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Research Article

Regularity Criterion for the Nematic Liquid Crystal Flows in Terms of Velocity

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We study the regularity criterion for the 3D nematic liquid crystal flows in the framework of anisotropic Lebesgue space. More precisely, we proved some sufficient conditions in terms of velocity or the fractional derivative of velocity in one direction.

1. Introduction

This paper is devoted to the regularity criterion for the three-dimensional nematic liquid crystal flows:

$$\begin{aligned} u_t + (u \cdot \nabla) u + \nabla p - \nu \Delta u &= -\lambda \nabla \cdot (\nabla d \odot \nabla d), \\ & x \in R^3, \quad t > 0, \\ d_t + (u \cdot \nabla) d = \gamma (\Delta d - f(d)), & \quad x \in R^3, \ t > 0, \\ \nabla \cdot u = 0, & \quad x \in R^3, \ t > 0, \end{aligned} \tag{1}$$

with initial data

$$(u, d)|_{t=0} = (u_0, d_0), \quad x \in R^3, \tag{2}$$

where $u(x,t)$ is the velocity field, $d(x,t)$ represents the macroscopic average of the nematic liquid crystal orientation field, and $p(x,t)$ is the scalar pressure. The symbol $\nabla d \odot \nabla d$ denotes a matrix whose (i,j)th entry is given by $\partial_i d \cdot \partial_j d$ for $1 \le i, j \le 3$; here $f(d) = (1/\varepsilon^2)(|d|^2 - 1)d$. Since the sizes of the viscosity constants $\nu, \lambda, \gamma, \varepsilon$ do not play important roles in our proof, for simplicity, we assume all these positive constants to be one.

The hydrodynamic theory of liquid crystals was established by Ericksen and Leslie [1–4]; the model (1) is a simplified system of Ericksen-Leslie model which was first introduced by Lin in [5], and one of the most significant works is given by Lin and Liu [6]; more precisely, they established global existence for weak solutions and classical solutions. Recently, Liu et al. in [7] established the regularity criterion for (1) as follows:

$$\int_0^T \|\partial_3 u(\tau)\|_{L^\alpha}^\beta d\tau < \infty, \quad \text{with } \frac{2}{\beta} + \frac{3}{\alpha} \le 1, \ \alpha > 3. \tag{3}$$

One may refer to some interesting and important regularity criteria of nematic liquid crystal flows studied by many authors (see, e.g., [8–13] and the references therein). When d is constant, the system (1) becomes the well-known Navier-Stokes equations. The regularity of solutions to the 3D NS equations has been widely investigated during the past fifty years; see, for example, [14–22] and so on. The aim of this paper is to establish a new regularity criterion by providing sufficient condition in terms of velocity or the fractional derivative of velocity in one direction in the framework of anisotropic Lebesgue space.

Throughout the paper, the norm of the Lebesgue spaces $L^p(R^3)$ is denoted by $\|\cdot\|_{L^p}$ and denoted the directional derivatives of a function ϕ by $\partial_i \phi = (\partial \phi / \partial x_i)$ $(i = 1, 2, 3)$, the symbol $\int f(x) dx = \int_{R^3} f(x) dx$, $\Lambda_i = \sqrt{-\partial_i^2}$, $\Lambda = \sqrt{-\Delta}$,

$\|\|\phi\|_{L_i^p}\|_{L_{j,k}^q} = (\int_{R^2}(\int_R |\phi(x)|^p dx_i)^{q/p} dx_j\, dx_k)^{1/q}$, and (i,j,k) belongs to the permutation group $S = \text{span}\{1,2,3\}$. Denote

$$E_1 = \left\{(\gamma,\alpha) \in (2,\infty]^2,\ \frac{1}{\gamma} + \frac{2}{\alpha} < 1\right\},$$
$$E_2 = \left\{\alpha \in (2,\infty],\ \frac{3-\alpha r}{\alpha} < 1,\ \frac{1-\alpha r}{\alpha - 2} < 1\right\}. \quad (4)$$

Theorem 1. *Let $(u_0, d_0) \in H^1(R^3) \times H^2(R^3)$ with the initial data $\text{div}\, u_0 = 0$, and let the pair (u,b) be the weak solution to the liquid crystal flows (1)-(2) on $[0,T)$ for some $0 < T < \infty$. If u satisfies*

$$\int_0^T \left\|\|u(\tau)\|_{L_i^\gamma}\right\|_{L_{j,k}^\alpha}^\beta d\tau < \infty, \ \text{with}\ \frac{2}{\beta} + \frac{2}{\alpha} + \frac{1}{\gamma} \le 1, \quad (\gamma,\alpha) \in E_1, \quad (5)$$

then (u,d) can be extended beyond T.

Theorem 2. *Let $(u_0, d_0) \in H^1(R^3) \times H^2(R^3)$ with the initial data $\text{div}\, u_0 = 0$, and let the pair (u,b) be the weak solution to the liquid crystal flows (1)-(2) on $[0,T)$ for some $0 < T < \infty$. If u satisfies*

$$\int_0^T \|\Lambda_i^r u(\tau)\|_{L^\alpha}^\beta d\tau < \infty, \ \text{with}$$
$$\frac{2}{\beta} + \frac{3}{\alpha} \le \begin{cases} 1 + \frac{1}{\alpha}, & \alpha \in (2,\infty), \quad \text{if } r \in \left(\frac{1}{\alpha}, 1\right], \\ \text{or} & \\ 1 + r, & \alpha \in E_2, \quad \text{if } r \in \left[0, \frac{1}{\alpha}\right), \end{cases} \quad (6)$$

then (u,d) can be extended beyond T.

Corollary 3. *Under the assumption of Theorem 2, if we fix $r = 1$, then the sufficient condition is that*

$$\int_0^T \|\partial_i u(\tau)\|_{L^\alpha}^\beta d\tau < \infty, \quad \text{with}\ \frac{2}{\beta} + \frac{3}{\alpha} \le 1 + \frac{1}{\alpha}, \quad \alpha \in (2,\infty). \quad (7)$$

Remark 4. Comparing with the corresponding results in [7], it is obvious that the conclusion of Corollary 3 is an improvement version of Theorem 1.1 in [7] in some sense.

2. The Proof of Theorems 1 and 2

In this section, we will prove Theorems 1 and 2. For convenience, we first recall the following three-dimensional Sobolev and Ladyzhenskaya inequalities in the whole space (see, e.g., [23–25]).

Lemma 5. *Let $2 \le q \le 6$, $2 \le p < \infty$, and $2 < r, s \le \infty$. There hold that*

$$\|\phi\|_{L^q} \le C\|\phi\|_{L^2}^{(6-q)/2q}\|\partial_1\phi\|_{L^2}^{(q-2)/2q}\|\partial_2\phi\|_{L^2}^{(q-2)/2q}\|\partial_3\phi\|_{L^2}^{(q-2)/2q},$$
$$\left\|\|\phi\|_{L_{i,j}^p}\right\|_{L_k^2} \le C\|\phi\|_{L^2}^{2/p}\|\partial_i\phi\|_{L^2}^{(p-2)/2p}\|\partial_j\phi\|_{L^2}^{(p-2)/2p},$$
$$\left\|\|\phi\|_{L_i^{2r/(r-2)}}\right\|_{L_{j,k}^{2s/(s-2)}} \le C\|\phi\|_{L^2}^{1-(1/r)-(2/s)}\|\partial_i\phi\|_{L^2}^{1/r}\|\partial_j\phi\|_{L^2}^{1/s}\|\partial_k\phi\|_{L^2}^{1/s}. \quad (8)$$

Proof of Theorem 1. Suppose that $[0,T^*)$ is the maximal interval of the existence of the local smooth solution. If $T^* \ge T$, then there is nothing to prove; on the other side, for $T^* < T$, our strategy is to show that

$$\limsup_{t\to T^*}\left(\|\nabla u(\cdot,t)\|_{L^2}^2 + \|\Delta d(\cdot,t)\|_{L^2}^2\right) \le C, \quad (9)$$

under the assumption (5). As a result, the interval $[0,T^*)$ cannot be a maximal interval of existence, which leads to a contradiction.

We multiply $(1)_1$ by u and integrate over R^3 and, similarly, multiply $(1)_2$ by $-\Delta d + f(d)$ and integrate over R^3 and then by adding two results above and using the fact that $\nabla\cdot(\nabla d \odot \nabla d) = \nabla(|\nabla d|^2/2) + \Delta d \nabla d$, we obtain

$$\frac{1}{2}\frac{d}{dt}\int\left(|u|^2 + |\nabla d|^2 + \frac{1}{2}\left(|d|^2 - 1\right)^2\right)dx + \int\left(|\nabla u|^2 + \left|\Delta d - f(d)\right|^2\right)dx = 0. \quad (10)$$

Here we used the facts that $\text{div}\, u = 0$ and $(u\cdot\nabla u, u) = (u, \nabla p) = (u\cdot\nabla d, f(d)) = (u, \nabla(|d|^2/2)) = 0$; here $(\cdot,\cdot)$ denotes the usual inner product of $L^2(R^3)$, which implies

$$\|u\|_{L^\infty(0,T;L^2)} + \|u\|_{L^2(0,T;H^1)} \le C. \quad (11)$$

Besides, we multiply $(1)_2$ by $|d|^4 d$ and integrate over R^3 and get

$$\frac{1}{6}\frac{d}{dt}\int|u|^6 dx + \int\left(5|d|^4|\nabla d|^2 + |d|^8\right)dx = \int|d|^6 dx, \quad (12)$$

which implies

$$\|d(\cdot,t)\|_{L^\infty(0,T;L^6)} \le C\|d_0\|_{L^6} \le C\|d_0\|_{H^1}. \quad (13)$$

Multiplying the first equation of (1) by $-\Delta u$ and integrating over R^3. Similarly, by taking Δ on both sides of the second equation of (1), by multiplying the resulting equation by Δd, by integrating over R^3, and then by adding two results above and taking the divergence-free condition $\text{div}\, u = 0$ into account, we obtain

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\left(\|\nabla u\|_{L^2}^2 + \|\Delta d\|_{L^2}^2\right) + \|\Delta u\|_{L^2}^2 + \|\nabla\Delta d\|_{L^2}^2 &= \int(u\cdot\nabla)u\cdot\Delta u\, dx \\ &\quad - 2\sum_{i=1}^3\int\nabla u_i\partial_i\nabla d\Delta d\, dx - \int\Delta f(d)\,\Delta d\, dx \\ &= I_1 + I_2 + I_3.\end{aligned} \quad (14)$$

In the following, we establish the bounds of I_1–I_3, for the first term I_1; thanks to Lemma 5 and using Young's inequality, we have

$$\begin{aligned} I_1 &\leq \int |u|\,|\nabla u|\,|\Delta u|\,dx \\ &\leq \left\| \|u\|_{L_3^\gamma} \right\|_{L_{1,2}^\alpha} \left\| \|\nabla u\|_{L_3^{2\gamma/(\gamma-2)}} \right\|_{L_{1,2}^{2\alpha/(\alpha-2)}} \|\Delta u\|_{L^2} \\ &\leq C \left\| \|u\|_{L_3^\gamma} \right\|_{L_{1,2}^\alpha} \|\nabla u\|_{L^2}^{1-(1/\gamma)-(2/\alpha)} \\ &\quad \times \|\partial_1 \nabla u\|_{L^2}^{1/\alpha} \|\partial_2 \nabla u\|_{L^2}^{1/\alpha} \|\partial_3 \nabla u\|_{L^2}^{1/\gamma} \|\Delta u\|_{L^2} \\ &\leq C \left\| \|u\|_{L_3^\gamma} \right\|_{L_{1,2}^\alpha} \|\nabla u\|_{L^2}^{1-(1/\gamma)-(2/\alpha)} \|\Delta u\|_{L^2}^{(1/\gamma)+(2/\alpha)+1} \\ &\leq \frac{1}{2} \|\Delta u\|_{L^2}^2 + C \left\| \|u\|_{L_3^\gamma} \right\|_{L_{1,2}^\alpha}^{2\alpha\gamma/(\alpha\gamma-2\gamma-\alpha)} \|\nabla u\|_{L^2}^2. \end{aligned} \tag{15}$$

For the second term I_2, similar to estimate of I_1, we have

$$\begin{aligned} I_2 &= -2 \sum_{i=1}^{3} \int \nabla u_i \partial_i \nabla d \Delta d\,dx \\ &= 2 \sum_{i,j=1}^{3} \int \left(u_i \partial_j \partial_i \partial_j d \Delta d\,dx + u_i \partial_i \partial_j d \partial_j \Delta d \right) dx \\ &\leq C \int |u| \left|\nabla^2 d\right| |\nabla \Delta d|\,dx \\ &\leq C \left\| \|u\|_{L_3^\gamma} \right\|_{L_{1,2}^\alpha} \|\Delta d\|_{L^2}^{1-(1/\gamma)-(2/\alpha)} \\ &\quad \times \|\partial_1 \Delta d\|_{L^2}^{1/\alpha} \|\partial_2 \Delta d\|_{L^2}^{1/\alpha} \|\partial_3 \Delta d\|_{L^2}^{1/\gamma} \|\nabla \Delta d\|_{L^2} \\ &\leq \frac{1}{4} \|\nabla \Delta d\|_{L^2}^2 + C \left\| \|u\|_{L_3^\gamma} \right\|_{L_{1,2}^\alpha}^{2\alpha\gamma/(\alpha\gamma-2\gamma-\alpha)} \|\Delta d\|_{L^2}^2. \end{aligned} \tag{16}$$

For the term I_3, using Hölder's inequality, Young's inequality, and (13), one has

$$\begin{aligned} I_3 &= -\int \Delta f(d)\, \Delta d\,dx = \int \nabla \left(|d|^2 d\right) \cdot \nabla \Delta d\,dx + \|\Delta d\|_{L^2}^2 \\ &= 3 \int |d|^2 \nabla d \cdot \nabla \Delta d\,dx + \|\Delta d\|_{L^2}^2 \\ &\leq C \|d\|_{L^6}^2 \|\nabla d\|_{L^6} \|\nabla \Delta d\|_{L^2} + \|\Delta d\|_{L^2}^2 \\ &\leq \frac{1}{4} \|\nabla \Delta d\|_{L^2}^2 + C \|\Delta d\|_{L^2}^2. \end{aligned} \tag{17}$$

Substituting the above estimates (15)–(17) into (14), we obtain

$$\begin{aligned} &\frac{d}{dt} \left(\|\nabla u\|_{L^2}^2 + \|\Delta d\|_{L^2}^2 \right) + \|\Delta u\|_{L^2}^2 + \|\nabla \Delta d\|_{L^2}^2 \\ &\quad \leq C \left(1 + \left\| \|u\|_{L_3^\gamma} \right\|_{L_{1,2}^\alpha}^{2\alpha\gamma/(\alpha\gamma-2\gamma-\alpha)} \right) \|\nabla u\|_{L^2}^2 + C \|\Delta d\|_{L^2}^2. \end{aligned} \tag{18}$$

Integrating (18) from 0 to t, using Hölder's inequality and Young's inequality, one has

$$\begin{aligned} &\|\nabla u\|_{L^2}^2 + \|\Delta d\|_{L^2}^2 + \int_0^t \left(\|\Delta u\|_{L^2}^2 + \|\nabla \Delta d\|_{L^2}^2 \right) d\tau \\ &\quad \leq C \int_0^t \left(1 + \left\| \|u\|_{L_3^\gamma} \right\|_{L_{1,2}^\alpha}^{2\alpha\gamma/(\alpha\gamma-2\gamma-\alpha)} \right) \\ &\qquad \times \left(\|\nabla u\|_{L^2}^2 + \|\Delta d\|_{L^2}^2 \right) d\tau + \|\nabla u_0\|_{L^2}^2 + \|\Delta d_0\|_{L^2}^2. \end{aligned} \tag{19}$$

Finally, applying Gronwall's inequality and using condition (5), then (u, d) can be extended beyond T. This completes the proof of Theorem 1. □

Proof of Theorem 2. When $r \in [0, 1/\alpha)$, combining Theorem 1 and using the following imbedding theorem, one can get the conclusion that

$$\left\| \|u\|_{L_i^{\alpha/(1-\alpha r)}} \right\|_{L_{j,k}^\alpha} \leq C \|\Lambda_i^r u\|_{L^\alpha}. \tag{20}$$

When $r \in (1/\alpha, 1)$, our strategy is to show that

$$\int_0^t \|\Lambda_i^r u(\tau)\|_{L^\alpha}^\beta d\tau < \infty, \quad \frac{2}{\beta} + \frac{3}{\alpha} \leq 1 + \frac{1}{\alpha}, \quad \alpha \in (2, \infty) \tag{21}$$

is a sufficient condition. We can verify that integral term $\int_0^t \| \|u(\tau)\|_{L_i^\infty} \|_{L_{j,k}^\delta}^{2\delta/(\delta-2)} d\tau$ satisfies the conditions of Theorem 1 with $\delta \in (2, \infty)$. Applying Lemma 5, Hölder's inequality, and the interpolation theorem, one can conclude that, for $\delta \in [\alpha(2r+1)/r\alpha, \alpha]$,

$$\begin{aligned} \left\| \|u\|_{L_i^\infty} \right\|_{L_{j,k}^\delta} &\leq C \left\| \|u\|_{L_i^2}^\theta \|\Lambda_i^r u\|_{L_i^\alpha}^{1-\theta} \right\|_{L_{j,k}^\delta} \\ &\leq C \left\| \|u\|_{L_i^2} \right\|_{L_{j,k}^p}^\theta \left\| \|\Lambda_i^r u\|_{L_i^\alpha} \right\|_{L_{j,k}^\alpha}^{1-\theta} \\ &\leq C \left\| \|u\|_{L_{j,k}^p} \right\|_{L_i^2}^\theta \|\Lambda_i^r u\|_{L^\alpha}^{1-\theta} \\ &\leq C \|u\|_{L^2}^{2\theta/p} \|\partial_j u\|_{L^2}^{(p-2)\theta/2p} \\ &\quad \times \|\partial_k u\|_{L^2}^{(p-2)\theta/2p} \|\Lambda_i^r u\|_{L^\alpha}^{1-\theta} \\ &\leq C \|u\|_{L^2}^{2\theta/p} \|\nabla u\|_{L^2}^{(p-2)\theta/p} \|\Lambda_i^r u\|_{L^\alpha}^{1-\theta}, \end{aligned} \tag{22}$$

where $1/\delta = (\theta/p) + ((1-\theta)/\alpha)$ with $\theta = 2(r\alpha - 1)/(2(r\alpha - 1) + \alpha)$ and we have used the fact that $\delta \geq ((2r+1)\alpha - 2)/\alpha r$ implies $p \geq 2$. Using Hölder's inequality, one has

$$\begin{aligned} &\int_0^t \left\| \|u(\tau)\|_{L_i^\infty} \right\|_{L_{j,k}^\delta}^{2\delta/(\delta-2)} d\tau \\ &\quad \leq C \int_0^t \|u\|_{L^2}^{4\delta\theta/p(\delta-2)} \|\nabla u\|_{L^2}^{2\delta\theta(p-2)/p(\delta-2)} \end{aligned}$$

$$
\begin{aligned}
&\times \left\|\Lambda_i^r u\right\|_{L^\alpha}^{2(1-\theta)\delta/(\delta-2)} d\tau \\
&\leq C\|u\|_{L_t^\infty L^2}^{4\delta\theta/p(\delta-2)} \|\nabla u\|_{L_t^2 L^2}^{2\delta\theta(p-2)/p(\delta-2)} \\
&\times \left(\int_0^t \left\|\Lambda_i^r u\right\|_{L^\alpha}^{(2(1-\theta)\delta/(\delta-2))\eta} d\tau\right)^{1/\eta},
\end{aligned} \tag{23}
$$

where $\eta = p(\delta-2)/(p(\delta-2)-\delta\theta(p-2))$.

According to the fact that $1/\delta = (\theta/p) + ((1-\theta)/\alpha)$ and $\theta = 2(r\alpha-1)/(2(r\alpha-1)+\alpha)$, we have

$$
\begin{aligned}
&\frac{2(1-\theta)\delta}{\delta-2}\eta \\
&= \frac{2p\delta(1-\theta)}{p(\delta-2)-\delta\theta(p-2)} = \frac{2\delta(1-\theta)}{\delta-2-\delta\theta(1-(2/p))} \\
&= \frac{2\delta(1-\theta)}{\delta-2-\delta\theta+2\delta(1/\delta)-2\delta((1-\theta)/\alpha)} \\
&= \frac{2\delta(1-\theta)}{\delta-\delta\theta-(2\delta(1-\theta)/\alpha)} = \frac{2(1-\theta)}{1-\theta-(2(1-\theta)/\alpha)} \\
&= \frac{2\alpha(1-\theta)}{\alpha-\alpha\theta-2(1-\theta)} = \frac{2\alpha}{\alpha-2}.
\end{aligned} \tag{24}
$$

This together with Theorem 1 gives the desired result of Theorem 2. □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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