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Research Article

Periodic Solutions of Multispecies Mutualism System with Infinite Delays

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We studied the delayed periodic mutualism system with Gilpin-Ayala effect. Some new and interesting sufficient conditions are obtained to guarantee the existence of periodic solution for the multispecies mutualism system with infinite delays. Our method is based on Mawhin's coincidence degree. To the best knowledge of the authors, there is no paper considering the existence of periodic solutions for n-species mutualism system with infinite delays.

1. Introduction

Recently, there are many papers considering the existence of periodic solutions for competitive Lotka-Volterra system based on Mawhin's coincidence degree theory (see [1–4]). But there are few papers considering the periodicity of mutualism system; for example, one can refer to [5–7]. However, the references mentioned above only considered two-dimensional mutualism system. To the best knowledge of the authors, there is no paper considering the existence of periodic solutions for n-species mutualism system. It should be noted that the method used in [5–7] is difficult to be extended to the n-dimensional system. So, we employ the method used in [2–4]. However, the problem considered in this paper is completely different from those mentioned above. On the other hand, the above-mentioned works considered the models with constant discrete delays or without delays. In practice, there will be a distribution of transmission delays. In this case, the transmission of species is no longer instantaneous and cannot be modelled with discrete delays. A more appropriate way is to incorporate distributed delays. Therefore, the studies of the model with distributed delays have more important significance than the ones of the model with discrete delays. Thus, in this paper, we considered the following mutualism system with distributed delays:

$$\begin{aligned}\dot{y}_i(t) = y_i(t)\Bigg[& r_i(t) - a_{ii}(t)\, y_i(t)^{\alpha_{ij}}(t) + \sum_{j=1, j\neq i}^{n} a_{ij}(t)\, y_j^{\alpha_{ij}}(t) \\ & + \sum_{j=1, j\neq i}^{n} b_{ij}(t)\, y_j^{\beta_{ij}}\left(t-\tau_{ij}\right) \\ & + \sum_{j=1, j\neq i}^{n} c_{ij}(t)\int_0^{\infty} H_{ij}(s)\, y_j^{\gamma_{ij}}(t+s)\,ds\Bigg], \\ & i = 1, 2, \dots n,\end{aligned} \tag{1}$$

where r_i, a_{ij}, b_{ij}, and c_{ij}, $i, j = 1, 2, \dots, n$, are ω-periodic functions, that is, $r_i(t+\omega) = r_i(t)$, $a_{ij}(t+\omega) = a_{ij}(t)$, $b_{ij}(t+\omega) = b_{ij}(t)$, and $c_{ij}(t+\omega) = c_{ij}(t)$, and α_{ij}, β_{ij}, and γ_{ij} are constants, $i, j = 1, 2, \dots, n$. From biological view, r_i, a_{ij}, b_{ij}, and c_{ij}, $i, j = 1, 2, \dots, n$, are nonnegative, $\int_0^{\infty} H_{ij}(s)ds = 1$, and a_{ii} is positive. System (1) is associated with the IVP as follows:

$$y_i(t_0) = y_i^0, \quad y_i^0 > 0, \ i = 1, 2, \ldots n. \tag{2}$$

2. Existence of Periodic Solutions

For convenience, we introduce some notations, definitions, and lemmas. If $g(t)$ is a continuous ω-periodic function defined on $\mathbf{R}$, denote

$$\underline{g} = \min_{t\in[0,\omega]} |g(t)|, \qquad \overline{g} = \max_{t\in[0,\omega]} |g(t)|,$$
$$m(g) = \frac{1}{\omega}\int_0^\omega g(t)\,dt. \tag{3}$$

We also denote the spectral radius of the matrix $\mathscr{A}$ by $\rho(\mathscr{A})$. Denote

$$X = \left\{x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \in C^1(\mathbf{R}, \mathbf{R}^n) \mid x(t+\omega) = x(t) \ \ \forall t \in \mathbf{R}\right\},$$
$$Z = \left\{x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T \in C(\mathbf{R}, \mathbf{R}^n) \mid x(t+\omega) = x(t) \ \ \forall t \in \mathbf{R}\right\}. \tag{4}$$

Lemma 1 (see [8]). *Let $\Omega \subset X$ be an open and bounded set. Let L be a Fredholm mapping of index zero and N be L-compact on $\overline{\Omega}$ (i.e., $QN(\overline{\Omega})$ is bounded and $K_P(I-Q)N : \overline{\Omega} \to X$ is compact). Assume*

(i) *for each $\lambda \in (0,1)$, $x \in \partial\Omega \cap \operatorname{Dom} L$, $Lx \neq \lambda Nx$;*

(ii) *for each $x \in \partial\Omega \cap \operatorname{Ker} L$, $QNx \neq 0$ and $\deg\{JQN, \Omega \cap \operatorname{Ker} L, 0\} \neq 0$.*

Then $Lx = Nx$ has at least one solution in $\overline{\Omega} \cap \operatorname{Dom} L$.

Definition 2 (see [9, 10]). A real $n \times n$ matrix $\mathscr{A} = (a_{ij})$ is said to be an M-matrix if $a_{ij} \le 0$, $i, j = 1, 2, \ldots, n$, $i \neq j$, and $\mathscr{A}^{-1} \ge 0$.

Lemma 3 (see [9, 10]). *Let $\mathscr{A} \ge 0$ be an $n \times n$ matrix and $\rho(\mathscr{A}) < 1$; then $(E_n - \mathscr{A})^{-1} \ge 0$, where E_n denotes the identity matrix of size n.*

Theorem 4. *Assume the following.*

(H_1) *The algebraic equation*

$$f(u) := \left(m(r_i) - m(a_{ii})u_i^{\alpha_{ii}} + \sum_{j=1}^n m(a_{ij})u_j^{\alpha_{ij}} + \sum_{j=1}^n m(b_{ij})u_j^{\beta_{ij}} + \sum_{j=1, j\neq i}^n m(c_{ij})u_j^{\gamma_{ij}}\right)_{n\times 1} = 0 \tag{5}$$

has finite solutions $(u_1^*, u_2^*, \ldots, u_n^*)^T \in \mathbf{R}_+^n$ with $u_i^* > 0$ and $\sum_{u^*} \operatorname{sgn} J_f(u^*) \neq 0$.

(H_2) *$\alpha_{ji} \le \alpha_{ii}$, $\beta_{ji} \le \alpha_{ii}$, $\gamma_{ji} \le \gamma_{ii}$ $(j \neq i)$, $i, j = 1, 2, \ldots, n$.*

(H_3) *$\rho(\mathscr{K}) < 1$, where $\mathscr{K} = (\Gamma_{ij})_{n\times n}$ and*

$$\Gamma_{ij} = \begin{cases} 0, & i = j, \\ \dfrac{\overline{a}_{ij} + \overline{b}_{ij} + \overline{c}_{ij}}{\underline{a}_{jj}}, & i \neq j. \end{cases} \tag{6}$$

Then system (1) *has at least one positive ω-periodic solution.*

Proof. Note that every solution $y(t) = (y_1(t), y_2(t), \ldots y_n(t))^T$ of system (1) with the initial value condition is positive. Make the change of variables

$$y_i(t) = e^{x_i(t)}, \quad i = 1, 2, \ldots, n. \tag{7}$$

Then system (1) is the same as

$$\begin{aligned} x_i(t) &= r_i(t) - a_{ii}(t)e^{\alpha_{ij}}(t) + \sum_{j=1, j\neq i}^n a_{ij}(t)e^{\alpha_{ij}}(t) \\ &\quad + \sum_{j=1, j\neq i}^n b_{ij}(t)e^{\beta_{ij}}(t - \tau_{ij}) \\ &\quad + \sum_{j=1, j\neq i}^n c_{ij}(t)\int_0^\infty H_{ij}(s)e^{\gamma_{ij}}(t+s)\,ds, \\ &\qquad i = 1, 2, \ldots n. \end{aligned} \tag{8}$$

Obviously, if system (8) has at least one ω-periodic solution, then system (1) has at least one ω-periodic solution. To prove Theorem 4, we should find an appropriate open set Ω satisfying Lemma 1. We divide the proof into three steps.

Step 1. We verify that (i) of Lemma 1 is satisfied. For any $x(t) \in X$, by periodicity, it is easy to check that

$$\begin{aligned} \Delta_i(x,t) &= r_i(t) - a_{ii}(t)e^{\alpha_{ij}}(t) + \sum_{j=1, j\neq i}^n a_{ij}(t)e^{\alpha_{ij}}(t) \\ &\quad + \sum_{j=1, j\neq i}^n b_{ij}(t)e^{\beta_{ij}}(t - \tau_{ij}) \\ &\quad + \sum_{j=1, j\neq i}^n c_{ij}(t)\int_0^\infty H_{ij}(s)e^{\gamma_{ij}}(t+s)\,ds. \end{aligned} \tag{9}$$

And define $L : \operatorname{Dom} L \subset X \to Z$ and $N : X \to Z$ as follows:

$$X \ni x(t) \longrightarrow (Lx)(t) = \frac{dx(t)}{dt} \in Z,$$
$$X \ni x(t) \longrightarrow (Nx)(t) = ((Nx)_1(t), (Nx)_2(t), \ldots, (Nx)_n(t))^T \in Z, \tag{10}$$

where

$$(Nx)_i(t) = \Delta_i(x,t), \quad i = 1, 2, \ldots, n. \tag{11}$$

The projectors are defined by $P : X \to X$ and $Q : Z \to Z$ by

$$Px(t) = \frac{1}{\omega}\int_0^\omega x(t)\,dt, \qquad Qz(t) = \frac{1}{\omega}\int_0^\omega z(t)\,dt, \qquad x \in X, \quad z \in Z. \tag{12}$$

It is easy to follow that L is a Fredholm mapping of index zero. Furthermore, the generalized inverse (to L) $K_P : \operatorname{Im} L \to \operatorname{Dom} L \cap \operatorname{Ker} P$ exists, which is given by

$$K_P(y) = \int_0^t y(s)\,ds - \frac{1}{\omega}\int_0^\omega\int_0^t y(s)\,ds\,dt. \tag{13}$$

Then $QN : X \to Z$ and $K_P(I-Q)N : X \to X$ are defined by

$$\begin{aligned} QNx &= \left(\frac{1}{\omega}\int_0^\omega \Delta_1(x,t)\,dt, \frac{1}{\omega}\int_0^\omega \Delta_2(x,t)\,dt, \ldots, \frac{1}{\omega}\int_0^\omega \Delta_n(x,t)dt\right)^T, \\ K_P(I-Q)Nx &= (\Psi_1(x,t), \Psi_2(x,t), \ldots, \Psi_n(x,t))^T, \end{aligned} \tag{14}$$

where

$$\begin{aligned} \Psi_k(x,t) &= \int_0^t \Delta_k(x,s)\,ds - \frac{1}{\omega}\int_0^\omega\int_0^t \Delta_k(x,s)\,ds\,dt \\ &\quad - \left(\frac{t}{\omega} - \frac{1}{2}\right)\int_0^\omega \Delta_k(x,s)\,ds, \quad k = 1,2,\ldots,n. \end{aligned} \tag{15}$$

Using similar arguments to Step 1 in [2], it is easy to show that $(K_P(I-Q)Nx)(\overline{\Omega})$ is relatively compact in the space $(X, \|\cdot\|_1)$.

Step 2. In this step, we are in a position to search for an appropriate open bounded subset Ω satisfying condition (i) of Lemma 1. Specifically, our aim is to search for an appropriate h_i defined by Ω in Step 1 such that Ω satisfies condition (i) of Lemma 1. To this end, assume that $x(t) \in X$ is a solution of the equation $Lx = \lambda Nx$ for each $\lambda \in (0,1)$; that is,

$$\begin{aligned} \dot{x}_i(t) = \lambda\Bigg[& r_i(t) - a_{ii}(t)e^{\alpha_{ij}}(t) + \sum_{j=1,j\neq i}^n a_{ij}(t)e^{\alpha_{ij}}(t) \\ &+ \sum_{j=1,j\neq i}^n b_{ij}(t)e^{\beta_{ij}}\left(t-\tau_{ij}\right) \\ &+ \sum_{j=1,j\neq i}^n c_{ij}(t)\int_0^\infty H_{ij}(s)e^{\gamma_{ij}}(t+s)\,ds\Bigg], \\ & i = 1,2,\ldots n. \end{aligned} \tag{16}$$

Since $x(t) \in X$, each $x_i(t), i = 1,2,\ldots,n$, as components of $x(t)$, is continuously differentiable and ω-periodic. In view of continuity and periodicity, there exists $t_i \in [0,\omega]$ such that $x_i(t_i) = \max_{t\in[0,\omega]}|x_i(t)|$, $i = 1,2,\ldots,n$. Accordingly, $\dot{x}_i(t_i) = 0$ and we arrive at

$$\begin{aligned} & r_i(t_i) - a_{ii}(t_i)e^{\alpha_{ii}x_i(t_i)} + \sum_{j=1,j\neq i}^n a_{ij}(t_i)e^{\alpha_{ij}x_j(t_i)} \\ &\quad + \sum_{j=1,j\neq i}^n b_{ij}(t_i)e^{\beta_{ij}x_j(t_i-\tau_{ij})} \\ &\quad + \sum_{j=1,j\neq i}^n c_{ij}(t)\int_0^\infty H_{ij}(s)e^{\gamma_{ij}x_j(t_i+s)}ds = 0, \\ &\qquad i = 1,2,\ldots,n. \end{aligned} \tag{17}$$

That is,

$$\begin{aligned} & a_{ii}(t_i)e^{\alpha_{ii}x_i(t_i)} \\ &= r_i(t_i) + \sum_{j=1,j\neq i}^n a_{ij}(t_i)e^{x_j(t_i)} + \sum_{j=1,j\neq i}^n b_{ij}(t_i)e^{\beta_{ij}x_j(t_i-\tau_{ij})} \\ &\quad + \sum_{j=1,j\neq i}^n c_{ij}(t)\int_0^\infty H_{ij}(s)e^{\gamma_{ij}x_j(t_i+s)}ds, \quad i = 1,2,\ldots n. \end{aligned} \tag{18}$$

Noticing that $x_j(t_j) = \max_{t\in[0,\omega]}|x_j(t)|$ implies

$$x_j\left(t_i - \tau_{ij}\right) \le x_j\left(t_j\right). \tag{19}$$

It follows from (H_2) that

$$\begin{aligned} & \underline{a}_{ii}e^{\alpha_{ii}x_i(t_i)} \\ &\le \left|a_{ii}(t_i)e^{\alpha_{ii}x_i(t_i)}\right| \\ &= \Bigg|r_i(t_i) + \sum_{j=1,j\neq i}^n a_{ij}(t_i)e^{\alpha_{ij}x_j(t_i)} + \sum_{j=1,j\neq i}^n b_{ij}(t_i)e^{\beta_{ij}x_j(t_i-\tau_{ij})} \\ &\qquad + \sum_{j=1,j\neq i}^n c_{ij}(t)\int_0^\infty H_{ij}(s)e^{\gamma_{ij}x_j(t_i+s)}ds\Bigg| \\ &\le \overline{r}_i + \sum_{j=1,j\neq i}^n \overline{a}_{ij}e^{\alpha_{ij}x_j(t_i)} + \sum_{j=1,j\neq i}^n \overline{b}_{ij}e^{\beta_{ij}x_j(t_i-\tau_{ij})} \\ &\quad + \sum_{j=1,j\neq i}^n \overline{c}_{ij}(t)e^{\gamma_{ij}x_j(t_i+s)}\int_0^\infty H_{ij}(s)\,ds \\ &= \overline{r}_i + \sum_{j=1,j\neq i}^n \overline{a}_{ij}e^{\alpha_{ij}x_j(t_i)} + \sum_{j=1,j\neq i}^n \overline{b}_{ij}e^{\beta_{ij}x_j(t_i-\tau_{ij})} \\ &\quad + \sum_{j=1,j\neq i}^n \overline{c}_{ij}(t)e^{\gamma_{ij}x_j(t_i+s)} \\ &\le \overline{r}_i + \sum_{j=1,j\neq i}^n \overline{a}_{ij}e^{\alpha_{jj}x_j(t_j)} + \sum_{j=1,j\neq i}^n \overline{b}_{ij}e^{\alpha_{jj}x_j(t_j)} \end{aligned}$$

$$+\sum_{j=1,j\neq i}^{n}\overline{c}_{ij}(t)\,e^{\alpha_{jj}x_j(t_j)}$$
$$=\overline{r}_i+\sum_{j=1,j\neq i}^{n}\left(\overline{a}_{ij}+\overline{b}_{ij}+\overline{c}_{ij}\right)e^{\alpha_{jj}x_j(t_j)}. \tag{20}$$

Here we used (H_2). Letting $(\underline{a}_{ii}+\underline{b}_{ii})e^{\alpha_{ii}x_i(t_i)}=z_i(t_i)$, it follows from (20) that

$$z_i(t_i)\le\overline{r}_i+\sum_{j=1,j\neq i}^{n}\left(\overline{a}_{ij}+\overline{b}_{ij}+\overline{c}_{ij}\right)\underline{a}_{jj}^{-1}z_j(t_j), \tag{21}$$

or

$$z_i(t_i)-\sum_{j=1,j\neq i}^{n}\frac{\overline{a}_{ij}+\overline{b}_{ij}+\overline{c}_{ij}}{\underline{a}_{jj}}z_j(t_j)\le\overline{r}_i, \tag{22}$$

which implies

$$\begin{pmatrix} 1 & -\dfrac{\overline{a}_{12}+\overline{b}_{12}+\overline{c}_{12}}{\underline{a}_{22}} & \cdots & -\dfrac{\overline{a}_{1n}+\overline{b}_{1n}+\overline{c}_{1n}}{\underline{a}_{nn}} \\ -\dfrac{\overline{a}_{21}+\overline{b}_{21}+\overline{c}_{21}}{\underline{a}_{11}} & 1 & \cdots & -\dfrac{\overline{a}_{2n}+\overline{b}_{2n}+\overline{c}_{2n}}{\underline{a}_{nn}} \\ \cdots & \cdots & \cdots & \cdots \\ -\dfrac{\overline{a}_{n1}+\overline{b}_{n1}+\overline{c}_{n1}}{\underline{a}_{11}} & -\dfrac{\overline{a}_{n2}+\overline{b}_{n2}+\overline{c}_{n2}}{\underline{a}_{22}} & \cdots & 1 \end{pmatrix}$$
$$\times\begin{pmatrix} z_1(t_1)\\ z_2(t_2)\\ \cdots\\ z_n(t_n)\end{pmatrix}\le\begin{pmatrix}\overline{r}_1\\ \overline{r}_2\\ \cdots\\ \overline{r}_n\end{pmatrix}. \tag{23}$$

Set $D=(D_1,D_2,\ldots,D_n)^T=(\overline{r}_1,\overline{r}_2,\ldots,\overline{r}_n)^T$. It follows from (23) that

$$(E-\mathscr{K})\,(z_1(t_1),z_2(t_2),\ldots,z_n(t_n))^T\le D. \tag{24}$$

In view of $\rho(\mathscr{K})<1$ and Lemma 3, $(E_n-\mathscr{K})^{-1}\ge 0$. Let

$$H=(\widetilde{h}_1,\widetilde{h}_2,\ldots,\widetilde{h}_n)^T:=(E-\mathscr{K})^{-1}D\ge 0. \tag{25}$$

Then it follows from (24) and (25) that

$$(z_1(t_1),z_2(t_2),\ldots,z_n(t_n))^T\le H,\ \text{ or}$$
$$z_i(t_i)\le\widetilde{h}_i,\quad i=1,2,\ldots,n, \tag{26}$$

which implies

$$|x_i(t)|_0=\max_{t\in[0,\omega]}|x_i(t)|=x_i(t_i)\le\frac{1}{\alpha_{ii}}\ln\frac{\widetilde{h}_i}{\underline{a}_{ii}+\underline{b}_{ii}},\quad i=1,2,\ldots,n. \tag{27}$$

On the other hand, it follows from (25) that

$$(E-\mathscr{K})\,H=D,\quad\text{or}\quad H=\mathscr{K}H+D,\ \text{ that is,}$$
$$\widetilde{h}_i=\sum_{j=1}^{n}\Gamma_{ij}\widetilde{h}_j+D_i,\quad i=1,2,\ldots,n. \tag{28}$$

Estimating (16), by using (26) and (28), we have

$$\begin{aligned}|\dot{x}_i(t)|_0&=\lambda\Bigg|r_i(t)-a_{ii}(t)\,e^{\alpha_{ii}x_i(t)}+\sum_{j=1,j\neq i}^{n}a_{ij}(t)\,e^{\alpha_{ij}x_j(t)}\\&\quad+\sum_{j=1,j\neq i}^{n}b_{ij}(t)\,e^{\beta_{ij}x_j(t-\tau_{ij})}\\&\quad+\sum_{j=1,j\neq i}^{n}c_{ij}(t)\int_0^{\infty}H_{ij}(s)\,e^{\gamma_{ij}x_j(t_i+s)}ds\Bigg|_0\\&\le\overline{r}_i+\Bigg|a_{ii}(t)\,e^{\alpha_{ii}x_i(t_i)}+\sum_{j=1,j\neq i}^{n}a_{ij}(t)\,e^{\alpha_{ij}x_j(t)}\\&\quad+\sum_{j=1,j\neq i}^{n}b_{ij}(t)\,e^{\beta_{ij}x_j(t_j)}\\&\quad+\sum_{j=1,j\neq i}^{n}c_{ij}(t)\int_0^{\infty}H_{ij}(s)\,e^{\gamma_{ij}x_j(t_i+s)}ds\Bigg|_0\\&=\overline{r}_i+\sum_{j=1,j\neq i}^{n}\frac{\overline{a}_{ij}+\overline{b}_{ij}+\overline{c}_{ij}}{\underline{a}_{jj}}z_j(t_j)+\frac{\overline{a}_{ii}}{\underline{a}_{ii}}z_i(t_i)\\&\le D_i+\sum_{j=1}^{n}\Gamma_{ij}\widetilde{h}_j+\frac{\overline{a}_{ii}}{\underline{a}_{ii}}z_i(t_i)\le\widetilde{h}_i+\frac{\overline{a}_{ii}}{\underline{a}_{ii}}\widetilde{h}_i\\&=\left[1+\frac{\overline{a}_{ii}}{\underline{a}_{ii}}\right]\widetilde{h}_i.\end{aligned} \tag{29}$$

We can choose a large enough real number ($d>1$) such that

$$\frac{1}{\alpha_{ii}}\ln\frac{d\widetilde{h}_i}{\underline{a}_{ii}}>\frac{1}{\alpha_{ii}}\ln\frac{\widetilde{h}_i}{\underline{a}_{ii}}+\left[1+\frac{\overline{a}_{ii}}{\underline{a}_{ii}}\right]\widetilde{h}_i. \tag{30}$$

Let $h_i=(1/\alpha_{ii})\ln(d\widetilde{h}_i/\underline{a}_{ii})$. Then, for any solution of $Lx=\lambda Nx$, we have $|x_i(t)|_1=|x_i(t)|_0+|\dot{x}_i(t)|_0\le\ln(\widetilde{h}_i/\underline{a}_{ii})+2\widetilde{h}_i<h_i$ for all $i=1,2,\ldots,n$. Obviously, h_i are independent of λ and the choice of $x(t)$. Thus, taking $h_i=\ln(d\widetilde{h}_i/\underline{a}_{ii})$, the open subset Ω satisfies that $Lx\neq\lambda Nx$ for each $\lambda\in(0,1)$, $x\in\partial\Omega\cap\operatorname{Dom}L$; that is, the open subset Ω satisfies the assumption (i) of Lemma 1.

Using similar arguments to Step 3 in [2], it is not difficult to show that, for each $x\in\partial\Omega\cap\operatorname{Ker}L$, $QNx\neq 0$ and $\deg\{JQN,\Omega\cap\operatorname{Ker}L,0\}\neq 0$.

Hence, by Lemma 1, system (8) has at least one positive ω-periodic solution in $\operatorname{Dom}L\cap\overline{\Omega}$. By (7), system (1) has at least one positive ω-periodic solution, denoted by $\widetilde{y}(t)$. This completes the proof of Theorem 4. □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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