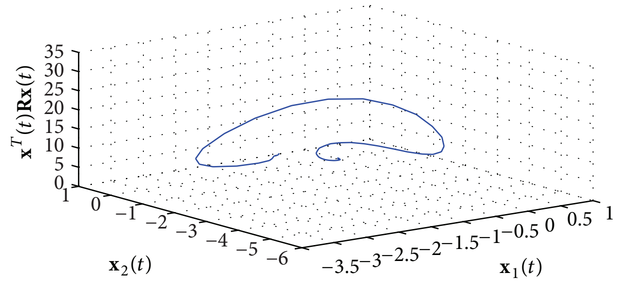

FIGURE 4: The graph of $\mathbf{x}^T(t)\mathbf{R}\mathbf{x}(t)$ $(t \in [0, T])$ of closed-loop controlled system.

This paper studied the delay-dependent resilient robust finite-time L_2-L_∞ control problem for a class of uncertain neutral time-delayed system with mixed time-varying delays. A state feedback controller is designed by using LMI technique and free weighting matrices, such that the closed-loop controlled system is FTB and satisfies the input-output L_2-L_∞ performance matrices. The simulation results verify the effectiveness of the design method. We will consider the finite-time observer for neutral time-delayed system in the future.

Conflict of Interests

The authors declare that they have no conflict of interests regarding the publication of this paper.

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