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Research Article

Benjamin-Ono-Burgers-MKdV Equation for Algebraic Rossby Solitary Waves in Stratified Fluids and Conservation Laws

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In the paper, by using multiple-scale method, the Benjamin-Ono-Burgers-MKdV (BO-B-MKdV) equation is obtained which governs algebraic Rossby solitary waves in stratified fluids. This equation is first derived for Rossby waves. By analysis and calculation, some conservation laws are derived from the BO-B-MKdV equation without dissipation. The results show that the mass, momentum, energy, and velocity of the center of gravity of algebraic Rossby waves are conserved and the presence of a small dissipation destroys these conservations.

1. Introduction

Nonlinear waves are crucial for the dynamics of the ocean and atmosphere [1–4]; Rossby waves hold a central position in nonlinear waves. In the past decades, much attention has been paid to the Rossby solitary waves. A number of analytical studies of evolution equations governing such waves have been carried out. According to the governing equation, Rossby solitary waves could roughly be divided into two categories: one is the classical solitary waves and the evolution of Rossby waves obeys the KdV type equation, such as KdV equation [5, 6], MKdV equation [7, 8], and Boussinesq equation [9]; the outstanding feature of this type solitary waves is that they are very stable and such a set of solitary waves are called soliton; the other is the algebraic solitary waves and the behavior of Rossby waves is governed by an integrodifferential equation, including Benjamin-Ono (BO) equation [10, 11], Intermediate-Long-Wave (ILW) equation [12], and Boussinesq-BO equation [13]; furthermore, the waveform of the algebraic solitary waves vanishes algebraically as $|x| \to \infty$.

The aim of the paper is to derive a new equation which governs the behavior of algebraic Rossby solitary waves. From the quasigeostrophic potential vorticity equation in stratified fluids, a new equation (BO-B-MKdV) is generated and is suitable for describing the evolution of Rossby solitary waves. The BO-B-MKdV equation includes dissipation effect and dispersion effect. It is a meaningful expanding for the results in [6, 10]. Based on the BO-B-MKdV equation, the conservation laws associated with the equation and dissipation effect are discussed and some conserved quantities of Rossby solitary waves are obtained.

2. Mathematics Model

The adiabatic potential vorticity equation is in the following form [14]:

$$\left(\frac{\partial}{\partial t}+\frac{\partial \Psi}{\partial x}\frac{\partial}{\partial y}-\frac{\partial \Psi}{\partial y}\frac{\partial}{\partial x}\right)\left[\nabla^2\Psi+f+\frac{f}{\rho_s}\left(\frac{\rho_s}{s}\frac{\partial \Psi}{\partial z}\right)\right]=0, \tag{1}$$

where Ψ is the dimensionless stream function; $s = N^2/f$, $N(z)$ is the Brunt-Vaisala frequency and is a measure of stability of the stratification; f is called Coriolis parameter; ρ_s is density; and $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ denotes the two-dimensional Laplace operator.

The lower boundary condition can be obtained from the thermal equation including dissipation as follows:

$$\left(\frac{\partial}{\partial t}+\frac{\partial \Psi}{\partial x}\frac{\partial}{\partial y}-\frac{\partial \Psi}{\partial y}\frac{\partial}{\partial x}\right)\frac{\partial \Psi}{\partial z}+\lambda s\left(\frac{\partial^2 \Psi}{\partial x^2}+\frac{\partial^2 \Psi}{\partial y^2}\right)=0, \tag{2}$$

where $\lambda > 0$ is dissipation coefficient.

In order to consider the role of nonlinearity, assume the following shear flows:

$$U=\begin{cases} u_1 & y<-L, \\ u(y,z) & -L \geq y \leq L, \\ u_2 & y>L, \end{cases} \tag{3}$$

where u_1, u_2 are constants. For simplicity, $u(y)$ is assumed to be smooth across $y = -L$ and $y = L$. In order to consider weakly nonlinear perturbation on a zonal flow, assume

$$\Psi=-\int^{y}\left(U(s,z)+\varepsilon^2\alpha\right)ds+\varepsilon\psi, \tag{4}$$

and then in the domain $[-L, L]$, we take $\lambda = \varepsilon^3\lambda_0$. Equations (1) and (2) can be written as the following perturbation equations:

$$\begin{gathered}\left[\frac{\partial}{\partial t}+\left(u+\varepsilon^2\alpha\right)\frac{\partial}{\partial x}+\varepsilon\left(\frac{\partial \psi}{\partial x}\frac{\partial}{\partial y}-\frac{\partial \psi}{\partial y}\frac{\partial}{\partial x}\right)\right] \\ \times\left[\nabla^2\psi+\frac{f}{\rho_s}\frac{\partial}{\partial z}\left(\frac{\rho_s}{s}\frac{\partial \psi}{\partial z}\right)\right]+\overline{\beta}\frac{\partial \psi}{\partial x}=0, \\ \left[\frac{\partial}{\partial t}+\left(u+\varepsilon^2\alpha\right)\frac{\partial}{\partial x}+\varepsilon\left(\frac{\partial \psi}{\partial x}\frac{\partial}{\partial y}-\frac{\partial \psi}{\partial y}\frac{\partial}{\partial x}\right)\right] \\ \times\frac{\partial \psi}{\partial z}-\frac{\partial u}{\partial z}\frac{\partial \psi}{\partial x}+\varepsilon^3\lambda_0 s\nabla^2\psi=0, \\ z=0,\end{gathered} \tag{5}$$

where $\overline{\beta} = \beta - (\partial^2 u/\partial y^2) - (f/\rho_s)(\partial/\partial z)((\rho_s/s)(\partial u/\partial z))$ and β is a constant. The upper boundary condition is as follows:

$$\rho_s\psi \longrightarrow 0, \qquad z \longrightarrow \infty. \tag{6}$$

In the domains $(-\infty, -L)$ and (L, ∞), the parameter β is smaller than that in the domain $[-L, L]$; here we assume $\beta = 0$ for $|y| > L$. Then, the governing equations in these areas are

$$\begin{gathered}\left[\frac{\partial}{\partial t}+\left(u_{1,2}+\varepsilon^2\alpha\right)\frac{\partial}{\partial x}+\varepsilon\left(\frac{\partial \psi}{\partial x}\frac{\partial}{\partial y}-\frac{\partial \psi}{\partial y}\frac{\partial}{\partial x}\right)\right] \\ \times\left[\nabla^2\psi+\frac{f}{\rho_s}\frac{\partial}{\partial z}\left(\frac{\rho_s}{s}\frac{\partial \psi}{\partial z}\right)\right]=0, \quad |y|>L.\end{gathered} \tag{7}$$

Here, the subscript denotes that for $y < -L$ and for $y > L$, respectively.

In order to achieve a balance between nonlinearity and dispersion, we introduce the following stretching transformation and the perturbation expansion of ψ_{in} in the domain $[-L, L]$:

$$\begin{gathered}X=\varepsilon x, \qquad T=\varepsilon^3 t, \qquad y=y, \qquad z=z; \\ \psi_{\text{in}}=\psi_1(X,y,T)+\varepsilon\psi_2(X,y,T)+\cdots.\end{gathered} \tag{8}$$

Separating ψ_1 as $\psi_1 = A(X,T)\phi_1(y,z)$, defining the operator L as

$$L=\frac{\partial^2}{\partial y^2}+\frac{f}{\rho_s}\frac{\partial}{\partial z}\left(\frac{\rho_s}{s}\frac{\partial}{\partial z}\right)+\frac{\overline{\beta}}{u}, \tag{9}$$

then substituting (8) into (5), we get $O(\varepsilon)$:

$$\begin{gathered}L\left(\phi_1(y,z)\right)=0; \\ u\frac{\partial \phi_1}{\partial z}-\frac{\partial u}{\partial z}\phi_1=0, \quad z=0; \\ \rho_s\phi_1 \longrightarrow 0, \qquad z \longrightarrow \infty.\end{gathered} \tag{10}$$

Equation (10) is an eigenvalue problem and describes the space structure of the wave along direction. $A(X,T)$ is the unknown amplitude in the order $O(\varepsilon)$ and needs to solve higher order equations.

Assuming $\psi_2 = (1/2)A^2(X,T)\phi_2(y,z)$, proceeding to $O(\varepsilon^2)$, we obtain

$$\begin{gathered}L\left(\phi_2(y,z)\right)=\left(\frac{\overline{\beta}}{u}\right)_y\frac{\phi_1^2}{u}; \\ \rho_s\phi_2 \longrightarrow 0, \qquad z \longrightarrow \infty; \\ u\frac{\partial \phi_2}{\partial z}-\frac{\partial u}{\partial z}\phi_2=-\left(\phi_1\frac{\partial^2 \phi_1}{\partial y\partial z}-\frac{\partial \phi_1}{\partial y}\frac{\partial \phi_1}{\partial z}\right), \quad z=0.\end{gathered} \tag{11}$$

To $O(\varepsilon^3)$, we have

$$\begin{aligned} uL\left(\frac{\partial \psi_3}{\partial X}\right)&=\left(\frac{\partial A}{\partial T}+\alpha\frac{\partial A}{\partial X}\right)\frac{\overline{\beta}\phi_1}{u} \\ &\quad+A^2\frac{\partial A}{\partial X}\left\{\frac{\phi_1\phi_2}{2u}\left(\frac{\overline{\beta}}{u}\right)_y-\frac{\phi_1^3}{2}\left[\frac{1}{u}\left(\frac{\overline{\beta}}{u}\right)_y\right]_y\right\} \\ &\quad-u\phi_1\frac{\partial^3 A}{\partial X^3}; \\ u\frac{\partial}{\partial z}\frac{\partial \psi_3}{\partial X}-\frac{\partial u}{\partial z}\frac{\partial \psi_3}{\partial X} &=-u\left(\frac{\partial A}{\partial T}+\alpha\frac{\partial A}{\partial X}\right)\frac{\partial \phi_1}{\partial z}-A^2\frac{\partial A}{\partial X} \\ &\quad\times\left(\frac{\phi_1}{2}\frac{\partial^2 \phi_2}{\partial y\partial z}-\frac{\partial \phi_1}{\partial y}\frac{\partial \phi_2}{\partial z}+\phi_2\frac{\partial^2 \phi_1}{\partial y\partial z}\right. \\ &\quad\left.-\frac{1}{2}\frac{\partial \phi_2}{\partial y}\frac{\partial \phi_1}{\partial z}\right)+A\lambda_0 s\frac{\partial^2 \phi_1}{\partial y^2}, \quad z=0; \\ \rho_s\psi_3 &\longrightarrow 0, \qquad z \longrightarrow \infty.\end{aligned} \tag{12}$$

Multiplying both the sides of the first equation of (12) by $\rho_s\phi_1/u$ and integrating it over y and z lead to

$$\begin{aligned}
&\int_0^{+\infty}\frac{\partial}{\partial X}\rho_s\left[\phi_1\frac{\partial}{\partial y}\psi_3-\psi_3\frac{\partial}{\partial y}\phi_1\right]\Bigg|_{-L}^{L}dz \\
&\quad+\int_{-L}^{L}\frac{\partial}{\partial X}\frac{f\rho_s}{s}\left[\phi_1\frac{\partial}{\partial z}\psi_3-\psi_3\frac{\partial}{\partial z}\phi_1\right]\Bigg|_0^{+\infty}dy \\
&=\left(\frac{\partial}{\partial T}+\alpha\frac{\partial}{\partial X}\right)A\int_{-L}^{L}\int_0^{+\infty}\rho_s\frac{\phi_1^2}{u^2}\overline{\beta}\,dy\,dz \\
&\quad-\frac{\partial^3}{\partial X^3}\int_{-L}^{L}\rho_s\phi_1^2dy\,dz+A^2\frac{\partial A}{\partial X} \\
&\quad\times\int_{-L}^{L}\int_0^{+\infty}\left\{\frac{\rho_s\phi_1^2\phi_2}{2u^2}\left(\frac{\overline{\beta}}{u}\right)_y\right. \\
&\qquad\left.-\frac{\rho_s\phi_1^4}{2u}\left[\frac{1}{u}\left(\frac{\overline{\beta}}{u}\right)_y\right]_y\right\}dy\,dz.
\end{aligned}\tag{13}$$

In (13), if the boundary conditions of ϕ_1 and ψ_3 are known, the equation governing the amplitude A will be determined. So in the following, we will consider the boundary conditions of ϕ_1 and ψ_3.

For the two external regions of $|y| > L$, consider the following transformations:

$$T=\varepsilon^3 t,\qquad \xi=x,\qquad y=y,\qquad Z=\varepsilon z\tag{14}$$

and the external stream function ψ_{ex} is set to

$$\psi_{\text{ex}}=\tilde{\psi}(\xi,y,Z,T,\varepsilon).\tag{15}$$

Substituting (14) and (15) into (7) based on the lowest-order equation of the external region, we can obtain

$$\left(\frac{\partial^2}{\partial\xi^2}+\frac{\partial^2}{\partial y^2}\right)\tilde{\psi}(\xi,y,Z,T,\varepsilon)=0.\tag{16}$$

The solution of (16) satisfies

$$\begin{aligned}
\tilde{\psi}&=\frac{\pm P}{\pi}\int_{-\infty}^{+\infty}\tilde{\psi}\Big|_{\substack{\xi=\xi'\\ y=\pm L}}\frac{(y\mp L)\,d\xi'}{(y\mp L)^2+(\xi-\xi')^2},\\
\frac{\partial\tilde{\psi}}{\partial y}&=\frac{\pm P}{\pi}\int_{-\infty}^{+\infty}\tilde{\psi}\Big|_{\substack{\xi=\xi'\\ y=\pm L}}\frac{\left[(\xi-\xi')^2-(y\mp L)^2\right]d\xi'}{\left[(y\mp L)^2+(\xi-\xi')^2\right]^2},
\end{aligned}\tag{17}$$

where the upper and the lower signs denote that for $y > L$ and for $y < -L$, respectively, and P stands for the principal value of the integration. Assuming that the inner solution matches smoothly with the outer solutions at $y = \pm L$, then we obtain

$$\begin{aligned}
\left(\psi_1+\varepsilon\psi_2+\varepsilon^2\psi_3\right)\Big|_{y=\pm L}&=\tilde{\psi}\Big|_{y=\pm L}+O\left(\varepsilon^3\right),\\
\frac{\partial\left(\psi_1+\varepsilon\psi_2+\varepsilon^2\psi_3\right)}{\partial y}\Bigg|_{y=\pm L}&=\frac{\partial\tilde{\psi}}{\partial y}\Bigg|_{y=\pm L}+O\left(\varepsilon^3\right).
\end{aligned}\tag{18}$$

Then by employing (18), we get

$$\begin{aligned}
&A\phi_1(\pm L)=\tilde{\psi}\big|_{y=\pm L},\qquad \psi_2\big|_{y=\pm L}=\psi_3\big|_{y=\pm L}=0,\\
&\frac{\partial\tilde{\psi}}{\partial y}=\mp\varepsilon^2\phi_1(\pm L)\frac{\partial^2\mathcal{J}(A(X,T))}{\partial X^2},\\
&\frac{\partial\psi_3}{\partial y}\bigg|_{y=\pm L}=\mp\phi_1(\pm L)\frac{\partial^2\mathcal{J}(A(X,T))}{\partial X^2},
\end{aligned}\tag{19}$$

where $\mathcal{J}(A(X,T)) \equiv (P/\pi)\int_{-\infty}^{+\infty}A(X',T)\ln|X-X'|dX'$. Combining (13) with (19), with the help of (10), (11), and (12), yields

$$\begin{aligned}
&\frac{\partial A}{\partial T}+\alpha\frac{\partial A}{\partial X}+a_1A^2\frac{\partial A}{\partial X}+a_2\frac{\partial^3 A}{\partial X^3}+a_3A\\
&\quad+a_4\frac{\partial^3}{\partial X^3}\mathcal{J}(A(X,T))=0,
\end{aligned}\tag{20}$$

and then (20) can be rewritten as follows:

$$\begin{aligned}
&\frac{\partial A}{\partial T}+\alpha\frac{\partial A}{\partial X}+a_1A^2\frac{\partial A}{\partial X}+a_2\frac{\partial^3 A}{\partial X^3}+a_3A\\
&\quad+a_4\frac{\partial^2}{\partial X^2}\mathcal{H}(A(X,T))=0,
\end{aligned}\tag{21}$$

where

$$\begin{aligned}
a&=\left\{\int_{-L}^{L}\left[\int_0^{+\infty}\frac{\rho_2\overline{\beta}}{u^2}\phi_1^2dz-\frac{f\rho_s}{su}\phi_1\frac{\partial\phi_1}{\partial z}\bigg|_{z=0}\right]dy\right\}^{-1},\\
a_1&=a\int_{-L}^{L}\left[\int_0^{+\infty}\frac{\rho_s\phi_1^2\phi_2}{2u^2}\left(\frac{\overline{\beta}}{u}\right)_y-\frac{\rho_s\phi_1^4}{2u}\left(\frac{1}{u}\left(\frac{\overline{\beta}}{u}\right)_y\right)_y dz\right.\\
&\qquad-\frac{f\rho_s}{su}\phi_1\left(\frac{\phi_1}{2u}\frac{\partial^2\phi_2}{\partial y\,\partial z}-\frac{1}{2u}\frac{\partial\phi_2}{\partial y}\frac{\partial\phi_1}{\partial z}\right.\\
&\qquad\left.\left.-\frac{1}{u}\frac{\partial\phi_1}{\partial y}\frac{\partial\phi_2}{\partial z}+\frac{\phi_2}{u}\frac{\partial^2\phi_1}{\partial y\,\partial z}\bigg|_{z=0}\right]dy,\right.\\
a_2&=-a\int_{-L}^{L}\int_0^{+\infty}\rho_s\phi_1^2dz\,dy,\\
a_3&=-a\int_{-L}^{L}\frac{\lambda_0 f\rho_s\phi_1}{u}\frac{\partial^2\phi_1}{\partial y^2}\bigg|_{z=0}dy,\\
a_4&=a\int_0^{+\infty}\rho_s\phi_1^2\Big|_{-L}^{L}dz,\\
&\mathcal{H}(A(X,T))\equiv\frac{P}{\pi}\int_{-\infty}^{+\infty}\frac{A(X',T)}{(X-X')}dX'
\end{aligned}\tag{22}$$

is the well-known Hilbert transform.

Equation (21) is an integrodifferential equation including dissipation effect and dispersion effect. In the absence of dissipation effect and $a_4 = 0$, (21) degenerates to the MKdV equation; in the absence of dissipation effect $a_3 A$ and dispersion effect A_{XXX}, (21) degenerates to the generalized BO equation. Because the term $a_3 A$ expresses the dissipation effect and has the same physical meaning with the term $\partial^2 A/\partial X^2$ in Burgers equation, so we call (21) BO-B-MKdV equation. As we know that the BO-B-MKdV equation is first obtained here. This equation is greatly different from the common equation to describe algebraic Rossby solitary waves, such as the BO equation [10] and the BO-Burgers equation; it includes the dissipation effect and dispersion effect and has stronger nonlinearity.

3. Conservation Laws

Conservation laws are a common feature of mathematical physics and describe physical properties that remain constant throughout the various processes that occur in the physical world. In physics, "to conserve" something means "to result in no net loss of" that particular component. It is very important in the analysis of unsteady problems of wave propagation. As is known, some famous soliton equations have some conserved quantities. For example, it is also proved that the BO equation has four conservation laws in [15]. One of the goals of the present paper is to investigate the following questions. Has the BO-B-MKdV equation also conservation laws in the absence of dissipation effect? How to change these conservation quantities in the presence of dissipation effect?

Here, we assume that when $|X| \to \infty$, A, A_X, A_{XX}, and $A_{XXX} \to 0$. First, (21) can be expressed as follows:

$$\frac{\partial A}{\partial T} = -\frac{\partial}{\partial X}\left[\alpha A + \frac{1}{3}a_1 A^3 + a_2 \frac{\partial^2 A}{\partial X^2} + a_4 \frac{\partial}{\partial X}\mathscr{H}\left(A\left(X,T\right)\right)\right] - a_3 A. \tag{23}$$

Integrating (23) with respect to X over $(-\infty, +\infty)$, then we have

$$C_1 = \int_{-\infty}^{+\infty} A\,dX = \exp\left(-a_3 T\right)\int_{-\infty}^{+\infty} A\left(X,0\right)dX. \tag{24}$$

From (24), we find that C_1 decreases exponentially with the increasing of time T and the coefficient a_3, while a_3 relates to the dissipation coefficient λ_0. This shows that the dissipation effect causes the mass of solitary waves to decrease exponentially. When the dissipation effect is absent, the mass of the solitary waves is conserved.

In the following, we multiply (21) by $A(X,T)$ and reorganize the terms to obtain

$$\frac{1}{2}\frac{\partial A^2}{\partial T} = -\frac{\partial}{\partial X}\left[\frac{1}{2}\alpha A^2 + \frac{1}{4}a_1 A^4 + a_2 A\frac{\partial^2 A}{\partial X^2} + \frac{1}{2}A_X^2 + a_4 A\frac{\partial}{\partial X}\mathscr{H}\left(A\left(X,T\right)\right)\right] - A_X\mathscr{H}\left(A_X\right) - a_3 A^2. \tag{25}$$

Based on the property of the Hilbert operator $\mathscr{H}$: $\int_{-\infty}^{+\infty} f(X)\mathscr{H}(f(X))dX = 0$, in which $f(X)$ is an arbitrary function vanishing at infinity and carrying on the integration with respect to X over $(-\infty, +\infty)$ leads to

$$C_2 = \int_{-\infty}^{+\infty} A^2 dX = \exp\left(-2a_3 T\right)\int_{-\infty}^{+\infty} A^2\left(X,0\right)dX. \tag{26}$$

Equation (26) shows that the momentum of the solitary waves is conserved without dissipation. Because of the dissipation effect, the momentum of the solitary waves also decreases exponentially with the increasing of time T and the dissipative coefficient λ_0. The rate of decline of momentum is faster than the rate of mass.

From (24) and (26), we have obtained the conservation of mass and momentum of the solitary waves, now without regard to the dissipation effect, by adding $(A^2 - (a_4/a_1)\mathscr{H}(A_X)) \times$ (21) to $(\partial/\partial X)$ (21) $\times [A_X + (a_4/a_1)\mathscr{H}(A)]$ and integrating it, by virtue of the relation

$$\frac{\partial^2\mathscr{H}\left(A\right)}{\partial X^2} = \mathscr{H}\left(\frac{\partial^2 A}{\partial X^2}\right), \qquad \int_{-\infty}^{+\infty}\left(u\mathscr{H}v + v\mathscr{H}u\right)dX = 0, \tag{27}$$

after tedious calculation, we obtain

$$\frac{dC_3}{dT} = \frac{d}{dT}\int_{-\infty}^{+\infty}\left[\frac{1}{3}A^3 + \frac{1}{2}A_X^2 + \frac{a_4}{a_1}\frac{\partial A}{\partial X}\mathscr{H}\left(A\right)\right]dX = 0. \tag{28}$$

C_3 is regarded as the energy of the solitary waves. So we can conclude that the energy of the solitary waves is conserved without dissipation.

Finally, let us define a quantity related to the phase of the solitary waves:

$$\widetilde{C}_4 = \frac{d}{dT}\int_{-\infty}^{+\infty} XA\,dX. \tag{29}$$

Then, employing the momentum of the solitary waves C_2 is a time-invariant quantity and the above assumptions A, A_X, A_{XX}, and A_{XXX} vanish as $|X| \to \infty$ as well as $\lambda_0 = 0$; we are easy to deduce $d\widetilde{C}_4/dT = 0$. We construct the velocity of the center of gravity for the ensemble of such waves $C_4 = \widetilde{C}_4/C_1$ [15]. Then, because C_1 and $\widetilde{C}_4$ are time-invariant quantities, we are easy to obtain $dC_4/dT = 0$; that is, the velocity of the center of gravity is conserved without dissipation.

4. Conclusions

In this paper, a new governing equation is derived by the multiple-scale method to describe the amplitude of algebraic Rossby solitary waves in stratified fluids under the influence of dissipation. By analysis and calculation, we obtain four conserved quantities as mass, momentum, energy, and velocity of the center of gravity of algebraic Rossby solitary waves without dissipation and draw the conclusion that the dissipation effect causes the mass, the momentum, the energy, and the velocity of the center of gravity to vary. In fact, after the above four conservation laws are given, we can wonder whether there exist other conservation laws and whether there is no limit as KdV equation, which remain to be studied in the future. In addition, we will also explore to study the blocking phenomenon in the ocean and atmosphere by using the mathematical model that we establish in this paper in the future.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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