Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2014, Article ID 181598, 6 pages
http://dx.doi.org/10.1155/2014/181598

Research Article

Best Proximity Point for α-ψ-Proximal Contractive Multimaps

Muhammad Usman Ali,[1] Tayyab Kamran,[2] and Naseer Shahzad[3]

[1] *Department of Mathematics, School of Natural Sciences, National University of Sciences and Technology H-12, Islamabad 44000, Pakistan*
[2] *Department of Mathematics, Quaid-i-Azam University, Islamabad 45320, Pakistan*
[3] *Department of Mathematics, King Abdulaziz University, Jeddah 21589, Saudi Arabia*

Correspondence should be addressed to Naseer Shahzad; nshahzad@kau.edu.sa

Received 27 April 2014; Accepted 21 June 2014; Published 24 July 2014

Academic Editor: Poom Kumam

We extend the notions of α-ψ-proximal contraction and α-proximal admissibility to multivalued maps and then using these notions we obtain some best proximity point theorems for multivalued mappings. Our results extend some recent results by Jleli and those contained therein. Some examples are constructed to show the generality of our results.

1. Introduction and Preliminaries

Samet et al. [1] introduced the notion of α-ψ-contractive type mappings and proved some fixed point theorems for such mappings in the frame work of complete metric spaces. Karapınar and Samet [2] generalized α-ψ-contractive type mappings and obtained some fixed point theorems for generalized α-ψ-contractive type mapping. Some interesting multivalued generalizations of α-ψ-contractive type mappings are available in [3–12]. Recently, Jleli and Samet [13] introduced the notion of α-ψ-proximal contractive type mappings and proved some best proximity point theorems. Many authors obtained best proximity point theorems in different setting; see, for example, [13–35]. Abkar and Gbeleh [16] and Al-Thagafi and Shahzad [18, 20] investigated best proximity points for multivalued mappings. The purpose of this paper is to extend the results of Jleli and Samet [13] for nonself multivalued mappings. To demonstrate generality of our main result we have constructed some examples.

Let (X, d) be a metric space. For $A, B \subset X$, we use the following notations: $\text{dist}(A, B) = \inf\{d(a, b) : a \in A, b \in B\}$, $D(x, B) = \inf\{d(x, b) : b \in B\}$, $A_0 = \{a \in A : d(a, b) = \text{dist}(A, B) \text{ for some } b \in B\}$, $B_0 = \{b \in B : d(a, b) = \text{dist}(A, B) \text{ for some } a \in A\}$, $2^X \setminus \emptyset$ is the set of all nonempty subsets of X, $CL(X)$ is the set of all nonempty closed subsets of X, and $K(X)$ is the set of all nonempty compact subsets of X. For every $A, B \in CL(X)$, let

$$H(A, B) = \begin{cases} \max\left\{\sup_{x\in A} d(x, B), \sup_{y\in B} d(y, A)\right\} & \text{if the maximum exists;} \\ \infty & \text{otherwise.} \end{cases} \quad (1)$$

Such a map H is called the generalized Hausdorff metric induced by d. A point $x^* \in X$ is said to be the best proximity point of a mapping $T : A \to B$ if $d(x^*, Tx^*) = \text{dist}(A, B)$. When $A = B$, the best proximity point reduces to fixed point of the mapping T.

Definition 1 (see [28]). Let (A, B) be a pair of nonempty subsets of a metric space (X, d) with $A_0 \neq \emptyset$. Then the pair (A, B) is said to have the weak P-property if and only if, for any $x_1, x_2 \in A$ and $y_1, y_2 \in B$,

$$\begin{aligned} d(x_1, y_1) &= \text{dist}(A, B) \\ d(x_2, y_2) &= \text{dist}(A, B) \end{aligned} \implies d(x_1, x_2) \leq d(y_1, y_2). \quad (2)$$

Example 2. Let $X = \{(0,1),(1,0),(0,3),(3,0)\}$, endowed with the usual metric d. Let $A = \{(0,1),(1,0)\}$ and $B = \{(0,3),(3,0)\}$. Then for

$$d((0,1),(0,3)) = \text{dist}(A,B), \quad d((1,0),(3,0)) = \text{dist}(A,B), \tag{3}$$

we have

$$d((0,1),(1,0)) < d((0,3),(3,0)). \tag{4}$$

Also, $A_0 \neq \emptyset$. Thus, the pair (A,B) satisfies weak P-property.

Definition 3 (see [13]). Let $T : A \to B$ and $\alpha : A \times A \to [0,\infty)$. We say that T is an α-proximal admissible if

$$\left.\begin{array}{l}\alpha(x_1,x_2) \geq 1\\ d(u_1,Tx_1) = \text{dist}(A,B)\\ d(u_2,Tx_2) = \text{dist}(A,B)\end{array}\right\} \Longrightarrow \alpha(u_1,u_2) \geq 1, \tag{5}$$

where $x_1, x_2, u_1, u_2 \in A$.

Example 4. Let $X = \mathbb{R}\times\mathbb{R}$, endowed with the usual metric d. Let a be any fixed positive real number, $A = \{(a,y) : y \in \mathbb{R}\}$ and $B = \{(0,y) : y \in \mathbb{R}\}$. Define $T : A \to B$ by

$$T(a,y) = \begin{cases}\left(0,\dfrac{y}{4}\right) & \text{if } y \geq 0\\ (0,4y) & \text{if } y < 0.\end{cases} \tag{6}$$

Define $\alpha : A \times A \to [0,\infty)$ by

$$\alpha((a,x),(a,y)) = \begin{cases}2 & \text{if } x, y \geq 0\\ 0 & \text{otherwise.}\end{cases} \tag{7}$$

Let $w_1 = (a,y_1)$, $w_2 = (a,y_2)$, $w_3 = (a,y_3)$, and $w_4 = (a,y_4)$ be arbitrary points from A satisfying

$$\alpha(w_1,w_2) = 2, \tag{8}$$

$$d(w_3,Tw_1) = a = \text{dist}(A,B), \quad d(w_4,Tw_2) = a = \text{dist}(A,B). \tag{9}$$

It follows from (8) that $y_1, y_2 \geq 0$. Further, from (9), $y_3 = y_1/4$ and $y_4 = y_2/4$, which implies that $y_3, y_4 \geq 0$. Hence, $\alpha(w_3,w_4) = 2$. Therefore, T is an α-proximal admissible map.

Let Ψ denote the set of all functions ψ: $[0,\infty) \to [0,\infty)$ satisfying the following properties:

(a) ψ is monotone nondecreasing;

(b) $\sum_{n=1}^{\infty}\psi^n(t) < \infty$ for each $t > 0$.

Definition 5 (see [13]). A nonself mapping $T : A \to B$ is said to be an α-ψ-proximal contraction, if

$$\alpha(x,y)\,d(Tx,Ty) \leq \psi(d(x,y)) \quad \forall x, y \in A, \tag{10}$$

where $\alpha : A \times A \to [0,\infty)$ and $\psi \in \Psi$.

Example 6. Let us consider Example 4 again with $\psi(t) = t/2$ for each $t \geq 0$. Then it is easy to see that, for each $w_1, w_2 \in A$, we have

$$\alpha(w_1,w_2)\,d(Tw_1,Tw_2) \leq \frac{1}{2}\left|w_1 - w_2\right| = \psi(d(w_1,w_2)). \tag{11}$$

Thus, T is an α-ψ-proximal contraction.

The following are main results of [13].

Theorem 7 (see [13], Theorem 3.1). *Let A and B be two nonempty closed subsets of a complete metric space (X,d) such that A_0 is nonempty. Let $\alpha : A \times A \to [0,\infty)$ and $\psi \in \Psi$. Suppose that $T : A \to B$ be a mappings satisfying the following conditions:*

(i) *$T(A_0) \subseteq B_0$ and (A,B) satisfies the P-property;*

(ii) *T is an α-proximal admissible;*

(iii) *there exist elements $x_0, x_1 \in A_0$ such that*

$$d(x_1,Tx_0) = \text{dist}(A,B), \quad \alpha(x_0,x_1) \geq 1; \tag{12}$$

(iv) *T is a continuous α-ψ-proximal contraction.*

Then there exists an element $x^* \in A_0$ such that $d(x^*,Tx^*) = \text{dist}(A,B)$.

(C) If $\{x_n\}$ is a sequence in A such that $\alpha(x_n,x_{n+1}) \geq 1$ for all n and $x_n \to x \in A$ as $n \to \infty$, then there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $\alpha(x_{n_k},x) \geq 1$ for all k.

Theorem 8 (see [13], Theorem 3.2). *Let A and B be two nonempty closed subsets of a complete metric space (X,d) such that A_0 is nonempty. Let $\alpha : A \times A \to [0,\infty)$ and $\psi \in \Psi$. Suppose that $T : A \to B$ is a mapping satisfying the following conditions:*

(i) *$T(A_0) \subseteq B_0$ and (A,B) satisfies the P-property;*

(ii) *T is an α-proximal admissible;*

(iii) *there exist elements x_0 and $x_1 \in A_0$ such that*

$$d(x_1,Tx_0) = \text{dist}(A,B), \quad \alpha(x_0,x_1) \geq 1; \tag{13}$$

(iv) *property (C) holds and T is an α-ψ-proximal contraction.*

Then there exists an element $x^* \in A_0$ such that $d(x^*,Tx^*) = \text{dist}(A,B)$.

Definition 9 (see [16]). An element $x^* \in A$ is said to be the best proximity point of a multivalued nonself mapping T, if $D(x^*,Tx^*) = \text{dist}(A,B)$.

2. Main Result

We start this section by introducing following definition.

Definition 10. Let A and B be two nonempty subsets of a metric space (X,d). A mapping $T : A \to 2^B \setminus \emptyset$ is called α-proximal admissible if there exists a mapping $\alpha : A \times A \to [0,\infty)$ such that

$$\left.\begin{array}{l}\alpha(x_1,x_2) \geq 1\\ d(u_1,y_1) = \operatorname{dist}(A,B)\\ d(u_2,y_2) = \operatorname{dist}(A,B)\end{array}\right\} \Longrightarrow \alpha(u_1,u_2) \geq 1, \quad (14)$$

where $x_1, x_2, u_1, u_2 \in A$, $y_1 \in Tx_1$, and $y_2 \in Tx_2$.

Definition 11. Let A and B be two nonempty subsets of a metric space (X,d). A mapping $T : A \to CL(B)$ is said to be an α-ψ-proximal contraction, if there exist two functions $\psi \in \Psi$ and $\alpha : A \times A \to [0,\infty)$ such that

$$\alpha(x,y) H(Tx,Ty) \leq \psi(d(x,y)), \quad \forall x,y \in A. \quad (15)$$

Lemma 12 (see [5]). *Let (X,d) be a metric space and $B \in CL(X)$. Then for each $x \in X$ with $d(x,B) > 0$ and $q > 1$, there exists an element $b \in B$ such that*

$$d(x,b) < qd(x,B). \quad (16)$$

Now we are in position to state and prove our first result.

Theorem 13. *Let A and B be two nonempty closed subsets of a complete metric space (X,d) such that A_0 is nonempty. Let $\alpha : A \times A \to [0,\infty)$ and $\psi \in \Psi$ be a strictly increasing map. Suppose that $T : A \to CL(B)$ is a mapping satisfying the following conditions:*

(i) *$Tx \subseteq B_0$ for each $x \in A_0$ and (A,B) satisfies the weak P-property;*

(ii) *T is an α-proximal admissible;*

(iii) *there exist elements $x_0, x_1 \in A_0$ and $y_1 \in Tx_0$ such that*

$$d(x_1,y_1) = \operatorname{dist}(A,B), \quad \alpha(x_0,x_1) \geq 1; \quad (17)$$

(iv) *T is a continuous α-ψ-proximal contraction.*

Then there exists an element $x^* \in A_0$ such that $D(x^*,Tx^*) = \operatorname{dist}(A,B)$.

Proof. From condition (iii), there exist elements $x_0, x_1 \in A_0$ and $y_1 \in Tx_0$ such that

$$d(x_1,y_1) = \operatorname{dist}(A,B), \quad \alpha(x_0,x_1) \geq 1. \quad (18)$$

Assume that $y_1 \notin Tx_1$; for otherwise x_1 is the best proximity point. From condition (iv), we have

$$\begin{aligned} 0 &< d(y_1,Tx_1) \leq H(Tx_0,Tx_1)\\ &\leq \alpha(x_0,x_1) H(Tx_0,Tx_1) \leq \psi(d(x_0,x_1)). \end{aligned} \quad (19)$$

For $q > 1$, it follows from Lemma 12 that there exists $y_2 \in Tx_1$ such that

$$0 < d(y_1,y_2) < qd(y_1,Tx_1). \quad (20)$$

From (19) and (20), we have

$$0 < d(y_1,y_2) < qd(y_1,Tx_1) \leq q\psi(d(x_0,x_1)). \quad (21)$$

As $y_2 \in Tx_1 \subseteq B_0$, there exists $x_2 \neq x_1 \in A_0$ such that

$$d(x_2,y_2) = \operatorname{dist}(A,B); \quad (22)$$

for otherwise x_1 is the best proximity point. As (A,B) satisfies the weak P-property, from (18) and (22), we have

$$0 < d(x_1,x_2) \leq d(y_1,y_2). \quad (23)$$

From (21) and (23), we have

$$0 < d(x_1,x_2) < qd(y_1,Tx_1) \leq q\psi(d(x_0,x_1)). \quad (24)$$

Since ψ is strictly increasing, we have $\psi(d(x_1,x_2)) < \psi(q\psi(d(x_0,x_1)))$. Put $q_1 = \psi(q\psi(d(x_0,x_1)))/\psi(d(x_1,x_2))$. Also, we have $\alpha(x_0,x_1) \geq 1$, $d(x_1,y_1) = \operatorname{dist}(A,B)$, and $d(x_2, y_2) = \operatorname{dist}(A,B)$. Since T is an α-proximal admissible, then $\alpha(x_1,x_2) \geq 1$. Thus we have

$$d(x_2,y_2) = \operatorname{dist}(A,B), \quad \alpha(x_1,x_2) \geq 1. \quad (25)$$

Assume that $y_2 \notin Tx_2$; for otherwise x_2 is the best proximity point. From condition (iv), we have

$$\begin{aligned} 0 &< d(y_2,Tx_2) \leq H(Tx_1,Tx_2)\\ &\leq \alpha(x_1,x_2) H(Tx_1,Tx_2) \leq \psi(d(x_1,x_2)). \end{aligned} \quad (26)$$

For $q_1 > 1$, it follows from Lemma 12 that there exists $y_3 \in Tx_2$ such that

$$0 < d(y_2,y_3) < q_1 d(y_2,Tx_2). \quad (27)$$

From (26) and (27), we have

$$\begin{aligned} 0 &< d(y_2,y_3) < q_1 d(y_2,Tx_2) \leq q_1 \psi(d(x_1,x_2))\\ &= \psi(q\psi(d(x_0,x_1))). \end{aligned} \quad (28)$$

As $y_3 \in Tx_2 \subseteq B_0$, there exists $x_3 \neq x_2 \in A_0$ such that

$$d(x_3,y_3) = \operatorname{dist}(A,B); \quad (29)$$

for otherwise x_2 is the best proximity point. As (A,B) satisfies the weak P-property, from (25) and (29), we have

$$0 < d(x_2,x_3) \leq d(y_2,y_3). \quad (30)$$

From (28) and (30), we have

$$\begin{aligned} 0 &< d(x_2,x_3) < q_1 d(y_2,Tx_2) \leq q_1 \psi(d(x_1,x_2))\\ &= \psi(q\psi(d(x_0,x_1))). \end{aligned} \quad (31)$$

Since ψ is strictly increasing, we have $\psi(d(x_2, x_3)) < \psi^2(q\psi(d(x_0, x_1)))$. Put $q_2 = \psi^2(q\psi(d(x_0, x_1)))/\psi(d(x_2, x_3))$. Also, we have $\alpha(x_1, x_2) \geq 1$, $d(x_2, y_2) = \text{dist}(A, B)$, and $d(x_3, y_3) = \text{dist}(A, B)$. Since T is an α-proximal admissible then $\alpha(x_2, x_3) \geq 1$. Thus, we have

$$d(x_3, y_3) = \text{dist}(A, B), \quad \alpha(x_2, x_3) \geq 1. \tag{32}$$

Continuing in the same way, we get sequences $\{x_n\}$ in A_0 and $\{y_n\}$ in B_0, where $y_n \in Tx_{n-1}$ for each $n \in \mathbb{N}$ such that

$$d(x_{n+1}, y_{n+1}) = \text{dist}(A, B), \quad \alpha(x_n, x_{n+1}) \geq 1, \tag{33}$$

$$d(y_{n+1}, y_{n+2}) < \psi^n(q\psi(d(x_0, x_1))). \tag{34}$$

As $y_{n+2} \in Tx_{n+1} \subseteq B_0$, there exists $x_{n+2} \neq x_{n+1} \in A_0$ such that

$$d(x_{n+2}, y_{n+2}) = \text{dist}(A, B). \tag{35}$$

Since (A, B) satisfies the weak P-property form (33) and (35), we have $d(x_{n+1}, x_{n+2}) \leq d(y_{n+1}, y_{n+2})$. Then from (34), we have

$$d(x_{n+1}, x_{n+2}) < \psi^n(q\psi(d(x_0, x_1))). \tag{36}$$

For $n > m$ we have

$$d(x_n, x_m) \leq \sum_{i=n}^{m-1} d(x_i, x_{i+1}) < \sum_{i=n}^{m-1} \psi^{i-1}(q\psi(d(x_0, x_1))). \tag{37}$$

Hence, $\{x_n\}$ is a Cauchy sequence in A. Similarly, we show that $\{y_n\}$ is a Cauchy sequence in B. Since A and B are closed subsets of a complete metric space, there exist x^* in A and y^* in B such that $x_n \to x^*$ and $y_n \to y^*$ as $n \to \infty$. By (35), we conclude that $d(x^*, y^*) = \text{dist}(A, B)$ as $n \to \infty$. Since T is continuous and $y_n \in Tx_{n-1}$, we have $y^* \in Tx^*$. Hence, $\text{dist}(A, B) \leq D(x^*, Tx^*) \leq d(x^*, y^*) = \text{dist}(A, B)$. Therefore, x^* is the best proximity point of the mapping T. □

Theorem 14. *Let A and B be two nonempty closed subsets of a complete metric space (X, d) such that A_0 is nonempty. Let $\alpha : A \times A \to [0, \infty)$ and $T : A \to K(B)$ be mappings satisfying the following conditions:*

(i) *$Tx \subseteq B_0$ for each $x \in A_0$ and (A, B) satisfies the weak P-property;*

(ii) *T is an α-proximal admissible;*

(iii) *there exist elements $x_0, x_1 \in A_0$ and $y_1 \in Tx_0$ such that*

$$d(x_1, y_1) = \text{dist}(A, B), \quad \alpha(x_0, x_1) \geq 1; \tag{38}$$

(iv) *T is a continuous α-ψ-proximal contraction.*

Then there exists an element $x^* \in A_0$ such that $D(x^*, Tx^*) =$ $\text{dist}(A, B)$.

Theorem 15. *Let A and B be two nonempty closed subsets of a complete metric space (X, d) such that A_0 is nonempty. Let $\alpha : A \times A \to [0, \infty)$ and $\psi \in \Psi$ be a strictly increasing map. Suppose that $T : A \to CL(B)$ is a mapping satisfying the following conditions:*

(i) *$Tx \subseteq B_0$ for each $x \in A_0$ and (A, B) satisfies the weak P-property;*

(ii) *T is an α-proximal admissible;*

(iii) *there exist elements $x_0, x_1 \in A_0$ and $y_1 \in Tx_0$ such that*

$$d(x_1, y_1) = \text{dist}(A, B), \quad \alpha(x_0, x_1) \geq 1; \tag{39}$$

(iv) *property (C) holds and T is an α-ψ-proximal contraction.*

Then there exists an element $x^* \in A_0$ such that $D(x^*, Tx^*) =$ $\text{dist}(A, B)$.

Proof. Following the proof of Theorem 13, there exist Cauchy sequences $\{x_n\}$ in A and $\{y_n\}$ in B such that (33) holds and $x_n \to x^* \in A$ and $y_n \to y^* \in B$ as $n \to \infty$. From the condition (C), there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $\alpha(x_{n_k}, x^*) \geq 1$ for all k. Since T is an α-ψ-proximal contraction, we have

$$\begin{aligned} H(Tx_{n_k}, Tx^*) &\leq \alpha(x_{n_k}, x^*) H(Tx_{n_k}, Tx^*) \\ &\leq \psi(d(x_{n_k}, x^*)), \quad \forall k. \end{aligned} \tag{40}$$

Letting $k \to \infty$ in the above inequality, we get $Tx_{n_k} \to Tx^*$. By continuity of the metric d, we have

$$d(x^*, y^*) = \lim_{k \to \infty} d(x_{n_k+1}, y_{n_k+1}) = \text{dist}(A, B). \tag{41}$$

Since $y_{n_k+1} \in T_{x_{n_k}}$, $y_{n_k} \to y^*$, and $Tx_{n_k} \to Tx^*$, then $y^* \in Tx^*$. Hence, $\text{dist}(A, B) \leq D(x^*, Tx^*) \leq d(x^*, y^*) = \text{dist}(A, B)$. Therefore, x^* is the best proximity point of the mapping T. □

Theorem 16. *Let A and B be two nonempty closed subsets of a complete metric space (X, d) such that A_0 is nonempty. Let $\alpha : A \times A \to [0, \infty)$ and $T : A \to K(B)$ be mappings satisfying the following conditions:*

(i) *$Tx \subseteq B_0$ for each $x \in A_0$ and (A, B) satisfies the weak P-property;*

(ii) *T is an α-proximal admissible;*

(iii) *there exist elements $x_0, x_1 \in A_0$ and $y_1 \in Tx_0$ such that*

$$d(x_1, y_1) = \text{dist}(A, B), \quad \alpha(x_0, x_1) \geq 1; \tag{42}$$

(iv) *property (C) holds and T is an α-ψ-proximal contraction.*

Then there exists an element $x^* \in A_0$ such that $D(x^*, Tx^*) =$ $\text{dist}(A, B)$.

Example 17. Let $X = [0, \infty) \times [0, \infty)$ be endowed with the usual metric d. Suppose that $A = \{(1/2, x) : 0 \leq x < \infty\}$ and $B = \{(0, x) : 0 \leq x < \infty\}$. Define $T : A \to CL(B)$ by

$$T\left(\frac{1}{2}, a\right) = \begin{cases} \left\{\left(0, \dfrac{x}{2}\right) : 0 \leq x \leq a\right\} & \text{if } a \leq 1 \\ \left\{\left(0, x^2\right) : 0 \leq x \leq a^2\right\} & \text{if } a > 1, \end{cases} \tag{43}$$

and $\alpha : A \times A \to [0, \infty)$ by

$$\alpha(x, y) = \begin{cases} 1 & \text{if } x, y \in \left\{\left(\dfrac{1}{2}, a\right) : 0 \leq a \leq 1\right\} \\ 0 & \text{otherwise.} \end{cases} \tag{44}$$

Let $\psi(t) = t/2$ for all $t \geq 0$. Notice that $A_0 = A$, $B_0 = B$, and $Tx \subseteq B_0$ for each $x \in A_0$. Also, the pair (A, B) satisfies the weak P-property. Let $x_0, x_1 \in \{(1/2, x) : 0 \leq x \leq 1\}$; then $Tx_0, Tx_1 \subseteq \{(0, x/2) : 0 \leq x \leq 1\}$. Consider $y_1 \in Tx_0$, $y_2 \in Tx_1$, and $u_1, u_2 \in A$ such that $d(u_1, y_1) = \operatorname{dist}(A, B)$ and $d(u_2, y_2) = \operatorname{dist}(A, B)$. Then we have $u_1, u_2 \in \{(1/2, x) : 0 \leq x \leq 1/2\}$. Hence, T is an α-proximal admissible map. For $x_0 = (1/2, 1) \in A_0$ and $y_1 = (0, 1/2) \in Tx_0$ in B_0, we have $x_1 = (1/2, 1/2) \in A_0$ such that $d(x_1, y_1) = \operatorname{dist}(A, B)$ and $\alpha(x_0, x_1) = 1$. If $x, y \in \{(1/2, a) : 0 \leq a \leq 1\}$, then we have

$$\alpha(x, y) H(Tx, Ty) = \frac{|x - y|}{2} = \frac{1}{2} d(x, y) = \psi(d(x, y)); \tag{45}$$

for otherwise

$$\alpha(x, y) H(Tx, Ty) \leq \psi(d(x, y)). \tag{46}$$

Hence, T is an α-ψ-proximal contraction. Moreover, if $\{x_n\}$ is a sequence in A such that $\alpha(x_n, x_{n+1}) = 1$ for all n and $x_n \to x \in A$ as $n \to \infty$, then there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $\alpha(x_{n_k}, x) = 1$ for all k. Therefore, all the conditions of Theorem 15 hold and T has the best proximity point.

Example 18. Let $X = [0, \infty) \times [0, \infty)$ be endowed with the usual metric d. Let $a > 1$ be any fixed real number, $A = \{(a, x) : 0 \leq x < \infty\}$ and $B = \{(0, x) : 0 \leq x < \infty\}$. Define $T : A \to CL(B)$ by

$$T(a, x) = \left\{\left(0, b^2\right) : 0 \leq b \leq x\right\} \tag{47}$$

and $\alpha : A \times A \to [0, \infty)$ by

$$\alpha((a, x), (a, y)) = \begin{cases} 1 & \text{if } x = y = 0 \\ \dfrac{1}{a(x + y)} & \text{otherwise.} \end{cases} \tag{48}$$

Let $\psi(t) = (1/a)t$ for all $t \geq 0$. Notice that $A_0 = A$, $B_0 = B$, and $Tx \subseteq B_0$ for each $x \in A_0$. If $w_1 = (a, y_1), w_2 = (a, y_2) \in A$ with either $y_1 \neq 0$ or $y_2 \neq 0$ or both are nonzero, we have

$$\begin{aligned} \alpha(w_1, w_2) H(Tw_1, Tw_2) &= \frac{1}{a(y_1 + y_2)} \left|(y_1)^2 - (y_2)^2\right| \\ &= \frac{1}{a} |y_1 - y_2| = \psi(d(w_1, w_2)); \end{aligned} \tag{49}$$

for otherwise

$$\alpha(w_1, w_2) H(Tw_1, Tw_2) = 0 = \psi(d(w_1, w_2)). \tag{50}$$

For $x_0 = (a, 1/2a) \in A_0$ and $y_1 = (0, 1/4a^2) \in Tx_0$ in B_0, we have $x_1 = (a, 1/4a^2) \in A_0$ such that $d(x_1, y_1) = a = \operatorname{dist}(A, B)$ and $\alpha(x_0, x_1) > 1$. Furthermore, it is easy to see that remaining conditions of Theorem 13 also hold. Thus, T has the best proximity point.

3. Consequences

From results of previous section, we immediately obtain the following results.

Corollary 19. *Let A and B be two nonempty closed subsets of a complete metric space (X, d) such that A_0 is nonempty. Let $\alpha : A \times A \to [0, \infty)$ and $T : A \to B$ be mappings satisfying the following conditions:*

(i) *$T(A_0) \subseteq B_0$ and (A, B) satisfies the weak P-property;*

(ii) *T is an α-proximal admissible;*

(iii) *there exist elements x_0 and x_1 in A_0 such that*

$$d(x_1, Tx_0) = \operatorname{dist}(A, B), \quad \alpha(x_0, x_1) \geq 1; \tag{51}$$

(iv) *T is a continuous α-ψ-proximal contraction.*

Then there exists an element $x^* \in A_0$ such that $d(x^*, Tx^*) = \operatorname{dist}(A, B)$.

Corollary 20. *Let A and B be two nonempty closed subsets of a complete metric space (X, d) such that A_0 is nonempty. Let $\alpha : A \times A \to [0, \infty)$ and $T : A \to B$ be mappings satisfying the following conditions:*

(i) *$T(A_0) \subseteq B_0$ and (A, B) satisfies the weak P-property;*

(ii) *T is an α-proximal admissible;*

(iii) *there exist elements x_0 and x_1 in A_0 such that*

$$d(x_1, Tx_0) = \operatorname{dist}(A, B), \quad \alpha(x_0, x_1) \geq 1; \tag{52}$$

(iv) *property (C) holds and T is an α-ψ-proximal contraction.*

Then there exists an element $x^* \in A_0$ such that $d(x^*, Tx^*) = \operatorname{dist}(A, B)$.

Remark 21. Note that Corollaries 19 and 20 generalize Theorems 7 and 8 in Section 1, respectively.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This paper was funded by the Deanship of Scientific Research (DSR), King Abdulaziz University, Jeddah. The third author, therefore, acknowledges with thanks DSR for financial support. The authors would like to thank the anonymous reviewers for their valuable comments.

References

[1] B. Samet, C. Vetro, and P. Vetro, "Fixed point theorems for α-ψ-contractive type mappings," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 75, no. 4, pp. 2154–2165, 2012.

[2] E. Karapınar and B. Samet, "Generalized α-ψ contractive type mappings and related fixed point theorems with applications," *Abstract and Applied Analysis*, vol. 2012, Article ID 793486, 17 pages, 2012.

[3] J. H. Asl, S. Rezapour, and N. Shahzad, "On fixed points of α-ψ-contractive multifunctions," *Fixed Point Theory and Applications*, vol. 2012, article 212, 2012.

[4] B. Mohammadi, S. Rezapour, and N. Shahzad, "Some results on fixed points of $(\alpha$-$\psi)$-ciric generalized multifunctions," *Fixed Point Theory and Applications*, vol. 2013, aricle 24, 2013.

[5] M. U. Ali and T. Kamran, "On (α^*, ψ)-contractive multi-valued mappings," *Fixed Point Theory and Applications*, vol. 2013, article 137, 2013.

[6] P. Amiri, S. Rezapour, and N. Shahzad, "Fixed points of generalized $(\alpha$-$\psi)$-contractions," *Revista de la Real Academia de Ciencias Exactas, Fisicas y Naturales. Serie A. Matematicas*, 2013.

[7] G. Minak and I. Altun, "Some new generalizations of Mizoguchi-Takahashi type fixed point theorem," *Journal of Inequalities and Applications*, vol. 2013, article 493, 2013.

[8] M. U. Ali, T. Kamran, W. Sintunavarat, and P. Katchang, "Mizoguchi-Takahashi's fixed point theorem with α, η functions," *Abstract and Applied Analysis*, vol. 2013, Article ID 418798, 4 pages, 2013.

[9] C. M. Chen and E. Karapinar, "Fixed point results for the α-Meir-Keeler contraction on partial Hausdorff metric spaces," *Journal of Inequalities and Applications*, vol. 2013, no. 410, 2013.

[10] M. U. Ali, T. Kamran, and E. Karapınar, "(α,ψ,ξ)-contractive multivalued mappings," *Fixed Point Theory and Applications*, vol. 2014, article 7, 2014.

[11] M. U. Ali, T. Kamran, and E. Karapinar, "A new approach to (α, ψ)-contractive nonself multivalued mappings," *Journal of Inequalities and Applications*, vol. 2014, article 71, 2014.

[12] M. U. Ali, Q. Kiran, and N. Shahzad, "Fixed point theorems for multi-valued mappings involving α-function," *Abstract and Applied Analysis*, vol. 2014, Article ID 409467, 6 pages, 2014.

[13] M. Jleli and B. Samet, "Best proximity points for $(\alpha$-$\psi)$-proximal contractive type mappings and applications," *Bulletin des Sciences Mathématiques*, vol. 137, no. 8, pp. 977–995, 2013.

[14] A. Abkar and M. Gabeleh, "Best proximity points for asymptotic cyclic contraction mappings," *Nonlinear Analysis*, vol. 74, no. 18, pp. 7261–7268, 2011.

[15] A. Abkar and M. Gabeleh, "Best proximity points for cyclic mappings in ordered metric spaces," *Journal of Optimization Theory and Applications*, vol. 151, no. 2, pp. 418–424, 2011.

[16] A. Abkar and M. Gbeleh, "The existence of best proximity points for multivalued non-self mappings," *Revista de la Real Academia de Ciencias Exactas, Fisicas y Naturales. Serie A. Matematicas*, vol. 107, no. 2, pp. 319–325, 2012.

[17] M. A. Alghamdi and N. Shahzad, "Best proximity point results in geodesic metric spaces," *Fixed Point Theory and Applications*, vol. 2012, article 234, 2012.

[18] M. A. Al-Thagafi and N. Shahzad, "Best proximity pairs and equilibrium pairs for Kakutani multimaps," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 70, no. 3, pp. 1209–1216, 2009.

[19] M. A. Al-Thagafi and N. Shahzad, "Convergence and existence results for best proximity points," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 70, no. 10, pp. 3665–3671, 2009.

[20] M. A. Al-Thagafi and N. Shahzad, "Best proximity sets and equilibrium pairs for a finite family of multimaps," *Fixed Point Theory and Applications*, vol. 2008, Article ID 457069, 10 pages, 2008.

[21] M. Derafshpour, S. Rezapour, and N. Shahzad, "Best proximity points of cyclic φ-contractions in ordered metric spaces," *Topological Methods in Nonlinear Analysis*, vol. 37, no. 1, pp. 193–202, 2011.

[22] C. di Bari, T. Suzuki, and C. Vetro, "Best proximity points for cyclic Meir-Keeler contractions," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 69, no. 11, pp. 3790–3794, 2008.

[23] A. A. Eldred and P. Veeramani, "Existence and convergence of best proximity points," *Journal of Mathematical Analysis and Applications*, vol. 323, no. 2, pp. 1001–1006, 2006.

[24] J. Markin and N. Shahzad, "Best proximity points for relatively u-continuous mappings in Banach and hyperconvex spaces," *Abstract and Applied Analysis*, vol. 2013, Article ID 680186, 5 pages, 2013.

[25] S. Rezapour, M. Derafshpour, and N. Shahzad, "Best proximity points of cyclic ϕ-contractions on reflexive Banach spaces," *Fixed Point Theory and Applications*, vol. 2010, Article ID 946178, 7 pages, 2010.

[26] S. S. Basha, N. Shahzad, and R. Jeyaraj, "Best proximity point theorems for reckoning optimal approximate solutions," *Fixed Point Theory and Applications*, vol. 2012, article 202, 2012.

[27] C. Vetro, "Best proximity points: convergence and existence theorems for p-cyclic mappings," *Nonlinear Analysis. Theory, Methods & Applications*, vol. 73, no. 7, pp. 2283–2291, 2010.

[28] J. Zhang, Y. Su, and Q. Cheng, "A note on "a best proximity point theorem for Geraghty-contractions"," *Fixed Point Theory and Applications*, vol. 2013, article 83, 2013.

[29] C. Mongkolkeha and P. Kumam, "Best proximity point theorems for generalized cyclic contractions in ordered metric spaces," *Journal of Optimization Theory and Applications*, vol. 155, no. 1, pp. 215–226, 2012.

[30] W. Sintunavarat and P. Kumam, "Coupled best proximity point theorem in metric spaces," *Fixed Point Theory and Applications*, vol. 2012, article 93, 2012.

[31] H. K. Nashine, C. Vetro, and P. Kumam, "Best proximity point theorems for rational proximal contractions," *Fixed Point Theory and Applications*, vol. 2013, article 95, 2013.

[32] Y. J. Cho, A. Gupta, E. Karapinar, P. Kumam, and W. Sintunavarat, "Tripled best proximity point theorem in metric spaces," *Mathematical Inequalities & Applications*, vol. 16, no. 4, pp. 1197–1216, 2013.

[33] C. Mongkolkeha, C. Kongban, and P. Kumam, "Existence and uniqueness of best proximity points for generalized almost contractions," *Abstract and Applied Analysis*, vol. 2014, Article ID 813614, 11 pages, 2014.

[34] P. Kumam, P. Salimi, and C. Vetro, "Best proximity point results for modified α-proximal C-contraction mappings," *Fixed Point Theory and Applications*, vol. 2014, article 99, 2014.

[35] V. Pragadeeswarar, M. Marudai, P. Kumam, and K. Sitthithakerngkiet, "The existence and uniqueness of coupled best proximity point for proximally coupled contraction in a complete ordered metric space," *Abstract and Applied Analysis*, vol. 2014, Article ID 274062, 7 pages, 2014.