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Research Article

Stability Analysis of a Class of Switched Nonlinear Systems with an Improved Average Dwell Time Method

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This paper investigates the stability of switched nonlinear (SN) systems in two cases: (1) all subsystems are globally asymptotically stable (GAS), and (2) both GAS subsystems and unstable subsystems coexist, and it proposes a number of new results on the stability analysis. Firstly, an improved average dwell time (ADT) method is presented for the stability of such switched system by extending our previous dwell time method. In particular, an improved mode-dependent average dwell time (MDADT) method for the switched systems whose subsystems are quadratically stable (QS) is also obtained. Secondly, based on the improved ADT and MDADT methods, several new results to the stability analysis are obtained. It should be pointed out that the obtained results have two advantages over the existing ones; one is that the improved ADT method simplifies the conditions of the existing ADT method, the other is that the obtained lower bound of ADT (τ_a^*) is also smaller than that obtained by other methods. Finally, illustrative examples are given to show the correctness and the effectiveness of the proposed methods.

1. Introduction

Switched systems arise in various fields of real life world, such as manufacturing, communication networks, autopilot design, automotive engine control, computer synchronization, traffic control, and chemical processes. In the past two decades, increasing attention has been paid to the analysis and synthesis of switched systems due to their significance in both theory and applications, and many significant results have been obtained for the analysis and design of switched systems, see [1–11] and references therein. For the switched systems, there are several important problems to be investigated, such as stability analysis and control design. Stability analysis has been a very important and hot issue since the switched systems came into being, and a lot of efforts have been devoted to it. For the stability analysis problem, there are two famous methods, that is, Common Lyapunov Function (CLF) method [4, 6] and Multiple Lyapunov Functions (MLF) method [11]. For the CLF method, for a given switched system, it is very difficult to determine whether all the subsystems share a CLF or not, even for the switched linear (SL) systems. Regarding the MLF method, it is well known that the switched system is GAS for any switching signal if the time between consecutive switching (i.e., dwell time) is sufficiently large when all the subsystems are stable. Also, some results have appeared in recent works to compute lower bounds of the dwell time for ensuring the stability [12–14]. But, how to obtain the minimum dwell time (MDT) for a given switched system has had no general method so far, even for the SL systems. As pointed out in [13], the ADT switching is a class of restricted switching signals which means that the number of switches in a finite interval is bounded and the average dwell time between consecutive switching is not less than a constant. It was well known that the ADT scheme characterizes a large class of stable switching signals than dwell time scheme, and its extreme case is the arbitrary switching. Thus, the ADT method is very important not only in theory, but also in practice, and considerable attention has been paid, and a lot of efforts have been done to take advantage of the ADT method to investigate the stability and stabilization problems both in linear and nonlinear systems.

However, on the one hand, almost all the results mentioned above are concerned with the stability of the switched linear or nonlinear systems with stable subsystems; see [12–15] and the references therein. Although the results in [12] deal with the SL system with stable and unstable subsystems, the ADT method used in such paper has two disadvantages: one is this ADT method is only used for the SL system, the other is the lower bound of ADT τ_a^* obtained by the results in [12] has much conservative property. Therefore, to obtain an improved ADT method for the SN systems with stable and unstable subsystem is very important, and to investigate this problem is of value both in theory and in practice. On the other hand, as the authors in [16] point out that the property in the ADT switching that the average time interval between any two consecutive switching is at least τ_a^*, which is independent of the system modes, is probably still not anticipated. In order to solve this problem, they obtain MDADT method, which can reduce the conservative property of ADT.

Motivated by the above reasons, we extend our previous results in [17, 18] to investigate the stability of SN system in both cases: one is where all subsystems are GAS, the other is where both GAS and unstable subsystems coexist. Firstly, we obtained an improved ADT method for studying the stability of such SN system and an improved MDADT method for a class of SN systems which have QS property inspired by the study of MDADT method. Secondly, based on which, some new stability analysis results for the SN system are obtained, which have some advantages over the existing result [12]. Finally, illustrative examples are studied by using the results obtained in this paper. The study of examples shows that our analysis methods work very well in analyzing the stability of SN systems with GAS subsystems or both GAS and unstable subsystems.

The rest of the paper is organized as follows. Section 2 presents the problem formulation of this paper, and Section 3 gives the main results. In Section 4, illustrative examples are given to support our new results, which is followed by the conclusion in Section 5.

2. Problem Formulation

Consider the SN system described as

$$\dot{x} = f_{\sigma(t)}(x), \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, the map $\sigma(t) : [t_0, \infty) \to \mathcal{I} = \{1, 2, \dots, N\}$ is a piecewise right-continuous function, called the switching law or switching path, which will be determined later, and $\sigma(t) = i$ means that the ith subsystem is active, and $f_i(x) = (f_{1i}(x), f_{2i}(x), \dots, f_{ni}(x))^T$ is smooth, $i \in \mathcal{I}$. Throughout this paper, we assume that there are no jumps in the state at the switching instants and that a finite number of switches occur on every bounded time interval. Let $x(t)$ denote the trajectory of the system (1) starting from x_0, $x_0 = x(t_0)$.

If $f_i(x) = A_i x$, $i \in \mathcal{I}$, the SN system (1) becomes

$$\dot{x} = A_{\sigma(t)} x, \tag{2}$$

where A_i is a real matrix, $i \in \mathcal{I}$.

For an arbitrary switching path $\sigma(t) = i_m \in \mathcal{I}$ $(t \in [t_m, t_{m+1}),\ m = 0, 1, 2, 3, \dots)$, $\{t_m\}_{m=0}^{+\infty}$ is called the switching time sequence, which is assumed to satisfy

$$t_0 < t_1 < t_2 < \cdots < t_m < \cdots < +\infty. \tag{3}$$

Let $\tau_k = t_k - t_{k-1}$ denote the dwell time, $k = 1, 2, \dots$.

For the development of this paper, we introduce several definitions.

Definition 1 (see [19]). Let $N_\sigma(\tau, t)$ denote the number of switching of $\sigma(t)$ over the interval $[\tau, t)$, for given N_0, $\tau_a > 0$,

$$N_\sigma(\tau, t) \le N_0 + \frac{t - \tau}{\tau_a}, \tag{4}$$

where τ_a is called average dwell time and N_0 denotes the chatter bound.

Definition 2 (see [20, 21]). The SN system (1) with $\mathcal{I} = \{1\}$ is called quadratic stability, if there is a Lyapunov function $V(x) = x^T P x$ which ensures the system (1) is stable, where $P > 0$.

Definition 3 (see [16]). For a switching signal $\sigma(t)$ and any $T \ge t \ge 0$, let $N_{\sigma i}(t, T)$ be the switching numbers that the ith subsystem is activated over the interval $[t, T)$, and $T_i(t, T)$ denote the total running time of the ith subsystem over the interval $[t, T)$, $i \in \mathcal{I}$. We say that $\sigma(t)$ has a MDADT τ_{ai} if there exist positive numbers N_{0i} (we call N_{0i} the mode-dependent chatter bounds here) and τ_{ai} such that

$$N_{\sigma i}(t, T) \le N_{0i} + \frac{T_i(t, T)}{\tau_{ai}}, \quad \forall T \ge t \ge 0. \tag{5}$$

The objective of this paper is to investigate the stability of SN systems in two cases: all subsystems are GAS, and both GAS subsystems and unstable subsystems coexist.

3. Main Results

Firstly, we investigate the stability of SN system (1) whose subsystems are GAS and propose the following results.

Theorem 4. *Consider the SN system* (1), *if there exist* $\mathscr{C}^1$ *functions* $V_{\sigma(t)}(x) : \mathbb{R}^n \to \mathbb{R}_+$, *and two* $\mathscr{K}_\infty$ *class functions* k_1, k_2 *such that, for all* $i \in \mathcal{I}$

$$k_1(\|x\|) \le V_i(x) \le k_2(\|x\|), \tag{6}$$

$$\dot{V}_i(x)\Big|_{(i)} = \frac{\partial^T V_i(x)}{\partial x} f_i(x) \le -\lambda_i V_i(x), \tag{7}$$

where $\lambda_i > 0$, $i \in \mathcal{I}$, *then the SN system* (1) *is GAS for any switching signal with ADT*

$$\tau_a > \tau_a^* = \frac{a}{\lambda_{\min}}, \tag{8}$$

where

$$a = \ln \mu, \qquad \mu = \sup_{x \ne 0} \frac{k_2(\|x(t)\|)}{k_1(\|x(t)\|)},$$
$$\lambda_{\min} = \min_{i \in \mathcal{I}} \lambda_i. \tag{9}$$

Proof. Let $t_1, t_2, \ldots$, denote the time points at which switching occurs, and write p_j for the value of $\sigma(t)$ on $[t_{j-1}, t_j)$. Integrating the inequality (7) over the interval $[t_{j-1}, t_j)$, we obtain that

$$\ln V_{p_j}\left(x_j\right) - \ln V_{p_j}\left(x_{j-1}\right) \leq -\lambda_{p_j}\tau_j \tag{10}$$

and then

$$V_{p_j}\left(x_j\right) \leq e^{-\lambda_{p_j}\tau_j} V_{p_j}\left(x_{j-1}\right). \tag{11}$$

According to inequality (6), we obtain that

$$\begin{aligned} k_1\left(\left\|x_j\right\|\right) &\leq V_{p_j}\left(x_j\right) \leq e^{-\lambda_{p_j}\tau_j} V_{p_j}\left(x_{j-1}\right) \\ &\leq e^{-\lambda_{p_j}\tau_j} k_2\left(\left\|x_{j-1}\right\|\right) \\ &\leq e^{-\lambda_{p_j}\tau_j} \frac{k_2\left(\left\|x_{j-1}\right\|\right)}{k_1\left(\left\|x_{j-1}\right\|\right)} k_1\left(\left\|x_{j-1}\right\|\right) \\ &\leq \mu e^{-\lambda_{p_j}\tau_j} k_1\left(\left\|x_{j-1}\right\|\right). \end{aligned} \tag{12}$$

Then, for any t satisfying $t_0 < t_1 < \cdots < t_i \leq t < t_{i+1}$, we obtain

$$\begin{aligned} k_1\left(\left\|x_t\right\|\right) &\leq V_{p_{i+1}}\left(x_t\right) \leq e^{-\lambda_{p_{i+1}}(t-t_i)} V_{p_{i+1}}\left(x_{t_i}\right) \\ &\leq e^{-\lambda_{p_{i+1}}(t-t_i)} k_2\left(\left\|x_{t_i}\right\|\right) \\ &\leq \mu e^{-\lambda_{p_{i+1}}(t-t_i)} k_1\left(\left\|x_{t_i}\right\|\right) \\ &\ \ \vdots \\ &\leq \mu^{i+1} e^{-\lambda_{\min}(t-t_0)} k_1\left(\left\|x_0\right\|\right) \\ &= e^{(i+1)a-\lambda_{\min}(t-t_0)} k_1\left(\left\|x_0\right\|\right) \\ &= c e^{aN_\sigma(t_0,t)-\lambda_{\min}(t-t_0)} k_1\left(\left\|x_0\right\|\right), \end{aligned} \tag{13}$$

where $c = e^a$.

When $a = 0$, that is, $\mu = 1$, we can obtain that

$$k_1\left(\left\|x_t\right\|\right) \leq e^{-\lambda_{\min}(t-t_0)} k_1\left(\left\|x_0\right\|\right) \tag{14}$$

which implies that the switched system (1) is GAS for arbitrary switching signals.

When $a > 0$, according to (4), we obtain

$$aN_\sigma\left(t_0, t\right) - \lambda_{\min}\left(t - t_0\right) \leq aN_0 + \left(\frac{a}{\tau_a} - \lambda_{\min}\right)\left(t - t_0\right). \tag{15}$$

Substituting (15) into (13), we arrive at

$$k_1\left(\left\|x_t\right\|\right) \leq c e^{aN_0} e^{((a/\tau_a)-\lambda_{\min})(t-t_0)} k_1\left(\left\|x_0\right\|\right). \tag{16}$$

If $\tau_a > a/\lambda_{\min}$, which implies that the system (1) is GAS with the above ADT. □

Remark 5. In general, $\mu \geq 1$. Especially, if $\mu = 1$, which implies that $V_i(x) \equiv V(x)$, $i \in \mathcal{I}$, that is, $V(x)$ is a CLF for the switched system (1), and thus this system is GAS under arbitrary switching. It is also noted that the ADT method proposed in [12, 14, 15, 19] needs the conditions (6)-(7) and an additional condition as "$V_i(x) \leq \mu V_j(x), \mu \geq 1, i \neq j, i, j \in \mathcal{I}$". Comparing Theorem 4 with [12, 19], Theorem 4 needs fewer conditions and thus can be applied to a wider range of systems. Furthermore, for SL systems, the lower bound of ADT τ_a^* obtained by Theorem 4 is smaller than the lower bound of ADT τ_a' obtained in [19]. In fact,

$$\tau_a^* = \max_{i\in\mathcal{I}}\left\{\frac{\lambda_{\max}\left(P_i\right)}{\lambda_{\min}\left(P_i\right)\lambda_{\min}}\right\} \leq \max_{i,j\in\mathcal{I}}\left\{\frac{\lambda_{\max}\left(P_j\right)}{\lambda_{\min}\left(P_i\right)\lambda_{\min}}\right\} = \tau_a', \tag{17}$$

where $V_i(x) = x^T P_i x$ with $P_i > 0$ is the Lyapunov function for the ith subsystem, $i \in \mathcal{I}$.

With Theorem 4, we can obtain the following corollary.

Corollary 6. *For the SL system* (2), *if A_i is Hurwitz, $i \in \mathcal{I}$, then the switched system* (2) *is GAS for any ADT $\tau_a > \tau_a^*$, where τ_a^* is given as* (8).

Remark 7. The proof of Corollary 6 can be obtained using similar techniques in Theorem 4, so we omit it here. In addition, if $V_i(x) = x^T P_i x$, where $P_i > 0$, $i \in \mathcal{I}$, then we can obtain that

$$\mu = \max_{i\in\mathcal{I}} \frac{\lambda_{\max}\left(P_i\right)}{\lambda_{\min}\left(P_i\right)}. \tag{18}$$

As the authors in [16] point out that the property in the ADT switching, that the average time interval between any two consecutive switching is at least τ_a^*, which is independent of the system modes, is probably still not anticipated. Then, they obtain an MDADT method, which can reduce the conservative property of ADT. Inspired by the study in which, we extend our results to obtain an improved MDADT method for a class of SN systems which have quadratic stability property.

Then, we give the result in the following.

Theorem 8. *Consider the SN system* (1), *if there exist functions $V_i(x) = x^T P_i x$, where $P_i > 0$, and a class of real numbers $k_{1i} > 0$, $k_{2i} > 0$ such that, for all $i \in \mathcal{I}$*

$$k_{1i}\|x\|^2 \leq V_i(x) \leq k_{2i}\|x\|^2, \tag{19}$$

$$\dot{V}_i(x)\Big|_{(i)} = \frac{\partial^T V_i(x)}{\partial x} f_i(x) \leq -\lambda_i V_i(x), \tag{20}$$

where $\lambda_i > 0$, $i \in \mathcal{I}$, then the switched system (1) *is GAS for any switching signal with MDADT*

$$\tau_{ai} > \tau_{ai}^* = \frac{a_i}{\lambda_i}, \tag{21}$$

where

$$a_i = \ln \mu_i, \qquad \mu_i = \frac{k_{2i}}{k_{1i}}. \tag{22}$$

Proof. For any t satisfying $t_0 < t_1 < \cdots < t_i \le t < t_{i+1}$, we obtain

$$\begin{aligned}
\|x_t\|^2 &\le \frac{1}{k_{1\sigma(t_i)}} V_{\sigma(t)}(x_t) \le \frac{1}{k_{1\sigma(t_i)}} e^{-\lambda_{\sigma(t_i)}(t-t_i)} V_{\sigma(t)}(x_{t_i}) \\
&\le \frac{k_{2\sigma(t_i)}}{k_{1\sigma(t_i)}} e^{-\lambda_{\sigma(t_i)}(t-t_i)} \|x_{t_i}\|^2 \\
&= \mu_{\sigma(t_i)} e^{-\lambda_{\sigma(t)}(t-t_i)} \|x_{t_i}\|^2 \\
&\le \mu_{\sigma(t_i)} \mu_{\sigma(t_{i-1})} e^{-\lambda_{\sigma(t)}(t-t_i)-\lambda_{\sigma(t_i)}(t_i-t_{i-1})} \|x_{t_{i-1}}\|^2 \\
&\;\;\vdots \\
&\le \mu \prod_{i=1}^{N} e^{a_i N_{\sigma i}(t_0,t)-\lambda_i T_i(t_0,t)} \|x_0\|^2,
\end{aligned} \tag{23}$$

where $T_i(t_0, t)$ denotes the total activation time of the ith subsystem in the interval $[t_0, t)$.

When $a_i = 0$, that is, $\mu_i = 1$, $i \in \mathcal{I}$, we conclude from (23) that

$$\|x_t\|^2 \le \prod_{i=1}^{N} e^{-\lambda_i T_i(t_0,t)} \|x_0\|^2 \le e^{-\lambda_{\min}(t-t_0)} \|x_0\|^2, \tag{24}$$

where $\lambda_{\min}$ is given as (9), which implies that the switched system (1) is GAS for any MDADT.

When $a_i > 0$, $i \in \mathcal{I}$, according to (4), we obtain

$$a_i N_{\sigma i}(t_0,t) - \lambda_i T_i(t_0,t) \le a_i N_{0i} + \left(\frac{a_i}{\tau_{ai}} - \lambda_i\right) T_i(t_0,t). \tag{25}$$

Substituting (25) into (23), we arrive at

$$\begin{aligned}
\|x_t\|^2 &\le \mu \prod_{i=1}^{N} e^{a_i N_{0i} + ((a_i/\tau_{ai})-\lambda_i) T_i(t_0,t)} \|x_0\|^2 \\
&= \mu \alpha e^{-\lambda_{\min}(t-t_0)} \|x_0\|^2,
\end{aligned} \tag{26}$$

where $\alpha = \prod_{i=1}^{N} e^{a_i N_{0i}}$, $\lambda_{\min} = \min_{i=1}^{N}\{\lambda_i - (a_i/\tau_{ai})\}$. If $\tau_{ai} > a_i/\lambda_i$, and thus the switched system (1) is GAS for any MDADT $\tau_{ai} > \tau_{ai}^*$. □

With Theorem 8, we can obtain the following corollary.

Corollary 9. *For the SL system* (2)*, if A_i, $i \in \mathcal{I}$, is Hurwitz, then the switched system* (2) *is GAS for any MDADT $\tau_{ai} > \tau_{ai}^*$, where $\tau_{ai}^* > 0$ are given as* (21).

Next, we consider the SN systems in which both GAS and unstable subsystems exist. For the switching signal $\sigma(t)$ and any $t > \tau$, we let $T^u(\tau, t)$ (resp., $T^s(\tau, t)$) denote the total activation time of the unstable subsystems (resp., the GAS subsystems) on interval $[\tau, t)$. Then, we let $\mathcal{I} = \mathcal{I}_u \cup \mathcal{I}_s$, where $\mathcal{I}_u \cap \mathcal{I}_s = \emptyset$.

Next, we give the main results in the following.

Theorem 10. *Consider the SN system* (1)*, if there exist $\mathcal{C}^1$ functions $V_{\sigma(t)}(x) : \mathbb{R}^n \to \mathbb{R}$, two $\mathcal{K}_\infty$ class functions k_1, k_2 such that* (6)*, and*

$$\dot{V}_i(x)\Big|_{(i)} = \frac{\partial^T V_i(x)}{\partial x} f_i(x) \le \lambda_i V_i(x), \quad i \in \mathcal{I}_u, \tag{27}$$

$$\dot{V}_i(x)\Big|_{(i)} = \frac{\partial^T V_i(x)}{\partial x} f_i(x) \le -\lambda_i V_i(x), \quad i \in \mathcal{I}_s, \tag{28}$$

where $\lambda_i > 0$, $i \in \mathcal{I}$, then under the following switching law (S1) the switched system (1) *is GAS for any switching signal with ADT*

$$\tau_a > \tau_a^* = \frac{a}{\lambda^*}, \tag{29}$$

where a is given as (9)*, and $\lambda^* \in (0, \lambda_s)$ is an arbitrarily chosen number,*

$$\lambda_s = \min_{i=r+1}^{N} \lambda_i, \qquad \lambda_u = \min_{i=1}^{r} \lambda_i \tag{30}$$

and the switching law

(S1) *Determine the $\sigma(t)$ satisfying $T^s(t_0,t)/T^u(t_0,t) \ge (\lambda_u + \lambda^*)/(\lambda_s - \lambda^*)$ holds for any $t > t_0$.*

Proof. Let $t_1, t_2, \ldots,$ denote the time points at which switching occurs, and write p_j for the value of $\sigma(t)$ on $[t_{j-1}, t_j)$. Integrating the inequality (27) or (28) over the interval $[t_{j-1}, t_j)$, we obtain that

$$\ln V_{p_j}(x_j) - \ln V_{p_j}(x_{j-1}) \le \operatorname{sign}(p_j) \lambda_{p_j} \tau_j \tag{31}$$

and then

$$V_{p_j}(x_j) \le e^{\operatorname{sign}(p_j)\lambda_{p_j}\tau_j} V_{p_j}(x_{j-1}), \tag{32}$$

where $\operatorname{sign}(p_j) = 1$, if $1 \le p_j \le r$, $\operatorname{sign}(p_j) = -1$, if $r+1 \le p_j \le N$.

Thus,

$$\begin{aligned}
k_1(\|x_j\|) &\le V_{p_j}(x_j) \le e^{\operatorname{sign}(p_j)\lambda_{p_j}\tau_j} V_{p_j}(x_{j-1}) \\
&\le e^{\operatorname{sign}(p_j)\lambda_{p_j}\tau_j} k_2(\|x_{j-1}\|) \\
&\le e^{\operatorname{sign}(p_j)\lambda_{p_j}\tau_j} \frac{k_2(\|x_{j-1}\|)}{k_1(\|x_{j-1}\|)} k_1(\|x_{j-1}\|) \\
&\le e^{\operatorname{sign}(p_j)\lambda_{p_j}\tau_j} \mu k_1(\|x_{j-1}\|),
\end{aligned} \tag{33}$$

where μ is given as (9).

Then, for any t satisfying $t_0 < t_1 < \cdots < t_i \le t < t_{i+1}$, we obtain

$$\begin{aligned} k_1(\|x_t\|) &\le V_{p_{i+1}}(x_t) \le e^{\operatorname{sign}(p_{i+1})\lambda_{p_{i+1}}(t-t_i)} V_{p_{i+1}}(x_{t_i}) \\ &\le e^{\operatorname{sign}(p_{i+1})\lambda_{p_{i+1}}(t-t_0)} k_2(\|x_{t_i}\|) \\ &\le \mu e^{\operatorname{sign}(p_{i+1})\lambda_{p_{i+1}}(t-t_i)} k_1(\|x_{t_i}\|) \cdots \\ &\le \mu^{i+1} e^{\lambda_u T^u(t_0,t) - \lambda_s T^s(t_0,t)} k_1(\|x_0\|) \\ &= e^{(i+1)a + \lambda^+ T^u(t_0,t) - \lambda_s T^s(t_0,t)} k_1(\|x_0\|) \\ &= c e^{a N_\sigma(t_0,t) + \lambda_u T^u(t_0,t) - \lambda_s T^s(t_0,t)} k_1(\|x_0\|), \end{aligned} \tag{34}$$

where $c = e^a$.

According to the switching law (S1), that is,

$$\begin{aligned} &\lambda_u T^u(t_0,t) - \lambda_s T^s(t_0,t) \\ &\quad \le -\lambda^*(T^u(t_0,t) + T^s(t_0,t)) = -\lambda^*(t - t_0) \end{aligned} \tag{35}$$

we obtain from (35) that

$$k_1(\|x_t\|) \le c e^{a N_\sigma(t_0,t) - \lambda^*(t-t_0)} k_1(\|x_0\|). \tag{36}$$

When $a = 0$, that is, $\mu = 1$, we can obtain from (36) that

$$k_1(\|x_t\|) \le e^{-\lambda^*(t-t_0)} k_1(\|x_0\|) \tag{37}$$

which implies that the switched system (1) is GAS for arbitrary switching paths.

When $a > 0$, according to (4), we arrive at

$$a N_\sigma(t_0,t) - \lambda^*(t - t_0) \le a N_0 + \left(\frac{a}{\tau_a} - \lambda^*\right)(t - t_0) \tag{38}$$

and then

$$k_1(\|x_t\|) \le c e^{a N_0} e^{((a/\tau_a) - \lambda^*)(t-t_0)} k_1(\|x_0\|). \tag{39}$$

If $\tau_a > a/\lambda^*$, then under the following switching law (S1) the switched system (1) is GAS for the above ADT. □

According to Theorem 10, we can obtain the following corollary.

Corollary 11. *Consider the SL system* (2)*, if A_i is unstable, $i = 1, 2, \ldots, r$, and A_i is Hurwitz, $i = r+1, 2, \ldots, N$, then under the switching law (S1) the system* (2) *is GAS for any ADT $\tau_a > \tau_a^*$, where $\tau_a^* > 0$ is given as* (29).

Learning form [22], we obtain other results which can deal with some subsystems of the switched system being stable, while some subsystems are not.

Theorem 12. *Consider the SN system* (1)*, if there exist $\mathscr{C}^1$ functions $V_{\sigma(t)}(x) : \mathbb{R}^n \to \mathbb{R}$ and two $\mathscr{K}_\infty$ class functions k_1, k_2 such that* (6)*,* (27)*, and* (28)*. If there exist constants τ_0, $\rho \ge 0$ such that*

$$\rho < \frac{\lambda_s}{\lambda_s + \lambda_u}, \tag{40}$$

$$\forall t \ge 0 : T^u(t_0,t) \le \tau_0 + \rho t \tag{41}$$

then the switched system (1) *is GAS for any switching signal with ADT*

$$\tau_a > \tau_a^* = \frac{a}{\lambda_s - (\lambda_s + \lambda_u)\rho}, \tag{42}$$

where a is given as (9)*, and λ_s, λ_u are given as* (30).

Proof. The proof of Theorem 12 follows the lines of the proof of Theorem 10. Similar to Theorem 10, for any t satisfying $t_0 < t_1 < \cdots < t_i \le t < t_{i+1}$, we obtain

$$k_1(\|x_t\|) \le c e^{a N_\sigma(t_0,t) + \lambda_u T^u(t_0,t) - \lambda_s T^s(t_0,t)} k_1(\|x_0\|), \tag{43}$$

where $c = e^a$.

According to (41), we get

$$T^s(t_0,t) \ge (1 - \rho)(t - t_0) - \tau_0. \tag{44}$$

We obtain from (44) that

$$\begin{aligned} &k_1(\|x_t\|) \\ &\quad \le c e^{a N_\sigma(t_0,t) + \lambda_u[\tau_0 + \rho(t-t_0)] + \lambda_s[\tau_0 + (\rho-1)(t-t_0)]} k_1(\|x_0\|) \\ &\quad = c e^{a N_\sigma(t_0,t) + (\lambda_s+\lambda_u)\tau_0 - [\lambda_s - (\lambda_s+\lambda_u)\rho](t-t_0)} k_1(\|x_0\|). \end{aligned} \tag{45}$$

When $a = 0$, that is, $\mu = 1$, we can obtain from (45) that

$$k_1(\|x_t\|) \le c e^{(\lambda_s+\lambda_u)\tau_0} e^{-[\lambda_s - (\lambda_s+\lambda_u)\rho](t-t_0)} k_1(\|x_0\|) \tag{46}$$

which implies that the switched system (1) is GAS for arbitrary switching paths.

When $a > 0$, according to (4), we arrive at

$$\begin{aligned} &a N_\sigma(t_0,t) - [\lambda_s - (\lambda_s + \lambda_u)\rho](t - t_0) \\ &\quad \le a N_0 + \left\{\frac{a}{\tau_a} - [\lambda_s - (\lambda_s + \lambda_u)\rho]\right\}(t - t_0) \end{aligned} \tag{47}$$

and then

$$\begin{aligned} &k_1(\|x_t\|) \\ &\quad \le c e^{a N_0} e^{(\lambda_s+\lambda_u)\tau_0} e^{\{(a/\tau_a) - [\lambda_s - (\lambda_s+\lambda_u)\rho]\}(t-t_0)} k_1(\|x_0\|). \end{aligned} \tag{48}$$

If $\tau_a > \tau_a^*$, then the switched system (1) is GAS for the above ADT. □

According to Theorem 12, we can obtain the following corollary.

Corollary 13. *Consider the SL system* (2)*, if A_i is unstable, $i = 1, 2, \ldots, r$, and A_i is Hurwitz, $i = r + 1, 2, \ldots, N$, then the system* (2) *is GAS for any ADT $\tau_a > \tau_a^*$, where $\tau_a^* > 0$ is given as* (42).

4. Illustrative Examples

In this section, we give two illustrative examples to show how to use the results obtained in this paper to analyze the stability of switched linear and nonlinear system with stable and unstable subsystems.

Example 1. Consider the following SL system [12]

$$\dot{x} = A_i x, \tag{49}$$

where $i \in \mathcal{I} = \{1, 2\}$, and

$$A_1 = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}, \qquad A_2 = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix}. \tag{50}$$

Obviously, A_1 is unstable while A_2 is Hurwitz stable. In [12], the authors have obtained that $T^s(t_0, t)/T^u(t_0, t) \geq 9$ and $\tau_a^{*\prime} = 2.4$. Next, we will investigate the switched system (49) by our method.

It is easy to know that $V(x) = x^T x$ is a CLF for the switched system (49), and

$$\begin{aligned} \dot{V}\big|_{(1)} &= 2x_1\dot{x}_1 + 2x_2\dot{x}_2\big|_{(1)} \leq 8V(x), \\ \dot{V}\big|_{(2)} &= 2x_1\dot{x}_1 + 2x_2\dot{x}_2\big|_{(2)} \leq -2V(x). \end{aligned} \tag{51}$$

According to the above results, we obtain that $\lambda_u = 8$, $\lambda_s = 2$, and $a = 0$. Therefore, the ADT $\tau_a^* = 0$; that is, the ADT can be arbitrary. Next, we choose $\lambda^* = 0.5$ which are the same to those in [12]. Then, the switching law (S1) will require

$$\frac{T^s(t_0, t)}{T^u(t_0, t)} \geq \frac{\lambda_u + \lambda^*}{\lambda_s - \lambda^*} = \frac{8.5}{1.5} \approx 5.67. \tag{52}$$

According to Corollary 11, the system (49) is GAS under the above switching law (S1).

To illustrate the correctness of the above conclusion, we carry out some simulation results with the following choices. Initial Condition: $[x_1(0), x_2(0)] = [2, -3]$, and Switching Path:

$$\sigma(t) = \begin{cases} 1, & t \in [t_{2m}, t_{2m+1}), \ t_{2m+1} - t_{2m} = 0.1, \\ 2, & t \in [t_{2m+1}, t_{2m+2}), \ t_{2m+2} - t_{2m+1} = 0.6, \end{cases} \tag{53}$$

where $m = 0, 1, 2, \ldots$. The simulation result is given in Figure 1, which is the response of the state under the above path $\sigma(t)$.

It can be observed from Figure 1 that the trajectory $x(t)$ converges to origin quickly. The simulation shows that Corollary 11 is very effective in analyzing the stability for the SL systems with both unstable and GAS subsystems.

Remark. From this example, we show that the lower bound of ADT ($\tau_a^* = 0$) for the switched system (49) obtained by our method is smaller than the ADT ($\tau_a^{*\prime} = 2.4$) by the method in [12], which implies that our method for determining the lower bound of ADT of switched system has some advantages in some cases.

Example 2. Consider the following SN system

$$\dot{x} = f_i(x), \tag{54}$$

where $i \in \mathcal{I} = \{1, 2\}$, and

$$f_1(x) = \begin{pmatrix} -x_1 - x_1 x_2^2 \\ x_1^2 x_2 - 3x_2 \end{pmatrix}, \qquad f_2(x) = \begin{pmatrix} 2x_1 + 2x_2 \\ x_1 + 3x_2 \end{pmatrix}. \tag{55}$$

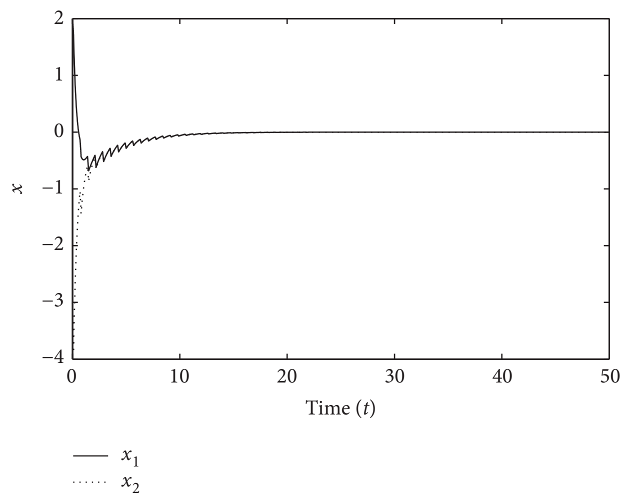

Figure 1: The state's response.

It is easy to know that $V(x) = x^T x$ is a CLF for the switched system (54), and

$$\begin{aligned} \dot{V}\big|_{(1)} &= 2x_1\dot{x}_1 + 2x_2\dot{x}_2\big|_{(1)} \leq -2V(x), \\ \dot{V}\big|_{(2)} &= 2x_1\dot{x}_1 + 2x_2\dot{x}_2\big|_{(2)} \leq 8V(x). \end{aligned} \tag{56}$$

According to the above results, we obtain that $\lambda_u = 8$, $\lambda_s = 2$ and $a = 0$. Therefore, the lower bound of ADT $\tau_a^* = 0$; that is, the ADT can be arbitrary. Next, we choose $\lambda^* = 0.8$. Then, the switching law (S1) will require

$$\frac{T^s(t_0, t)}{T^u(t_0, t)} \geq \frac{\lambda_u + \lambda^*}{\lambda_s - \lambda^*} = \frac{8.8}{1.2} \approx 7.33. \tag{57}$$

According to Theorem 10, the switched system (54) is GAS under the above switching law (S1).

To illustrate the correctness of the above conclusion, we carry out some simulation results with the following choices. Initial Condition: $[x_1(0), x_2(0)] = [-2.5, 3]$, and Switching Path:

$$\sigma(t) = \begin{cases} 2, & t \in [t_{2m}, t_{2m+1}), \ t_{2m+1} - t_{2m} = 0.2, \\ 1, & t \in [t_{2m+1}, t_{2m+2}), \ t_{2m+2} - t_{2m+1} = 1.6, \end{cases} \tag{58}$$

where $m = 0, 1, 2, \ldots$. The simulation result is given in Figure 2, which is the response of the state under the above path $\sigma(t)$.

It can be observed from Figure 2 that the trajectory $x(t)$ converges to origin quickly. The simulation shows that Theorem 10 is very effective in analyzing the stability for the SN systems with both unstable and GAS subsystems.

5. Conclusions

In conclusion, we have investigated the stability of SN systems in two cases: all subsystems are GAS and both GAS subsystems and unstable subsystems coexist, and we proposed a number of new results on the stability analysis. An

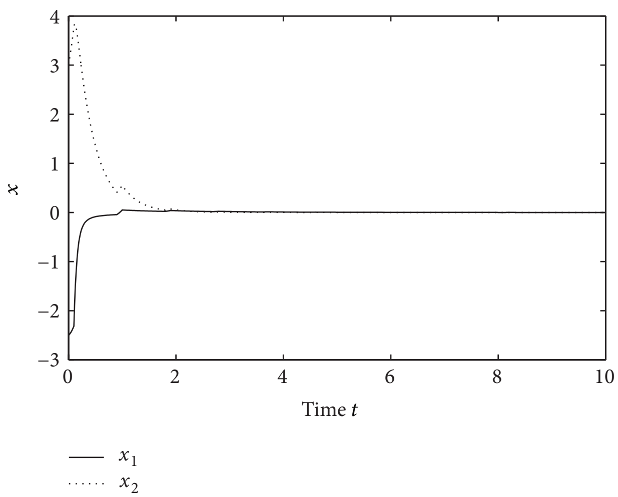

FIGURE 2: The state's response.

improved ADT method has been established for the stability of such switched system, and an improved MDADT method for the switched systems whose subsystems are QS also has been obtained. Based on which several new results to the stability analysis have been obtained. Comparing with the exiting corresponding results, not only the conditions of the improved ADT method are simplified, but also the obtained lower bound of ADT is smaller than that obtained by other methods. Finally, illustrative examples with numerical simulations have been studied by using the obtained results to show the effectiveness and correctness of the obtained results.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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