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Research Article

Discussion on "Multidimensional Coincidence Points" via Recent Publications

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We show that some definitions of multidimensional coincidence points are not compatible with the mixed monotone property. Thus, some theorems reported in the recent publications (Dalal et al., 2014 and Imdad et al., 2013) have gaps. We clarify these gaps and we present a new theorem to correct the mentioned results. Furthermore, we show how multidimensional results can be seen as simple consequences of our unidimensional coincidence point theorem.

1. Introduction and Preliminaries

In the sequel, X will be a nonempty set and $\preceq$ will represent a partial order on X. Given $n \in \mathbb{N}$ with $n \geq 2$, let denote by X^n the product space $X \times X \times \overset{(n)}{\cdots} \times X$ of n identical copies of X.

In [1], Guo and Lakshmikantham introduced the notion of *coupled fixed point* and, thus, they initiated the investigation of multidimensional fixed point theory.

Definition 1 (Guo and Lakshmikantham [1]). Let $F : X \times X \to X$ be a given mapping. We say that $(x, y) \in X \times X$ is a *coupled fixed point of* F if

$$F(x, y) = x, \qquad F(y, x) = y. \tag{1}$$

Following this initial paper [1], in 2006, Bhaskar and Lakshmikantham [2] obtained some coupled fixed point theorems for mapping $F : X \times X \to X$ (where X is a partially ordered metric space) by defining the notion of *mixed monotone mapping*.

Definition 2 (see [2]). Let $(X, \preceq)$ be a partially ordered set. A mapping $F : X \times X \to X$. F is said to have the *mixed monotone property* if $F(x, y)$ is monotone nondecreasing in x and is monotone nonincreasing in y; that is, for any $x, y \in X$,

$$\begin{aligned} x_1, x_2 \in X, \quad x_1 \preceq x_2 &\Longrightarrow F(x_1, y) \preceq F(x_2, y), \\ y_1, y_2 \in X, \quad y_1 \preceq y_2 &\Longrightarrow F(x, y_2) \preceq F(x, y_1). \end{aligned} \tag{2}$$

After that, Lakshmikantham and Ćirić [3] proved coupled fixed/coincidence point theorems for mappings $F : X \times X \to X$ and $g : X \to X$ by introducing the concept of the *mixed g-monotone property*. Inspired by these papers [2, 3], Berinde and Borcut defined tripled fixed points and established some tripled fixed point theorems.

Definition 3 (Berinde and Borcut [4]). Let $F : X^3 \to X$ be a given mapping. We say that $(x, y, x) \in X^3$ is a *tripled fixed point of* F if

$$F(x, y, z) = x, \qquad F(y, x, y) = y, \qquad F(z, y, x) = z. \tag{3}$$

Definition 4 (see [4]). Let $(X, \preceq)$ be a partially ordered set and $F : X^3 \to X$. We say that F has the *mixed monotone property* if $F(x,y,z)$ is monotone nondecreasing in x and z and it is monotone nonincreasing in y; that is, for any $x,y,z \in X$

$$\begin{aligned} &x_1,x_2 \in X, \quad x_1 \preceq x_2 \Longrightarrow F(x_1,y,z) \preceq F(x_2,y,z), \\ &y_1,y_2 \in X, \quad y_1 \preceq y_2 \Longrightarrow F(x,y_1,z) \succeq F(x,y_2,z), \\ &z_1,z_2 \in X, \quad z_1 \preceq z_2 \Longrightarrow F(x,y,z_1) \preceq F(x,y,z_2). \end{aligned} \tag{4}$$

As a natural extension, Karapınar [5] studied the quadruple case (see also [6, 7]).

Definition 5 (see [5]). An element $(x,y,z,w) \in X^4$ is called a *quadruple fixed point of* $F : X^4 \to X$ if

$$\begin{aligned} F(x,y,z,w) = x, \qquad F(y,z,w,x) = y, \\ F(z,w,x,y) = z, \qquad F(w,x,y,z) = w. \end{aligned} \tag{5}$$

Definition 6 (see [5]). Let $(X,\preceq)$ be a partially ordered set and $F : X^4 \to X$. We say that F has the *mixed monotone property* if $F(x,y,z,w)$ is monotone nondecreasing in x and z and it is monotone nonincreasing in y and w; that is, for any $x,y,z,w \in X$

$$\begin{aligned} &x_1,x_2 \in X, \quad x_1 \preceq x_2 \Longrightarrow F(x_1,y,z,w) \preceq F(x_2,y,z,w), \\ &y_1,y_2 \in X, \quad y_1 \preceq y_2 \Longrightarrow F(x,y_1,z,w) \succeq F(x,y_2,z,w), \\ &z_1,z_2 \in X, \quad z_1 \preceq z_2 \Longrightarrow F(x,y,z_1,w) \preceq F(x,y,z_2,w), \\ &w_1,w_2 \in X, \quad w_1 \preceq w_2 \Longrightarrow F(x,y,z,w_1) \succeq F(x,y,z,w_2). \end{aligned} \tag{6}$$

When a mapping $g : X \to X$ is involved, we have the notion of *coincidence point*. We will only recall the corresponding definitions in the quadruple case since they are similar in other dimensions.

Definition 7 (see [6]). An element $(x,y,z,w) \in X^4$ is called a *quadrupled coincident point of the mappings* $F : X^4 \to X$ and $g : X \to X$ if

$$\begin{aligned} gx = F(x,y,z,w), \qquad gy = F(y,z,w,x), \\ gz = F(z,w,x,y), \qquad gw = F(w,x,y,z). \end{aligned} \tag{7}$$

Definition 8 (see [6]). Let $(X,\preceq)$ be a partially ordered set and let $F : X^4 \to X$ and $g : X \to X$ be two mappings. We say F has the *mixed g-monotone property* if $F(x,y,z,w)$ is g-nondecreasing in x and z and is g-nonincreasing in y and w; that is, for any $x,y,z,w \in X$,

$$\begin{aligned} x_1,x_2 \in X, \quad & gx_1 \preceq gx_2 \\ & \Longrightarrow F(x_1,y,z,w) \preceq F(x_2,y,z,w), \\ y_1,y_2 \in X, \quad & gy_1 \preceq gy_2 \\ & \Longrightarrow F(x,y_1,z,w) \succeq F(x,y_2,z,w), \\ z_1,z_2 \in X, \quad & gz_1 \preceq gz_2 \\ & \Longrightarrow F(x,y,z_1,w) \preceq F(x,y,z_2,w), \\ w_1,w_2 \in X, \quad & gw_1 \preceq gw_2 \\ & \Longrightarrow F(x,y,z,w_1) \succeq F(x,y,z,w_2). \end{aligned} \tag{8}$$

It is very natural to extend the definition of 2-dimensional fixed point (coupled fixed point), 3-dimensional fixed point (tripled fixed point), and so on to multidimensional fixed point (n-tuple fixed point) (see, e.g., [8–19]). In this paper, we give some remarks on the notion of n-tuple fixed point given in several papers, such as Imdad et al. [9], Dalal et al. [8], and Ertürk and Karakaya [20, 21]. Notice that this paper can be considered as a continuation of Karapınar and Roldán [10, 22]. We note also that authors preferred to say "n-tuplet fixed point" [20, 21] or "n-tuplet fixed point" [8, 9] instead of "n-tuple fixed point".

Definition 9 (see [8, 9, 20]). An element $(x^1,x^2,x^3,\ldots,x^n) \in X^n$ is called an n-tuple fixed point of the mapping $F : X^n \to X$ if

$$\begin{aligned} x^1 &= F(x^1,x^2,x^3,\ldots,x^n), \\ x^2 &= F(x^2,x^3,\ldots,x^n,x^1), \\ x^3 &= F(x^3,\ldots,x^n,x^1,x^2), \\ &\vdots \\ x^n &= F(x^n,x^1,x^2,\ldots,x^{n-1}). \end{aligned} \tag{9}$$

Definition 10 (see [8, 9, 20]). Let $(X,\preceq)$ be a partially ordered set and let $F : X^n \to X$ be a mapping. We say F has the *mixed monotone property* if $F(x^1,x^2,x^3,\ldots,x^n)$ is nondecreasing in odd arguments and is nonincreasing in its even arguments; that is, for any $x^1,x^2,x^3,\ldots,x^n \in X$,

$$\begin{aligned} y_1,z_1 \in X, \quad & y_1 \preceq z_1 \\ & \Longrightarrow F(y_1,x^2,x^3,\ldots,x^n) \\ & \qquad \preceq F(z_1,x^2,x^3,\ldots,x^n), \\ y_2,z_2 \in X, \quad & y_2 \preceq z_2 \\ & \Longrightarrow F(x^1,y_2,x^3,\ldots,x^n) \succeq F(x^1,z_2,x^3,\ldots,x^n), \\ \vdots \\ y_n,z_n \in X, \quad & y_n \preceq z_n \\ & \Longrightarrow F(x^1,x^2,x^3,\ldots,y_n) \\ & \qquad \preceq F(x^1,x^2,x^3,\ldots,z_n), \text{ if } n \text{ is odd}, \\ y_n,z_n \in X, \quad & y_n \preceq z_n \\ & \Longrightarrow F(x^1,x^2,x^3,\ldots,y_n) \\ & \qquad \succeq F(x^1,x^2,x^3,\ldots,z_n), \text{ if } n \text{ is even}. \end{aligned} \tag{10}$$

Definition 11 (see [8, 9, 20]). An element $(x^1,x^2,x^3,\ldots,x^n) \in X^n$ is called an *n-tuple coincidence point of the mappings $F : X^n \to X$ and $g : X \to X$* if

$$\begin{aligned} gx^1 &= F(x^1,x^2,x^3,\ldots,x^n), \\ gx^2 &= F(x^2,x^3,\ldots,x^n,x^1), \end{aligned}$$

$$gx^3 = F\left(x^3, \dots, x^n, x^1, x^2\right),$$
$$\vdots$$
$$gx^n = F\left(x^n, x^1, x^2, \dots, x^{n-1}\right). \tag{11}$$

Definition 12 (see [8, 9, 20]). Let $(X, \preceq)$ be a partially ordered set and let $F : X^n \to X$ and $g : X \to X$ be two mappings. We say F has the *mixed g-monotone property* if $F(x^1, x^2, x^3, \dots, x^n)$ is g-nondecreasing in odd arguments and is g-nonincreasing in its even arguments; that is, for any $x^1, x^2, x^3, \dots, x^n \in X$,

$$\begin{aligned}
&y_1, z_1 \in X, \quad gy_1 \preceq gz_1 \\
&\qquad \Longrightarrow F\left(y_1, x^2, x^3, \dots, x^n\right) \preceq F\left(z_1, x^2, x^3, \dots, x^n\right), \\
&y_2, z_2 \in X, \quad gy_2 \preceq gz_2 \\
&\qquad \Longrightarrow F\left(x^1, y_2, x^3, \dots, x^n\right) \succeq F\left(x^1, z_2, x^3, \dots, x^n\right), \\
&\quad \vdots \\
&y_n, z_n \in X, \quad gy_n \preceq gz_n \\
&\qquad \Longrightarrow F\left(x^1, x^2, x^3, \dots, x^{n-1}, y_n\right) \\
&\qquad\qquad \preceq F\left(x^1, x^2, x^3, \dots, x^{n-1}, z_n\right), \quad \text{if } n \text{ is odd}, \\
&y_n, z_n \in X, \quad gy_n \preceq gz_n \\
&\qquad \Longrightarrow F\left(x^1, x^2, x^3, \dots, x^{n-1}, y_n\right) \\
&\qquad\qquad \succeq F\left(x^1, x^2, x^3, \dots, x^{n-1}, z_n\right), \\
&\qquad\qquad\qquad \text{if } n \text{ is even}.
\end{aligned} \tag{12}$$

Using these preliminaries, the following result was announced in [9]. Notice that in that paper, the authors used the notation $\prod_{i=1}^{r} X^i$ to refer to the product space X^r.

Theorem 13 (Imdad et al. [9], Theorem 13). *Let $(X, \preceq)$ be a partially ordered set equipped with a metric d such that (X, d) is a complete metric space. Assume that there is a function $\phi : [0, +\infty) \to [0, +\infty)$ with $\phi(t) < t$ and $\lim_{r \to t^+} \phi(t) < t$ for all $t > 0$. Further, suppose that $F : X^r \to X$ and $g : X \to X$ are two maps such that F has the mixed g-monotone property satisfying the following conditions:*

(i) $F(X^r) \subseteq g(X)$,

(ii) *g is continuous and monotonically increasing,*

(iii) *(g, F) is a commutating pair,*

(iv)

$$d\left(F\left(x^1, x^2, \dots, x^r\right), F\left(y^1, y^2, \dots, y^r\right)\right) \leq \phi\left(\frac{1}{r}\sum_{i=1}^{r} d\left(gx_i, gy_i\right)\right) \tag{13}$$

for all $x^1, x^2, \dots, x^r, y^1, y^2, \dots, y^r \in X$ with $gx^1 \preceq gy^1$, $gx^2 \succeq gy^2$, $gx^3 \preceq gy^3$,..., $gx^r \succeq gy^r$.

Also, suppose that either

(a) *F is continuous or*

(b) *X has the following properties:*

(b.1) *if a nondecreasing sequence $\{x_n\} \to x$, then $x_n \preceq x$ for all $n \geq 0$;*

(b.2) *if a nonincreasing sequence $\{x_n\} \to x$, then $x_n \succeq x$ for all $n \geq 0$.*

If there exist $x_0^1, x_0^2, x_0^3, \dots, x_0^r \in X$ such that

$$\begin{aligned}
gx_0^1 &\preceq F\left(x_0^1, x_0^2, x_0^3, \dots, x_0^r\right), \\
gx_0^2 &\succeq F\left(x_0^2, x_0^3, \dots, x_0^r, x_0^1\right), \\
gx_0^3 &\preceq F\left(x_0^3, \dots, x_0^r, x_0^1, x_0^2\right), \\
&\vdots \\
gx_0^r &\succeq F\left(x_0^r, x_0^1, x_0^2, \dots, x_0^{r-1}\right),
\end{aligned} \tag{14}$$

then F and g have a r-tupled coincidence point; that is, there exists $x^1, x^2, x^3, \dots, x^r \in X$ such that

$$\begin{aligned}
gx^1 &= F\left(x^1, x^2, x^3, \dots, x^r\right), \\
gx^2 &= F\left(x^2, x^3, \dots, x^r, x^1\right), \\
gx^3 &= F\left(x^3, \dots, x^r, x^1, x^2\right), \\
&\vdots \\
gx^r &= F\left(x^r, x^1, x^2, \dots, x^{r-1}\right).
\end{aligned} \tag{15}$$

Based on this theorem, Dalal et al. [8] extended the previous result to compatible mappings in the following sense.

Definition 14 (Dalal et al. [8]). Let (X, d) be a metric space provided with a partial order $\preceq$ and let $F : X^n \to X$ and $g : X \to X$ be two mappings. We will say that (F, g) is a *compatible pair* if

$$\begin{aligned}
&\lim_{n \to \infty} d\left(gF\left(x_n^1, x_n^2, x_n^3, \dots, x_n^r\right),\right. \\
&\qquad \left. F\left(gx_n^1, gx_n^2, gx_n^3, \dots, gx_n^r\right)\right) = 0, \\
&\lim_{n \to \infty} d\left(gF\left(x_n^2, x_n^3, \dots, x_n^r, x_n^1\right),\right. \\
&\qquad \left. F\left(gx_n^2, gx_n^3, \dots, gx_n^r, gx_n^1\right)\right) = 0,
\end{aligned}$$

$$\lim_{n\to\infty} d\left(gF\left(x_n^3,\ldots,x_n^r,x_n^1,x_n^2\right),\right.$$
$$\left.F\left(gx_n^3,\ldots,gx_n^r,gx_n^1,gx_n^2\right)\right)=0,$$
$$\vdots$$
$$\lim_{n\to\infty} d\left(gF\left(x_n^r,x_n^1,x_n^2,\ldots,x_n^{r-1}\right),\right.$$
$$\left.F\left(gx_n^r,gx_n^1,gx_n^2,\ldots,gx_n^{r-1}\right)\right)=0 \quad (16)$$

whenever $\{x_n^1\},\{x_n^2\},\{x_n^3\},\ldots,\{x_n^r\}$ are sequences in X such that

$$\begin{aligned}
&\lim_{n\to\infty} F\left(x_n^1,x_n^2,x_n^3,\ldots,x_n^r\right)=\lim_{n\to\infty} gx_n^1=x^1,\\
&\lim_{n\to\infty} F\left(x_n^2,x_n^3,\ldots,x_n^r,x_n^1\right)=\lim_{n\to\infty} gx_n^2=x^2,\\
&\lim_{n\to\infty} F\left(x_n^3,\ldots,x_n^r,x_n^1,x_n^2\right)=\lim_{n\to\infty} gx_n^3=x^3,\\
&\vdots\\
&\lim_{n\to\infty} F\left(x_n^r,x_n^1,x_n^2,\ldots,x_n^{r-1}\right)=\lim_{n\to\infty} gx_n^r=x^r,
\end{aligned} \quad (17)$$

for some $x^1,x^2,x^3,\ldots,x^r\in X$.

Theorem 15 (Dalal et al. [8], Theorem 3.2). *Let $(X,\preceq)$ be a partially ordered set equipped with a metric d such that (X,d) is a complete metric space. Assume that there is a function $\varphi:[0,\infty)\to[0,\infty)$ with $\varphi(t)<t$ and $\lim_{r\to t^+}\varphi(t)<t$ for all $t>0$. Further, let $F:X^r\to X$ and $g:X\to X$ be two maps such that F has the mixed g-monotone property satisfying the following conditions:*

(i) *$F(X^r)\subseteq g(X)$,*

(ii) *g is continuous and monotonically increasing,*

(iii) *the pair (g,F) is compatible,*

(iv)

$$d\left(F\left(x^1,x^2,\ldots,x^r\right),F\left(y^1,y^2,\ldots,y^r\right)\right)\le\varphi\left(\frac{1}{r}\sum_{i=1}^{r} d\left(gx_i,gy_i\right)\right) \quad (18)$$

for all $x^1,x^2,\ldots,x^r,y^1,y^2,\ldots,y^r\in X$ with $gx^1\preceq gy^1$, $gx^2\succeq gy^2$, $gx^3\preceq gy^3$,…, $gx^r\succeq gy^r$.

Also, suppose that either

(a) *F is continuous or*

(b) *X has the following properties:*

(b.1) *if a nondecreasing sequence $\{x_n\}\to x$, then $x_n\preceq x$ for all $n\ge 0$;*

(b.2) *if a nonincreasing sequence $\{x_n\}\to x$, then $x_n\succeq x$ for all $n\ge 0$.*

If there exists $x_0^1,x_0^2,x_0^3,\ldots,x_0^r\in X$ such that

$$\begin{aligned}
gx_0^1&\preceq F\left(x_0^1,x_0^2,x_0^3,\ldots,x_0^r\right),\\
gx_0^2&\succeq F\left(x_0^2,x_0^3,\ldots,x_0^r,x_0^1\right),\\
gx_0^3&\preceq F\left(x_0^3,\ldots,x_0^r,x_0^1,x_0^2\right),\\
&\vdots\\
gx_0^r&\succeq F\left(x_0^r,x_0^1,x_0^2,\ldots,x_0^{r-1}\right),
\end{aligned} \quad (19)$$

then F and g have a r-tupled coincidence point; that is, there exists $x^1,x^2,x^3,\ldots,x^r\in X$ such that

$$\begin{aligned}
gx^1&=F\left(x^1,x^2,x^3,\ldots,x^r\right),\\
gx^2&=F\left(x^2,x^3,\ldots,x^r,x^1\right),\\
gx^3&=F\left(x^3,\ldots,x^r,x^1,x^2\right),\\
&\vdots\\
gx^r&=F\left(x^r,x^1,x^2,\ldots,x^{r-1}\right).
\end{aligned} \quad (20)$$

2. Some Remarks

Firstly we notice that, in the case $n=3$, Definitions 9 and 11,

$$\begin{aligned}
gx^1&=F\left(x^1,x^2,x^3\right),\\
gx^2&=F\left(x^2,x^3,x^1\right),\\
gx^3&=F\left(x^3,x^1,x^2\right)
\end{aligned} \quad (21)$$

do not extend the notion of tripled coincidence point in the sense of Berinde and Borcut [4]. Therefore, their results are not extensions of well-known results in the tripled case. This fact shows that the odd case is not well-posed by Definitions 9 and 11 or, more precisely, the mixed monotone property is not useful to ensure the existence of coincidence points. In this sense, we have the following result.

Theorem 16. *Theorem 13 in [9] is not valid if n is odd.*

Proof. It is sufficient to examine the case $n=3$ to indicate the mentioned invalidity. It is evident that the illustrative proof for the case $n=3$ can be analogously extended to the case in which n is odd. We follow the lines of the proof of Theorem 3.1 in [8]. Let $x_0^1,x_0^2,x_0^3\in X$ be the initial points. We construct three recursive sequences $\{x_k^1\}$, $\{x_k^2\}$, and $\{x_k^3\}$ in the following way:

$$\begin{aligned}
gx_k^1&=F\left(x_{k-1}^1,x_{k-1}^2,x_{k-1}^3\right),\\
gx_k^2&=F\left(x_{k-1}^2,x_{k-1}^3,x_{k-1}^1\right),\\
gx_k^3&=F\left(x_{k-1}^3,x_{k-1}^1,x_{k-1}^2\right)\quad \forall k\in\mathbb{N},\ k\ge 1.
\end{aligned} \quad (22)$$

Due to the assumption, we derive that

$$\begin{aligned} gx_0^1 &\preceq F\left(x_0^1, x_0^2, x_0^3\right) = gx_1^1, \\ gx_0^2 &\succeq F\left(x_0^2, x_0^3, x_0^1\right) = gx_1^2, \\ gx_0^3 &\preceq F\left(x_0^3, x_0^1, x_0^2\right) = gx_1^3. \end{aligned} \tag{23}$$

Then, the authors concluded that these sequences verify, for all $k \geq 1$,

$$\begin{aligned} gx_{k-1}^1 &\preceq gx_k^1, \\ gx_{k-1}^2 &\succeq gx_k^2, \\ gx_{k-1}^3 &\preceq gx_k^3. \end{aligned} \tag{24}$$

Now, we will show that it is impossible to prove that $gx_1^2 \succeq gx_2^2$ because the mixed g-monotone property leads to contrary inequalities. Indeed, we derive the following inequalities:

$$gx_1^2 \preceq gx_0^2 \Longrightarrow F\left(x_1^2, x_0^3, x_0^1\right) \preceq F\left(x_0^2, x_0^3, x_0^1\right) = gx_1^2. \tag{25}$$

Furthermore,

$$gx_0^3 \preceq gx_1^3 \Longrightarrow F\left(x_1^2, x_0^3, x_0^1\right) \succeq F\left(x_1^2, x_1^3, x_0^1\right). \tag{26}$$

By combining the inequalities above, we conclude that

$$F\left(x_1^2, x_1^3, x_0^1\right) \preceq F\left(x_1^2, x_0^3, x_0^1\right) \preceq F\left(x_0^2, x_0^3, x_0^1\right) = gx_1^2. \tag{27}$$

Notice that in the third component the inequality is on the contrary

$$gx_0^1 \preceq gx_1^1 \Longrightarrow F\left(x_1^2, x_1^3, x_0^1\right) \preceq F\left(x_1^2, x_1^3, x_1^1\right) = gx_2^2. \tag{28}$$

Then, we find that

$$F\left(x_1^2, x_1^3, x_0^1\right) \preceq gx_1^2, \qquad F\left(x_1^2, x_1^3, x_0^1\right) \preceq gx_2^2. \tag{29}$$

Consequently, we cannot get the inequality $gx_1^2 \succeq gx_2^2$, since other possibilities yield to another cases in which points are not comparable. □

By using the same argument above, we also conclude that Corollaries 14 and 15 in [9] are not valid. Similarly, we may prove the following result.

Corollary 17. *Theorem 3.1 in [8] is not valid if n is odd.*

In Theorem 16, we investigate the case in which n is odd. But we must emphasize that, when n is even, the main results of Dalal et al. [8] are also very weak. To prove it, we show the following example inspired by [23].

Example 18. Let $X = \mathbb{R}$ be the set of all real numbers provided with its usual order $\leq$ and the Euclidean metric $d(x, y) = |x - y|$ for all $x, y \in X$. Let $F : X^4 \to X$ and $g : X \to X$ be the mappings given by

$$\begin{gathered} F(x, y, z, w) = \frac{x - 6y + z - w}{11} \quad \forall x, y, z, w \in X; \\ gx = \frac{10x}{11} \quad \forall x \in X. \end{gathered} \tag{30}$$

It is easy to check that the contractivity condition of Theorem 16 is not satisfied. Indeed, consider $x = a$, $y \leq b$, $z = c$, and $w = t$. Then, we have that

$$\begin{aligned} &d\left(F(x, y, z, w), F(a, b, c, t)\right) \\ &\quad = \left|\frac{x - 6y + z - w}{11} - \frac{a - 6b + c - t}{11}\right| = \frac{6|y - b|}{11}, \\ &\frac{d(ga, gx) + d(gy, gb) + d(gz, gc) + d(gw, gt)}{4} \\ &\quad = \frac{10|y - b|}{44}. \end{aligned} \tag{31}$$

Thus, it is impossible to find φ (as it was defined in [8]) such that

$$\begin{aligned} &d\left(F(x, y, z, w), F(a, b, c, d)\right) \\ &\quad \leq \varphi\left(\frac{d(ga, gx) + d(gy, gb) + d(gz, gc) + d(gw, gd)}{4}\right). \end{aligned} \tag{32}$$

However, it is clear that $(0, 0, 0, 0)$ is the only common n-tuple fixed point of F and g.

3. Corrected Versions of the Mentioned Theorems

For the sake of completeness and to conclude this paper, in this section, we state a corrected version of Theorem 3.1 in [8], which immediately leads to a corrected version of Theorem 13 in [9]. For this purpose, we recollect here some notations, definitions, and results from the literature (that can also be found in [10, 14–16]).

First at all, instead of Definitions 9 and 11, we recall here the concept of *multidimensional fixed/coincidence point* introduced by Roldán et al. in [13] (see also [14–16]), which is an extension of Berzig and Samet's notion given in [12].

Throughout this section, fix $n \in \mathbb{N}$ such that $n \geq 2$ and let $F : X^n \to X$ and $g : X \to X$ be two mappings. Fix a nontrivial partition $\{\mathsf{A}, \mathsf{B}\}$ of $\Lambda_n = \{1, 2, \ldots, n\}$; that is, A and B are nonempty subsets of Λ_n such that $\mathsf{A} \cup \mathsf{B} = \Lambda_n$ and $\mathsf{A} \cap \mathsf{B} = \emptyset$. We will denote

$$\begin{aligned} \Omega_{\mathsf{A},\mathsf{B}} &= \left\{\sigma : \Lambda_n \longrightarrow \Lambda_n : \sigma(\mathsf{A}) \subseteq \mathsf{A}, \sigma(\mathsf{B}) \subseteq \mathsf{B}\right\}, \\ \Omega'_{\mathsf{A},\mathsf{B}} &= \left\{\sigma : \Lambda_n \longrightarrow \Lambda_n : \sigma(\mathsf{A}) \subseteq \mathsf{B}, \sigma(\mathsf{B}) \subseteq \mathsf{A}\right\}. \end{aligned} \tag{33}$$

Henceforth, let $\sigma_1, \sigma_2, \ldots, \sigma_n : \Lambda_n \to \Lambda_n$ be n mappings from Λ_n into itself and let Υ be the n-tuple $(\sigma_1, \sigma_2, \ldots, \sigma_n)$.

Definition 19 (Roldán et al. [13, 16]). A point $(x_1, x_2, \ldots, x_n) \in X^n$ is called a *Υ-coincidence point of the mappings* $F : X^n \to X$ and $g : X \to X$ if

$$F\left(x_{\sigma_i(1)}, x_{\sigma_i(2)}, \ldots, x_{\sigma_i(n)}\right) = gx_i \quad \forall i \in \{1, 2, \ldots, n\}. \quad (34)$$

If g is the identity mapping on X, then $(x_1, x_2, \ldots, x_n) \in X^n$ is called a *Υ-fixed point of the mapping F*.

It is clear that the previous definition extends the notions of coupled, tripled, and quadruple fixed/coincidence points. In fact, if we represent a mapping $\sigma : \Lambda_n \to \Lambda_n$ throughout its ordered image, that is, $\sigma = (\sigma(1), \sigma(2), \ldots, \sigma(n))$, then

(i) Gnana-Bhaskar and Lakshmikantham's coupled fixed points occur when $n = 2$, $\sigma_1 = (1, 2)$, and $\sigma_2 = (2, 1)$;

(ii) Berinde and Borcut's tripled fixed points are associated with $n = 3$, $\sigma_1 = (1, 2, 3)$, $\sigma_2 = (2, 1, 2)$, and $\sigma_3 = (3, 2, 1)$;

(iii) Karapınar's quadruple fixed points are considered when $n = 4$, $\sigma_1 = (1, 2, 3, 4)$, $\sigma_2 = (2, 3, 4, 1)$, $\sigma_3 = (3, 4, 1, 2)$, and $\sigma_4 = (4, 1, 2, 3)$;

(iv) Berzig and Samet's multidimensional fixed points are given when $\mathsf{A} = \{1, 2, \ldots, m\}$ and $\mathsf{B} = \{m + 1, m + 2, \ldots, n\}$.

For more details see [13].

A partial order $\preceq$ on X can be extended to a partial order $\sqsubseteq$ on X^n in the following way. If $(X, \preceq)$ is a partially ordered space, $x, y \in X$ and $i \in \Lambda_n$, we will use the following notation:

$$x \preceq_i y \Longleftrightarrow \begin{cases} x \preceq y, & \text{if } i \in \mathsf{A}, \\ x \succeq y, & \text{if } i \in \mathsf{B}. \end{cases} \quad (35)$$

Consider on the product space X^n the following partial order: for $\mathsf{X} = (x_1, x_2, \ldots, x_n), \mathsf{Y} = (y_1, y_2, \ldots, y_n) \in X^n$,

$$\mathsf{X} \sqsubseteq \mathsf{Y} \Longleftrightarrow x_i \preceq_i y_i, \quad \forall i. \quad (36)$$

We say that two points X and Y are *comparable* if $\mathsf{X} \sqsubseteq \mathsf{Y}$ or $\mathsf{X} \sqsupseteq \mathsf{Y}$.

Using this partial order, the mixed g-monotone property is as follows.

Definition 20 (see [13]). Let $(X, \preceq)$ be a partially ordered space. We say that F has the *mixed $(g, \preceq)$-monotone property* (*with respect to* $\{\mathsf{A}, \mathsf{B}\}$) if F is g-monotone nondecreasing in arguments of A and g-monotone nonincreasing in arguments of B; that is, for all $x_1, x_2, \ldots, x_n, y, z \in X$ and all i,

$$\begin{aligned} & gy \preceq gz \\ & \Longrightarrow \begin{cases} F\left(x_1, \ldots, x_{i-1}, y, x_{i+1}, \ldots, x_n\right) \\ \quad \preceq F\left(x_1, \ldots, x_{i-1}, z, x_{i+1}, \ldots, x_n\right), & \text{if } i \in \mathsf{A}, \\ F\left(x_1, \ldots, x_{i-1}, y, x_{i+1}, \ldots, x_n\right) \\ \quad \succeq F\left(x_1, \ldots, x_{i-1}, z, x_{i+1}, \ldots, x_n\right), & \text{if } i \in \mathsf{B}. \end{cases} \end{aligned} \quad (37)$$

Remark 21 (see [10]). In order to ensure the existence of Υ-coincidence/fixed points, it is very important to assume that the mixed g-monotone property is compatible with the permutation of the variables; that is, the mappings of $\Upsilon = (\sigma_1, \sigma_2, \ldots, \sigma_n)$ should verify

$$\sigma_i \in \Omega_{\mathsf{A},\mathsf{B}} \quad \text{if } i \in \mathsf{A}, \qquad \sigma_i \in \Omega'_{\mathsf{A},\mathsf{B}} \quad \text{if } i \in \mathsf{B}. \quad (38)$$

Remark 22 (see [10]). Notice that, in fact, when n is even, Definitions 11 and 12 can be seen as particular cases of the previous definitions when A is the set of all odd numbers and B is the family of all even numbers in $\{1, 2, \ldots, n\}$ and the mappings $\sigma_1, \sigma_2, \ldots, \sigma_n$ are appropriate permutations of the variables.

The following definitions are usual in the field of fixed point theory.

Definition 23. An *ordered metric space* $(X, d, \preceq)$ is a metric space (X, d) provided with a partial order $\preceq$.

Definition 24 (see [2]). An ordered metric space $(X, d, \preceq)$ is said to be *nondecreasing-regular* (resp., *nonincreasing-regular*) if we have that $x_m \preceq x$ (resp., $x_m \succeq x$) for all $m \in \mathbb{N}$ when $\{x_m\} \subseteq X$ is any sequence verifying $\{x_m\} \to x$ and $x_m \preceq x_{m+1}$ (resp., $x_m \succeq x_{m+1}$) for all $m \in \mathbb{N}$. And $(X, d, \preceq)$ is said to be *regular* if it is both nondecreasing-regular and nonincreasing-regular.

Definition 25. Let $(X, \preceq)$ be a partially ordered set and let $T, g : X \to X$ be two mappings. We will say that T is *monotone $(g, \preceq)$-nondecreasing* if $Tx \preceq Ty$ for all $x, y \in X$ such that $gx \preceq gy$.

Remark 26. If T is $(g, \preceq)$-*nondecreasing* and $gx = gy$, then $Tx = Ty$. It follows from

$$\begin{aligned} gx = gy & \Longrightarrow \begin{Bmatrix} gx \preceq gy \\ gy \preceq gx \end{Bmatrix} \\ & \Longrightarrow \begin{Bmatrix} Tx \preceq Ty \\ Ty \preceq Tx \end{Bmatrix} \Longrightarrow Tx = Ty. \end{aligned} \quad (39)$$

Lemma 27 (see [16]). *Let (X, d) be a metric space and define $\Delta_n : X^n \times X^n \to [0, \infty)$, for all $A = (a_1, a_2, \ldots, a_n), B = (b_1, b_2, \ldots, b_n) \in X^n$, by*

$$\Delta_n(A, B) = \frac{1}{n} \sum_{i=1}^{n} d\left(a_i, b_i\right). \quad (40)$$

Then Δ_n is metric on X^n. And d is complete if, and only if, Δ_n is complete.

Lemma 28 (see [16]). *Let $(X, d, \preceq)$ be an ordered metric space and let $F : X^n \to X$ and $g : X \to X$ be two mappings. Let $\Upsilon = (\sigma_1, \sigma_2, \ldots, \sigma_n)$ be an n-tuple of mappings from $\{1, 2, \ldots, n\}$ into itself verifying $\sigma_i \in \Omega_{\mathsf{A},\mathsf{B}}$ if $i \in \mathsf{A}$ and*

$\sigma_i \in \Omega'_{\mathsf{A},\mathsf{B}}$ *if* $i \in \mathsf{B}$. *Define* $F_\Upsilon, G : X^n \to X^n$, *for all* $x_1, x_2, \ldots, x_n \in X$, *by*

$$\begin{aligned} F_\Upsilon(x_1, x_2, \ldots, x_n) = \big(& F\left(x_{\sigma_1(1)}, x_{\sigma_1(2)}, \ldots, x_{\sigma_1(n)}\right), \\ & F\left(x_{\sigma_2(1)}, x_{\sigma_2(2)}, \ldots, x_{\sigma_2(n)}\right), \ldots, \\ & F\left(x_{\sigma_n(1)}, x_{\sigma_n(2)}, \ldots, x_{\sigma_n(n)}\right)\big); \\ G(x_1, x_2, \ldots, x_n) = (& gx_1, gx_2, \ldots, gx_n). \end{aligned} \tag{41}$$

(1) *If F has the mixed $(g, \preceq)$-monotone property, then F_Υ is monotone $(G, \sqsubseteq)$-nondecreasing.*

(2) *If F is Δ_n-continuous, then F_Υ is also Δ_n-continuous.*

(3) *If g is d-continuous, then G is Δ_n-continuous.*

(4) *A point $(x_1, x_2, \ldots, x_n) \in X^n$ is a Υ-fixed point of F if, and only if, $(x_1, x_2, \ldots, x_n)$ is a fixed point of F_Υ.*

(5) *A point $(x_1, x_2, \ldots, x_n) \in X^n$ is a Υ-coincidence point of F and g if, and only if, $(x_1, x_2, \ldots, x_n)$ is a coincidence point of F_Υ and G.*

(6) *If $(X, d, \preceq)$ is regular, then $(X^n, \Delta^n, \sqsubseteq)$ is also regular.*

The commutativity and compatibility of the mappings are defined as follows.

Definition 29. We will say that two mappings $T, g : X \to X$ are *commuting* if $gTx = Tgx$ for all $x \in X$. We will say that $F : X^n \to X$ and $g : X \to X$ are *commuting* if $gF(x_1, x_2, \ldots, x_n) = F(gx_1, gx_2, \ldots, gx_n)$ for all $x_1, \ldots, x_n \in X$.

The following notion was introduced in order to avoid the necessity of commutativity.

Definition 30 (see [24–26]). Let $(X, d, \preceq)$ be an ordered metric space. Two mappings $T, g : X \to X$ are said to be *O-compatible* if

$$\lim_{m\to\infty} d(gTx_m, Tgx_m) = 0 \tag{42}$$

provided that $\{x_m\}$ is a sequence in X such that $\{gx_m\}$ is $\preceq$-monotone and

$$\lim_{m\to\infty} Tx_m = \lim_{m\to\infty} gx_m \in X. \tag{43}$$

The natural extension to an arbitrary number of variables is the following one.

Definition 31. Let $(X, d, \preceq)$ be an ordered metric space and let $F : X^n \to X$ and $g : X \to X$ be two mappings. Let $\Upsilon = (\sigma_1, \sigma_2, \ldots, \sigma_n)$ be an n-tuple of mappings from $\{1, 2, \ldots, n\}$ into itself verifying $\sigma_i \in \Omega_{\mathsf{A},\mathsf{B}}$ if $i \in \mathsf{A}$ and $\sigma_i \in \Omega'_{\mathsf{A},\mathsf{B}}$ if $i \in \mathsf{B}$. We will say that (F, g) is a (O, Υ)-*compatible pair* if

$$\begin{aligned} \lim_{m\to\infty} d\Big(& gF\left(x_m^{\sigma_i(1)}, x_m^{\sigma_i(2)}, x_m^{\sigma_i(3)}, \ldots, x_m^{\sigma_i(n)}\right), \\ & F\left(gx_m^{\sigma_i(1)}, gx_m^{\sigma_i(2)}, gx_m^{\sigma_i(3)}, \ldots, gx_m^{\sigma_i(n)}\right)\Big) = 0 \\ & \forall i \in \{1, 2, \ldots, n\} \end{aligned} \tag{44}$$

whenever $\{x_m^1\}, \{x_m^2\}, \ldots, \{x_m^n\}$ are sequences in X such that $\{gx_m^1\}, \{gx_m^2\}, \ldots, \{gx_m^n\}$ are $\preceq$-monotone and

$$\lim_{m\to\infty} F\left(x_m^{\sigma_i(1)}, x_m^{\sigma_i(2)}, x_m^{\sigma_i(3)}, \ldots, x_m^{\sigma_i(n)}\right) = \lim_{m\to\infty} gx_m^i \in X \quad \forall i \in \{1, 2, \ldots, n\}. \tag{45}$$

Notice that the previous definition is different from Definition 14 because we impose that the sequences $\{gx_m^1\}, \{gx_m^2\}, \ldots, \{gx_m^n\}$ are $\preceq$-monotone.

Lemma 32. *If F and g are (O, Υ)-compatible, then F_Υ and G are O-compatible.*

Inspired by Boyd and Wong's theorem [27], Mukherjea [28] introduced the following kind of control functions:

$$\Psi = \left\{\varphi : [0, \infty) \longrightarrow [0, \infty) : \varphi(t) < t, \ \lim_{r\to t^+} \varphi(r) < t \text{ for each } t > 0\right\}. \tag{46}$$

The following property is well-known.

Lemma 33. *Let $\varphi \in \Psi$ and let $\{a_m\} \subset [0, \infty)$ be a sequence. If $a_{m+1} \le \varphi(a_m)$ and $a_m \ne 0$ for all m, then $\{a_m\} \to 0$.*

Using this kind of control functions, we present the following theorem.

Theorem 34. *Let $(X, d, \preceq)$ be an ordered metric space and let $T, g : X \to X$ be two mappings such that the following properties are fulfilled;*

(i) *$T(X) \subseteq g(X)$;*

(ii) *T is monotone $(g, \preceq)$-nondecreasing;*

(iii) *there exists $x_0 \in X$ such that $gx_0 \preceq Tx_0$;*

(iv) *there exists $\varphi \in \Psi$ verifying*

$$d(Tx, Ty) \le \varphi(d(gx, gy)) \quad \forall x, y \in X \ \text{ such that } gx \preceq gy. \tag{47}$$

Also assume that, at least, one of the following conditions holds:

(a) *(X, d) is complete, T and g are continuous, and the pair (T, g) is O-compatible;*

(b) *(X, d) is complete and T and g are continuous and commuting;*

(c) *$(g(X), d)$ is complete and $(X, d, \preceq)$ is nondecreasing-regular;*

(d) *(X, d) is complete, $g(X)$ is closed, and $(X, d, \preceq)$ is nondecreasing-regular;*

(e) *(X, d) is complete, g is continuous and monotone $\preceq$-nondecreasing, the pair (T, g) is O-compatible, and $(X, d, \preceq)$ is nondecreasing-regular.*

Then T and g have, at least, a coincidence point.

Proof. We divide the proof into four steps.

Step 1. We claim that there exists a sequence $\{x_m\} \subseteq X$ such that $\{gx_m\}$ is $\preceq$-nondecreasing and $gx_{m+1} = Tx_m$ for all $m \geq 0$. Starting from $x_0 \in X$ given in (iii) and taking into account that $Tx_0 \in T(X) \subseteq g(X)$, there exists $x_1 \in X$ such that $Tx_0 = gx_1$. Then $gx_0 \preceq Tx_0 = gx_1$. Since T is $(g, \preceq)$-nondecreasing, $Tx_0 \preceq Tx_1$. Now $Tx_1 \in T(X) \subseteq g(X)$, so there exists $x_2 \in X$ such that $Tx_1 = gx_2$. Then $gx_1 = Tx_0 \preceq Tx_1 = gx_2$. Since T is $(g, \preceq)$-nondecreasing, $Tx_1 \preceq Tx_2$. Repeating this argument, there exists a sequence $\{x_m\}_{m\geq 0}$ such that

$$\begin{gathered} \{gx_m\} \text{ is } \preceq\text{-nondecreasing}, \\ gx_{m+1} = Tx_m \preceq Tx_{m+1} = gx_{m+2} \quad \forall m \geq 0. \end{gathered} \tag{48}$$

Now, let us define $a_m = d(gx_{m+1}, gx_{m+2})$ for all $m \geq 0$.

Step 2. We claim that $a_{m+1} \leq \phi(a_m)$ for all $m \geq 0$. Since $gx_{m+1} \preceq gx_{m+2}$ for all $m \geq 0$, it follows from (iv) that

$$\begin{aligned} a_{m+1} &= d(gx_{m+2}, y_{m+3}) = d(Tx_{m+1}, Tx_{m+2}) \\ &\leq \phi(d(gx_{m+1}, gx_{m+2})) = \phi(a_m). \end{aligned} \tag{49}$$

Step 3. We claim that $\{d(gx_m, gx_{m+1})\} \to 0$. We consider two possibilities.

(i) Suppose that there is $m_0 \in \mathbb{N}$ such that $a_{m_0} = 0$. Then $d(gx_{m_0+1}, gx_{m_0+2}) = a_{m_0} = 0$. Remark 26 guarantees that $a_{m_0+1} = d(gx_{m_0+2}, gx_{m_0+3}) = d(Tx_{m_0+1}, Tx_{m_0+2}) = 0$. By induction, the same reasoning proves that if there is $m_0 \in \mathbb{N}$ such that $a_{m_0} = 0$, then $a_m = 0$ for all $m \geq m_0$ and, in this case, it is clear that $\{a_m\} \to 0$.

(ii) Suppose that $a_m \neq 0$ for all m. In this case, $\{a_m\} \to 0$ by Lemma 33.

Step 4. We claim that $\{gx_m\}$ is a Cauchy sequence. Let us show that $\{gx_m\}$ is Cauchy reasoning by contradiction. Suppose that $\{gx_m\}$ is not Cauchy. Then there exist $\varepsilon_0 > 0$ and partial subsequences $\{gx_{n(k)}\}$ and $\{gx_{m(k)}\}$ verifying $k < n(k) < m(k) < n(k+1)$, $d(gx_{n(k)}, gx_{m(k)}) > \varepsilon_0$, and $d(gx_{n(k)}, gx_{m(k)-1}) \leq \varepsilon_0$ for all $k \geq 1$ ($m(k)$ is the least integer number, greater that $n(k)$, such that $d(gx_{n(k)}, gx_{m(k)}) > \varepsilon_0$). Since $n(k) \leq m(k) - 1 < m(k)$, we have $gx_{n(k)} \preceq gx_{m(k)-1} \preceq gx_{m(k)}$. By (e),

$$\begin{aligned} \varepsilon_0 &< d(gx_{n(k)}, gx_{m(k)}) \\ &\leq d(gx_{n(k)}, gx_{m(k)-1}) + d(gx_{m(k)-1}, gx_{m(k)}) \\ &\leq \varepsilon_0 + d(gx_{m(k)-1}, gx_{m(k)}), \end{aligned} \tag{50}$$

and using Step 3,

$$\begin{gathered} \lim_{k\to\infty} d(gx_{n(k)}, gx_{m(k)}) = \varepsilon_0, \\ \varepsilon_0 < d(gx_{n(k)}, gx_{m(k)}) \quad \forall k. \end{gathered} \tag{51}$$

Using the contractivity condition (iv),

$$\begin{aligned} &d(gx_{n(k)+1}, gx_{m(k)+1}) \\ &\quad = d(Tx_{n(k)}, Tx_{m(k)}) \leq \phi(d(gx_{n(k)}, gx_{m(k)})) \quad \forall k. \end{aligned} \tag{52}$$

Moreover

$$\begin{aligned} &d(gx_{n(k)}, gx_{m(k)}) \\ &\quad \leq d(gx_{n(k)}, gx_{n(k)+1}) + d(gx_{n(k)+1}, gx_{m(k)}) \\ &\quad \leq d(gx_{n(k)}, gx_{n(k)+1}) + d(gx_{n(k)+1}, gx_{m(k)+1}) \\ &\qquad + d(gx_{m(k)}, gx_{m(k)+1}) \\ &\quad \leq d(gx_{n(k)}, gx_{n(k)+1}) + \phi(d(gx_{n(k)}, gx_{m(k)})) \\ &\qquad + d(gx_{m(k)}, gx_{m(k)+1}). \end{aligned} \tag{53}$$

Taking limit as $k \to \infty$ in (53) and using $\phi \in \Psi$, Step 3, and (51), we get the contradiction

$$\begin{aligned} \varepsilon_0 &= \lim_{k\to\infty} d(gx_{n(k)}, gx_{m(k)}) \\ &\leq \lim_{k\to\infty} (d(gx_{n(k)}, gx_{n(k)+1}) + \phi(d(gx_{n(k)}, gx_{m(k)})) \\ &\qquad + d(gx_{m(k)}, gx_{m(k)+1})) \\ &= 0 + \lim_{t\to\varepsilon_0^+} \phi(t) + 0 < \varepsilon_0. \end{aligned} \tag{54}$$

This contradiction proves that, in any case, $\{gx_m\}$ is a Cauchy sequence. Now, we prove that T and g have a coincidence point distinguishing between cases (a)–(e).

Case (a). (X, d) is complete, T and g are continuous, and the pair (T, g) is O-compatible. As (X, d) is complete, there exists $z \in X$ such that $\{gx_m\} \to z$. Since $Tx_m = gx_{m+1}$ for all m, we also have that $\{Tx_m\} \to z$. As T and g are continuous, then $\{Tgx_m\} \to Tz$ and $\{ggx_m\} \to gz$. Taking into account that the pair (T, g) is O-compatible, we deduce that $\lim_{m\to\infty} d(gTx_m, Tgx_m) = 0$. In such a case, we conclude that

$$\begin{aligned} d(gz, Tz) &= \lim_{m\to\infty} d(ggx_{m+1}, Tgx_m) = 0 \\ &= \lim_{m\to\infty} d(gTx_m, Tgx_m) = 0; \end{aligned} \tag{55}$$

that is, z is a coincidence point of T and g.

Case (b). (X, d) is complete and T and g are continuous and commuting. It is obvious because (b) implies (a).

Case (c). $(g(X), d)$ is complete and $(X, d, \preceq)$ is nondecreasing-regular. As $\{gx_m\}$ is a Cauchy sequence in the complete space $(g(X), d)$, there is $y \in g(X)$ such that $\{gx_m\} \to y$. Let $z \in X$ be any point such that $y = gz$. In this case, $\{gx_m\} \to gz$. We are also going to show that $\{gx_m\} \to Tz$, so we will conclude that $gz = Tz$ (and z is a coincidence point of T and g).

Indeed, as $(X, d, \preceq)$ is regular and $\{gx_m\}$ is $\preceq$-nondecreasing and converging to gz, we deduce that $gx_m \preceq gz$ for all $m \geq 0$. Applying the contractivity condition (iv),

$$d(gx_{m+1}, Tz) = d(Tx_m, Tz) \leq \varphi(d(gx_m, gz)) \quad \forall m \geq 0. \tag{56}$$

We are going to show that

$$d(gx_{m+1}, Tz) \leq d(gx_m, gz) \quad \forall m \geq 1. \tag{57}$$

(i) If $d(gx_m, gz) \neq 0$, then $d(gx_{m+1}, Tz) \leq \phi(d(gx_m, gz)) < d(gx_m, gz)$ because $\phi \in \Psi$.

(ii) Suppose that there is some $m_0 \in \mathbb{N}$ such that $d(gx_{m_0}, gz) = 0$. Remark 26 guarantees that $d(Tx_{m_0}, Tz) = 0$. This proves that if there is some $m_0 \in \mathbb{N}$ such that $d(gx_{m_0}, gz) = 0$, then $d(Tx_{m_0}, Tz) = 0$, so (57) also holds.

In any case, (57) holds and this implies that $\{gx_m\}$ converges to Tz. This completes this case.

Case (d). (X, d) is complete, $g(X)$ is closed, and $(X, d, \preceq)$ is nondecreasing-regular. It follows from the fact that a closed subset of a complete metric space is also complete. Then, $(g(X), d)$ is complete and *Case (c)* is applicable.

Case (e). (X, d) is complete, g is continuous and monotone $\preceq$-nondecreasing, the pair (T, g) is O-compatible, and $(X, d, \preceq)$ is nondecreasing-regular. As (X, d) is complete, there exists $z \in X$ such that $\{gx_m\} \to z$. As $Tx_m = gx_{m+1}$ for all m, we also have that $\{Tx_m\} \to z$. As g is continuous, $\{ggx_m\} \to gz$. Furthermore, as the pair (T, g) is O-compatible, then

$$\lim_{m \to \infty} d(ggx_{m+1}, Tgx_m) = \lim_{m \to \infty} d(gTx_m, Tgx_m) = 0. \tag{58}$$

As $\{ggx_m\} \to gz$, the previous property means that $\{Tgx_m\} \to gz$. We are going to show that $\{Tgx_m\} \to Tz$ and this finishes the proof.

Indeed, since $\{gx_m\}$ is $\preceq$-nondecreasing, converges to z, and $(X, d, \preceq)$ is nondecreasing-regular, we have that $gx_m \preceq z$ for all $m \geq 0$. Moreover, as g is monotone $\preceq$-nondecreasing, we deduce that $ggx_m \preceq gz$ for all $m \geq 0$. Applying the contractivity condition (iv),

$$d(Tgx_m, Tz) \leq \varphi(d(ggx_m, gz)) \quad \forall m \geq 0. \tag{59}$$

We claim that

$$d(Tgx_m, Tz) \leq d(ggx_m, gz) \quad \forall m \geq 1. \tag{60}$$

(i) If $d(ggx_m, gz) \neq 0$, then $d(Tgx_m, Tz) \leq \phi(d(ggx_m, gz)) < d(ggx_m, gz)$ because $\phi \in \Psi$.

(ii) Suppose that there is some $m_0 \in \mathbb{N}$ such that $d(ggx_{m_0}, gz) = 0$. Remark 26 guarantees that $d(Tgx_{m_0}, Tz) = 0$. This proves that if there is some $m_0 \in \mathbb{N}$ such that $d(ggx_{m_0}, gz) = 0$, then $d(Tgx_{m_0}, Tz) = 0$, so (60) also holds.

In any case, (60) holds and this implies that $\{Tgx_m\}$ converges to Tz. This completes the proof. □

Inspired by Berinde's approach [23], we deduce the following result which removes the weakness of Theorem 3.1 in [8].

Corollary 35. *Let $(X, d, \preceq)$ be an ordered metric space, let $F : X^n \to X$ and $g : X \to X$ be two mappings, and let $\Upsilon = (\sigma_1, \sigma_2, \ldots, \sigma_n)$ be an n-tuple of mappings from $\{1, 2, \ldots, n\}$ into itself verifying $\sigma_i \in \Omega_{\mathsf{A},\mathsf{B}}$ if $i \in \mathsf{A}$ and $\sigma_i \in \Omega'_{\mathsf{A},\mathsf{B}}$ if $i \in \mathsf{B}$. Suppose that the following properties are fulfilled:*

(i) *$F(X^n) \subseteq g(X)$;*

(ii) *F has the mixed g-monotone property;*

(iii) *there exists $x_0^1, x_0^2, \ldots, x_0^n \in X$ such that $gx_0 \preceq_i F(x_0^{\sigma_i(1)}, x_0^{\sigma_i(2)}, \ldots, x_0^{\sigma_i(n)})$ for all $i \in \{1, 2, \ldots, n\}$;*

(iv) *there exists $\varphi \in \Psi$ verifying*

$$\begin{aligned} &\frac{1}{n}\sum_{i=1}^{n} d\left(F\left(x_{\sigma_i(1)}, x_{\sigma_i(2)}, \ldots, x_{\sigma_i(n)}\right),\right. \\ &\qquad \left.F\left(y_{\sigma_i(1)}, y_{\sigma_i(2)}, \ldots, y_{\sigma_i(n)}\right)\right) \\ &\quad \leq \varphi\left(\frac{1}{n}\sum_{i=1}^{n} d(gx_i, gy_i)\right) \end{aligned} \tag{61}$$

for all $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n \in X$ such that $gx_i \preceq_i gy_i$ for all $i \in \{1, 2, \ldots, n\}$.

Also assume that at least one of the following conditions holds;

(a) *(X, d) is complete, F and g are continuous, and the pair (F, g) is (O, Υ)-compatible;*

(b) *(X, d) is complete and F and g are continuous and commuting;*

(c) *$(g(X), d)$ is complete and $(X, d, \preceq)$ is regular;*

(d) *(X, d) is complete, $g(X)$ is closed, and $(X, d, \preceq)$ is regular;*

(e) *(X, d) is complete, g is continuous and monotone $\preceq$-nondecreasing, the pair (F, g) is (O, Υ)-compatible, and $(X, d, \preceq)$ is regular.*

Then F and g have, at least, a Υ-coincidence point.

Proof. Notice that the contractivity condition (61) means that

$$\Delta_n(F_\Upsilon \mathsf{X}, F_\Upsilon \mathsf{Y}) \leq \varphi(\Delta_n(\mathsf{X}, \mathsf{Y})) \tag{62}$$

for all $\mathsf{X} = (x_1, x_2, \ldots, x_n), \mathsf{Y} = (y_1, y_2, \ldots, y_n) \in X^n$ such that $G\mathsf{X} \sqsubseteq G\mathsf{Y}$. Therefore, it is only necessary to apply Theorem 34 to the mappings $F_\Upsilon, G : X^n \to X^n$ defined in Lemma 28. □

We now reconsider Example 18.

Example 36. Let $X = \mathbb{R}$ be the set of all real numbers provided with its usual order $\leq$ and the Euclidean metric $d(x, y) = |x-y|$ for all $x, y \in X$. Let $F : X^4 \to X$ and $g : X \to X$ be given by

$$F(x,y,z,w) = \frac{x-6y+z-w}{11} \quad \forall x,y,z,w \in X; \qquad gx = \frac{10x}{11} \quad \forall x \in X. \tag{63}$$

It is easy to check that the contractivity condition of Corollary 35 is satisfied successfully. Indeed, we have that

$$\begin{aligned}
&= \frac{1}{4}\sum_{i=1}^{4} d\left(F\left(x_{\sigma_i(1)}, x_{\sigma_i(2)}, x_{\sigma_i(3)}, x_{\sigma_i(4)}\right),\right. \\
&\qquad \left. F\left(y_{\sigma_i(1)}, y_{\sigma_i(2)}, y_{\sigma_i(3)}\right), y_{\sigma_i(4)}\right) \\
&\leq \frac{1}{4}\left[\frac{9}{11}|x-a| + \frac{9}{11}|y-b| \right. \\
&\qquad \left. + \frac{9}{11}|z-c| + \frac{9}{11}|w-d|\right] \\
&= \frac{9}{44}\left(|x-a| + |y-b| + |z-c| + |w-d|\right), \\
&\frac{d(ga,gx) + d(gy,gb) + d(gz,gc) + d(gw,gt)}{4} \\
&= \frac{10}{44}\left(|x-a| + |y-b| + |z-c| + |w-d|\right).
\end{aligned} \tag{64}$$

Thus, it is sufficient to take $\varphi(t) = 19/20$ (as it was defined in [8]) such that the contractive condition in Corollary 35 is satisfied.

Notice that $(0,0,0,0)$ is the only common n-tuple fixed point of F and g and

$$\frac{d(a,x) + d(y,b) + d(z,c) + d(w,t)}{4} = \frac{|y-b|}{4}. \tag{65}$$

Thus, it is impossible to find φ (as it was defined in [8]) such that

$$\begin{aligned}
&d\left(F(x,y,z,w), F(a,b,c,d)\right) \\
&\quad \leq \varphi\left(\frac{d(a,x) + d(y,b) + d(z,c) + d(w,d)}{4}\right).
\end{aligned} \tag{66}$$

However, it is clear that $(0,0,0,0)$ is the only common n-tuple fixed point of F and g.

4. Consequences

In this section, we can list some of the consequences of our main result (Theorem 34).

Corollary 37 (Ran and Reurings [29]). *Let $(X, \preccurlyeq)$ be an ordered set endowed with a metric d and $T : X \to X$ be a given mapping. Suppose that the following conditions hold:*

(a) *(X, d) is complete,*

(b) *T is nondecreasing (with respect to $\preccurlyeq$),*

(c) *T is continuous,*

(d) *there exists $x_0 \in X$ such that $x_0 \preccurlyeq Tx_0$,*

(e) *there exists a constant $k \in (0,1)$ such that $d(Tx, Ty) \leq kd(x, y)$ for all $x, y \in X$ with $x \succcurlyeq y$.*

Then T has a fixed point. Moreover, if for all $(x, y) \in X^2$ there exists $z \in X$ such that $x \preccurlyeq z$ and $y \preccurlyeq z$, one obtains uniqueness of the fixed point.

Nieto and Rodríguez-López [30] slightly modified the hypothesis of the previous result obtaining the following theorem.

Corollary 38 (Nieto and Rodríguez-López [30]). *Let $(X, \preccurlyeq)$ be an ordered set endowed with a metric d and $T : X \to X$ be a given mapping. Suppose that the following conditions hold:*

(a) *(X, d) is complete,*

(b) *T is nondecreasing (with respect to $\preccurlyeq$),*

(c) *if a nondecreasing sequence $\{x_m\}$ in X converges to some point $x \in X$, then $x_m \preccurlyeq x$ for all m,*

(d) *there exists $x_0 \in X$ such that $x_0 \preccurlyeq Tx_0$,*

(e) *there exists a constant $k \in (0,1)$ such that $d(Tx, Ty) \leq kd(x, y)$ for all $x, y \in X$ with $x \succcurlyeq y$.*

Then T has a fixed point. Moreover, if for all $(x, y) \in X^2$ there exists $z \in X$ such that $x \preccurlyeq z$ and $y \preccurlyeq z$, one obtains uniqueness of the fixed point.

Corollary 39 (Bhaskar and Lakshmikantham [2]). *Let $(X, \preccurlyeq)$ be a partially ordered set endowed with a metric d. Let $F : X \times X \to X$ be a given mapping. Suppose that the following conditions hold:*

(i) *(X, d) is complete;*

(ii) *F has the mixed monotone property;*

(iii) *F is continuous or X has the following properties:*

  (X_1) *if a nondecreasing sequence $\{x_n\}$ in X converges to some point $x \in X$, then $x_n \preccurlyeq x$ for all n,*

  (X_2) *if a decreasing sequence $\{y_n\}$ in X converges to some point $y \in X$, then $y_n \succcurlyeq y$ for all n;*

(iv) *there exists $x_0, y_0 \in X$ such that $x_0 \preccurlyeq F(x_0, y_0)$ and $y_0 \succcurlyeq F(y_0, x_0)$;*

(v) *there exists a constant $k \in (0,1)$ such that for all (x, y), $(u, v) \in X \times X$ with $x \succcurlyeq u$ and $y \preccurlyeq v$,*

$$d\left(F(x,y), F(u,v)\right) \leq \frac{k}{2}\left[d(x,u) + d(y,v)\right]. \tag{67}$$

Then F has a coupled fixed point $(x^*, y^*) \in X \times X$. Moreover, if for all $(x, y), (u, v) \in X \times X$ there exists $(z_1, z_2) \in X \times X$ such that $(x, y) \preccurlyeq_2 (z_1, z_2)$ and $(u, v) \preccurlyeq_2 (z_1, z_2)$, one has uniqueness of the coupled fixed point and $x^* = y^*$.

In [31] a version of the following result using a mapping g can be found.

Corollary 40 (Berinde and Borcut [32]). *Let $(X, \preccurlyeq)$ be a partially ordered set and suppose there is a metric d on X such that (X, d) is a complete metric space. Let $F : X \times X \times X \to X$ be a mapping having the mixed g-monotone property. Assume that there exist constants $j, k, \ell \in [0, 1)$ with $j + k + \ell < 1$ such that*

$$d(F(x, y, z), F(u, v, w)) \leq jd(x, u) + kd(y, v) + \ell d(z, w) \tag{68}$$

for all $x, y, z, u, v, w \in X$ with $x \preccurlyeq u$, $y \succcurlyeq v$, $z \preccurlyeq w$. Suppose either F is continuous or $(X, d, \preccurlyeq)$ has the following properties:

(a) *if a nondecreasing sequence $\{x_m\} \to x$, then $x_m \preccurlyeq x$ for all m;*

(b) *if a nondecreasing sequence $\{y_m\} \to y$, then $y_m \preccurlyeq y$ for all m.*

If there exists $x_0, y_0, z_0 \in X$ such that

$$\begin{gathered} x_0 \preccurlyeq F(x_0, y_0, z_0), \qquad y_0 \succcurlyeq F(y_0, x_0, y_0), \\ z_0 \preccurlyeq F(z_0, y_0, x_0), \end{gathered} \tag{69}$$

then there exists $x, y, z \in X$ such that

$$x = F(x, y, z), \qquad y = F(y, x, y), \qquad z = F(z, y, x). \tag{70}$$

A quadruple version was obtained by Karapınar and Luong in [33].

Corollary 41 (Karapınar and Luong [33]). *Let $(X, \preccurlyeq)$ be a partially ordered set and (X, d) be a complete metric space. Let $F : X \times X \times X \times X \to X$ be a mapping having the mixed monotone property. Assume that there exist constants $k \in [0, 1)$ such that*

$$\begin{aligned} & d(F(x, y, z, w), F(u, v, r, t)) \\ & \quad \leq \frac{k}{4}[d(x, u) + d(y, v) + d(z, r) + d(w, t)] \end{aligned} \tag{71}$$

for all $x, y, z, u, v, w \in X$ with $x \succcurlyeq u$, $y \preccurlyeq v$, $z \succcurlyeq r$ and $w \preccurlyeq t$. Suppose that there exists $x_0, y_0, z_0, w_0 \in X$ such that

$$\begin{aligned} x_0 \preccurlyeq F(x_0, y_0, z_0, w_0), \qquad & y_0 \succcurlyeq F(y_0, z_0, w_0, x_0), \\ z_0 \preccurlyeq F(z_0, w_0, x_0, y_0), \qquad & w_0 \succcurlyeq F(w_0, x_0, y_0, z_0). \end{aligned} \tag{72}$$

Suppose that either F is continuous or $(X, d, \preccurlyeq)$ has the following properties:

(a) *if a nondecreasing sequence $\{x_m\} \to x$, then $x_m \preccurlyeq x$ for all m;*

(b) *if a nondecreasing sequence $\{y_m\} \to y$, then $y_m \preccurlyeq y$ for all m.*

Then there exists $x, y, z, w \in X$ such that

$$\begin{aligned} F(x, y, z, w) = x, \qquad & F(y, z, w, x) = y, \\ F(z, w, x, y) = z, \qquad & F(w, x, y, z) = w. \end{aligned} \tag{73}$$

Later, Berzig and Samet extended the previous result to the multidimensional case in the following way.

Corollary 42 (Berzig and Samet [34]). *Let $(X, \preccurlyeq)$ be a partially ordered set and suppose there is a metric d on X such that (X, d) is a complete metric space. For N, m positive integers, $N \geq 2$, $1 \leq m < N$, let $F : X^N \to X$ be a continuous mapping having the m-mixed monotone property. Assume that there exist the constants $\delta_i \in [0, 1)$ with $\sum_{i=1}^N \delta_i < 1$ for which*

$$d(F(U), F(V)) \leq \sum_{i=1}^N \delta_i d(x_i, y_i) \tag{74}$$

for all $U = (x_1, \ldots, x_N), V = (y_1, \ldots, y_N) \in X^N$ such that

$$\begin{gathered} x_1 \preccurlyeq y_1, \ldots, x_m \preccurlyeq y_m, \\ x_{m+1} \succcurlyeq y_{m+1}, \ldots, x_N \succcurlyeq y_N. \end{gathered} \tag{75}$$

If there exists $U^{(0)} = (x_1^{(0)}, \ldots, x_N^{(0)}) \in X^N$ such that

$$\begin{aligned} & x_1^{(0)} \preccurlyeq F(x^{(0)}[\varphi_1(1:m)], x^{(0)}[\psi_1(m+1:N)]), \\ & \vdots \\ & x_m^{(0)} \preccurlyeq F(x^{(0)}[\varphi_m(1:m)], x^{(0)}[\psi_m(m+1:N)]), \\ & x_{m+1}^{(0)} \succcurlyeq F(x^{(0)}[\varphi_{m+1}(1:m)], x^{(0)}[\psi_{m+1}(m+1:N)]), \\ & \vdots \\ & x_N^{(0)} \succcurlyeq F(x^{(0)}[\varphi_N(1:m)], x^{(0)}[\psi_N(m+1:N)]), \end{aligned} \tag{76}$$

where $\varphi_1, \ldots, \varphi_m : \{1, \ldots, m\} \to \{1, \ldots, m\}$, $\psi_1, \ldots, \psi_m : \{m + 1, \ldots, N\} \to \{m + 1, \ldots, N\}$, $\varphi_{m+1}, \ldots, \varphi_N : \{1, \ldots, m\} \to \{m + 1, \ldots, N\}$, and $\psi_{m+1}, \ldots, \psi_N : \{m + 1, \ldots, N\} \to \{1, \ldots, m\}$, then there exists $(x_1, x_2, \ldots, x_N) \in X^N$ satisfying

$$\begin{aligned} & x_1 = F(x[\varphi_1(1:m)], x[\psi_1(m+1:N)]), \\ & \vdots \\ & x_m = F(x[\varphi_m(1:m)], x[\psi_m(m+1:N)]), \\ & x_{m+1} = F(x[\varphi_{m+1}(1:m)], x[\psi_{m+1}(m+1:N)]), \\ & \vdots \\ & x_N = F(x[\varphi_N(1:m)], x[\psi_N(m+1:N)]). \end{aligned} \tag{77}$$

Corollary 43 (Choudhury and Kundu [24], Theorem 3.1). *Let $(X, \preceq)$ be a partially ordered set and let there be a metric*

d on X such that (X,d) is a complete metric space. Let $\varphi : [0,\infty) \to [0,\infty)$ be such that $\varphi(t) < t$ and $\lim_{r\to t^+}\varphi(r) < t$ for each $t > 0$. Let $F : X^n \to X$ and $g : X \to X$ be two mappings such that F has the mixed g-monotone property and satisfies

$$d(F(x,y),F(u,v)) \le \varphi\left(\frac{d(gx,gu)+d(gy,gv)}{2}\right) \quad (78)$$

for all $x,y,u,v \in X$, with $gx \preceq gu$ and $gy \succeq gv$. Let $F(X\times X) \subseteq g(X)$, g be continuous and monotone increasing and F and g be compatible mappings. Also suppose

(a) *F is continuous, or*

(b) *X has the following properties:*

(b.1) *if a nondecreasing sequence $\{x_n\} \to x$, then $x_n \preceq x$ for all $n \ge 0$;*

(b.2) *if a nonincreasing sequence $\{y_n\} \to y$, then $y_n \succeq y$ for all $n \ge 0$.*

If there exists $x_0, y_0 \in X$ such that

$$gx_0 \preceq F(x_0,y_0), \qquad gy_0 \succeq F(y_0,x_0), \quad (79)$$

then there exists $x, y \in X$ such that

$$gx = F(x,y), \qquad gy = F(y,x); \quad (80)$$

that is, F and g have a coincidence point.

In the multidimensional case, we have the following result.

Corollary 44 (Wang [35], Theorem 3.4). *Let $(X,\preceq)$ be a partially ordered set and suppose there is a metric d on X such that (X,d) is a complete metric space. Let $G : X^n \to X^n$ and $T : X^n \to X^n$ be a G-isotone mapping for which there exists $\phi \in \Psi$ such that for all $Y \in X^n$, $V \in X^n$ with $G(Y) \sqsupseteq G(V)$,*

$$\rho_n(T(Y),T(V)) \le \phi(\rho_n(G(Y),G(V))), \quad (81)$$

where ρ_n is defined for all $Y = (y_1, y_2, \dots, y_n)$, $V = (v_1, v_2, \dots, v_n) \in X^n$ by

$$\rho_n(Y,V) = \frac{1}{n}\left[d(y_1,v_1) + d(y_2,v_2) + \cdots + d(y_n,v_n)\right]. \quad (82)$$

Suppose $T(X^n) \subseteq G(X^n)$ and also suppose either

(a) *T is continuous, G is continuous and commutes with T, or*

(b) *$(X,d,\preceq)$ is regular and $G(X^n)$ is closed.*

If there exists $Y_0 \in X^n$ such that $G(Y_0)$ and $T(Y_0)$ are $\sqsubseteq$-comparable, then T and G have a coincidence point.

We, finally, note that most of multidimensional fixed point theorems can be reduced to one-dimensional fixed point results. This observation and hence the initial results in this direction were given in [16, 36]. In particular, in [36], the authors proved that the first coupled fixed point result (Theorem 2.1 in [2]) is a consequence of Theorem 2.1 in [37]. On the other hand, in [16], the authors proved that the initial multidimensional fixed point result (Theorem 9 in [13]) can be derived from Theorem 2.1 in [37] either.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Authors' Contribution

All authors contributed equally and significantly in writing this paper. All authors read and approved the final paper.

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