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Research Article

Global Strong Solution to the Density-Dependent 2-D Liquid Crystal Flows

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The initial-boundary value problem for the density-dependent flow of nematic crystals is studied in a 2-D bounded smooth domain. For the initial density away from vacuum, the existence and uniqueness is proved for the global strong solution with the large initial velocity u_0 and small ∇d_0. We also give a regularity criterion $\nabla d \in L^p(0,T;L^q(\Omega))$ $((2/q)+(2/p)=1,\ 2<q\leq\infty)$ of the problem with the Dirichlet boundary condition $u=0$, $d=d_0$ on $\partial\Omega$.

1. Introduction and Main Results

Let $\Omega \subseteq \mathbb{R}^2$ be a bounded domain with smooth boundary $\partial\Omega$, and ν is the unit outward normal vector on $\partial\Omega$. We consider the global strong solution to the density-dependent incompressible liquid crystal flow [1–4] as follows:

$$\operatorname{div} u = 0, \tag{1}$$

$$\partial_t \rho + \operatorname{div}(\rho u) = 0, \tag{2}$$

$$\partial_t(\rho u) + \operatorname{div}(\rho u \otimes u) + \nabla\pi - \Delta u = -\nabla\cdot(\nabla d \odot \nabla d), \tag{3}$$

$$\partial_t d + u\cdot\nabla d - \Delta d = |\nabla d|^2 d, \tag{4}$$

in $(0,\infty)\times\Omega$ with initial and boundary conditions

$$(\rho, u, d)(\cdot, 0) = (\rho_0, u_0, d_0) \quad \text{in } \Omega, \tag{5}$$

$$u = 0, \quad \partial_\nu d = 0 \quad \text{on } \partial\Omega, \tag{6}$$

where ρ denotes the density, u the velocity, d the unit vector field that represents the macroscopic molecular orientations, and π the pressure. The symbol $\nabla d \odot \nabla d$ denotes a matrix whose (i,j)th entry is $\partial_i d \partial_j d$, and it is easy to find that $\nabla d \odot \nabla d = \nabla d^T \nabla d$.

When d is a given constant unit vector, then (1), (2), and (3) represent the well-known density-dependent Navier-Stokes system, which has received many studies; see [5–7] and references therein.

When $\rho \equiv 1$ and $\Omega := \mathbb{R}^2$, Xu and Zhang [8] proved global existence of weak solutions to the problem if $u_0 \in L^2, \nabla d_0 \in L^2, |d_0| = 1$, and

$$\exp\left(216\left(\|u_0\|_{L^2}^2 + \frac{1}{16}\right)^2\right)\|\nabla d_0\|_{L^2}^2 < \frac{1}{16}. \tag{7}$$

When $\rho \equiv 1$ and (6) is replaced by

$$u = 0, \qquad d = d_0 \quad \text{on } \partial\Omega. \tag{8}$$

Lin et al. [9] proved the global existence of weak solutions to the system (1)–(5) and (8), which are smooth away from at most finitely many singular times, and they also prove a regularity criterion

$$d \in L^2(0,T;H^2(\Omega)). \tag{9}$$

When $\rho = 1$ and the term $|\nabla d|^2$ in (4) is replaced by $(1-|d|^2)d$, then the problem has been studied in [10–15].

Very recently, Wen and Ding [16] proved the global existence and uniqueness of strong solutions to the problem (1)–(6) with small u_0 and ∇d_0 and the local strong solutions with large initial data when $\Omega \subseteq \mathbb{R}^2$ is a smooth bounded domain.

Fan et al. [17] studied the regularity criterion of the Cauchy problem (1)–(5) when $\Omega := \mathbb{R}^2$.

We will prove the following.

Theorem 1. *Let $0 < m \leq \rho_0 \leq M < \infty$, $\rho_0 \in W^{1,r}$ for some $r \in (2,\infty)$, $u_0 \in H_0^1 \cap H^2$, and $d_0 \in H^3$ with $\operatorname{div} u_0=0$, and $|d_0| = 1$ in Ω. If*

$$\|\nabla d_0\|_{L^2}^2 \exp\left[216\frac{C_0^2}{m}\left(\|\sqrt{\rho_0}u_0\|_{L^2}^2 + \frac{1}{8C_0^2}\right)^2\right] \leq \frac{1}{8C_0^2}, \quad (10)$$

with an absolute constant C_0 in (22), then the problem (1)–(6) has a unique global-in-time strong solution (ρ, u, d) satisfying

$$\|\rho\|_{L^\infty(0,T;W^{1,r})} \leq C, \quad \|\rho_t\|_{L^\infty(0,T;L^r)} \leq C,$$
$$\|u\|_{L^\infty(0,T;H^2)\cap L^2(0,T;W^{2,s})} \leq C, \quad \text{for some } s > 2, \quad (11)$$
$$\|d\|_{L^\infty(0,T;H^3)} \leq C.$$

Remark 2. When $\Omega := \mathbb{R}^2$, Theorem 1 is also correct, thus improving the result in [18], where u_0 and ∇d_0 are assumed to be small.

Next, we consider (1)–(4) with $\rho \equiv 1$ as follows:

$$\operatorname{div} u = 0, \quad (12)$$

$$\partial_t u + u\cdot\nabla u + \nabla\pi - \Delta u = -\nabla\cdot(\nabla d \odot \nabla d), \quad (13)$$

$$\partial_t d + u\cdot\nabla d - \Delta d = |\nabla d|^2 d, \quad (14)$$

$$u = 0, \quad d = d_0 \quad \text{on } \partial\Omega, \quad (15)$$

$$(u, d)(\cdot, 0) = (u_0, d_0) \quad \text{in } \Omega. \quad (16)$$

We will prove the following.

Theorem 3. *Let $u_0 \in L^2$ and $d_0 \in H^1$ with $\operatorname{div} u_0 = 0$ and $|d_0| = 1$ in Ω and $d_0 \in C^{2,\beta}(\partial\Omega)$ for some $\beta \in (0,1)$. If d satisfies*

$$\nabla d \in L^q(0,T;L^p), \quad \frac{2}{q} + \frac{2}{p} = 1, \quad 2 < p \leq \infty, \quad (17)$$

then the strong solution (u, d) can be extended beyond $T > 0$.

Remark 4. In [9], the authors prove the regularity criterion (9) for the problem (12)–(16), and our condition (17) is weaker than (9). Moreover, (17) is scaling invariant for (12)–(14).

2. Proof of Theorem 1

This section is devoted to the proof of Theorem 1. Since the local-in-time well-posedness has been proved in [16], we only need to establish a priori estimates. Also, by the local well-posedness result in [16], we note that ∇d is absolutely continuous on $[0,T]$ for any given $T > 0$.

By the maximum principle, it follows from (1) and (2) that

$$0 < m \leq \rho \leq M < \infty. \quad (18)$$

Testing (3) by u and using (1) and (2), we see that

$$\frac{1}{2}\frac{d}{dt}\int \rho u^2 dx + \int |\nabla u|^2 dx = -\int (u\cdot\nabla) d \cdot \Delta d\, dx. \quad (19)$$

Testing (4) by $-\Delta d - |\nabla d|^2 d$, using $|d| = 1$, we find that

$$\frac{1}{2}\frac{d}{dt}\int |\nabla d|^2 dx + \int \left|\Delta d + |\nabla d|^2 d\right|^2 dx = \int (u\cdot\nabla) d\cdot \Delta d\, dx. \quad (20)$$

Summing up (19) and (20) and integrating over $(0,T)$, we get

$$\int \left(\rho u^2 + |\nabla d|^2\right) dx + 2\int_0^T \int \left(|\nabla u|^2 + \left|\Delta d + |\nabla d|^2 d\right|\right) dx\, dt \leq \int \left(\rho_0 u_0^2 + |\nabla d_0|^2\right) dx. \quad (21)$$

Since $\partial_\nu d = 0$ on $(0,\infty)\times\partial\Omega$, we have the following Gagliardo-Nirenberg inequality:

$$\|\nabla d\|_{L^4}^2 \leq C_0 \|\nabla d\|_{L^2}\|\Delta d\|_{L^2}. \quad (22)$$

By (20) and the Ladyzhenskaya inequality in 2D, we derive

$$\begin{aligned}
&\frac{1}{2}\frac{d}{dt}\int |\nabla d|^2 dx + \int \left|\Delta d + |\nabla d|^2 d\right|^2 dx \\
&\leq \|u\|_{L^4}\|\nabla d\|_{L^4}\|\Delta d\|_{L^2} \\
&\leq \sqrt{2}\|u\|_{L^2}^{1/2}\|\nabla u\|_{L^2}^{1/2}\cdot\sqrt{C_0}\|\nabla d\|_{L^2}^{1/2}\|\Delta d\|_{L^2}^{3/2} \\
&\leq \frac{\|\Delta d\|_{L^2}^2}{8} + 216C_0^2\|u\|_{L^2}^2\|\nabla u\|_{L^2}^2\|\nabla d\|_{L^2}^2 \\
&\leq \frac{\|\Delta d\|_{L^2}^2}{8} + 216\frac{C_0^2}{m}\left(\|\sqrt{\rho_0}u_0\|_{L^2}^2 + \|\nabla d_0\|_{L^2}^2\right)\|\nabla u\|_{L^2}^2\|\nabla d\|_{L^2}^2.
\end{aligned} \quad (23)$$

On the other hand, since $(a+b)^2 \geq (a^2/2) - b^2$, we have

$$\begin{aligned}
\int \left|\Delta d + |\nabla d|^2 d\right|^2 dx &\geq \frac{\|\Delta d\|_{L^2}^2}{2} - \|\nabla d\|_{L^4}^4 \\
&\geq \frac{\|\Delta d\|_{L^2}^2}{2} - C_0^2\|\nabla d\|_{L^2}^2\|\Delta d\|_{L^2}^2.
\end{aligned} \quad (24)$$

If the initial data $\|\nabla d_0\|_{L^2}^2 < (1/C_0^2)(1/8)$, then there exists $T_1 > 0$ such that for any $t \in [0, T_1]$,

$$\|\nabla d(t)\|_{L^2}^2 \leq \frac{1}{C_0^2}\cdot\frac{1}{4}. \quad (25)$$

We denote by T_1^* the maximal time such that (25) holds on $[0, T_1^*]$. Therefore, by (23), (24), and (25), it follows that for any $t \in [0, T_1^*]$,

$$\frac{d}{dt}\int |\nabla d|^2 dx + \frac{1}{4}\|\Delta d\|_{L^2}^2 \leq 432\frac{C_0^2}{m}\left(\|\sqrt{\rho_0}u_0\|_{L^2}^2 + \|\nabla d_0\|_{L^2}^2\right)\|\nabla u\|_{L^2}^2\|\nabla d\|_{L^2}^2 \leq 432\frac{C_0^2}{m}\left(\|\sqrt{\rho_0}u_0\|_{L^2}^2 + \frac{1}{8C_0^2}\right)\|\nabla u\|_{L^2}^2\|\nabla d\|_{L^2}^2, \quad (26)$$

which gives

$$\|\nabla d(t)\|_{L^2}^2 + \frac{1}{4}\int_0^t \|\Delta d(\tau)\|_{L^2}^2 d\tau \leq \|\nabla d_0\|_{L^2}^2 \exp\left[432\frac{C_0^2}{m}\left(\|\sqrt{\rho_0}u_0\|_{L^2}^2 + \frac{1}{8C_0^2}\right) \times \int_0^{T_1^*}\|\nabla u\|_{L^2}^2 d\tau\right] \leq \|\nabla d_0\|_{L^2}^2 \exp\left[216\frac{C_0^2}{m}\left(\|\sqrt{\rho_0}u_0\|_{L^2}^2 + \frac{1}{8C_0^2}\right)^2\right] \leq \frac{1}{8C_0^2}, \quad (27)$$

which implies that $T_1^* = T$ if the initial data satisfies

$$\|\nabla d_0\|_{L^2}^2 \exp\left[216\frac{C_0^2}{m}\left(\|\sqrt{\rho_0}u_0\|_{L^2}^2 + \frac{1}{8C_0^2}\right)^2\right] \leq \frac{1}{8C_0^2}. \quad (28)$$

Let T^* be a maximal existence time for the solution (ρ, u, d). Then, (18), (21), and (27) ensure that $T^* = \infty$ by continuity argument.

Testing (3) by u_t, using (1), (18), (21), (22), $|d| = 1$, and the Gagliardo-Nirenberg inequalities, we obtain

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\int |\nabla u|^2 dx + \int \rho u_t^2 dx &= -\int \rho u\cdot\nabla u\cdot u_t dx - \int u_t\cdot\nabla d\cdot\Delta d\, dx \\ &\leq C\|\sqrt{\rho}u_t\|_{L^2}\left(\|u\|_{L^4}\|\nabla u\|_{L^4} + \|\nabla d\|_{L^4}\|\Delta d\|_{L^4}\right) \\ &\leq C\|\sqrt{\rho}u_t\|_{L^2}\left[\|u\|_{L^2}^{1/2}\|\nabla u\|_{L^2}\left(\|\Delta u\|_{L^2}^{1/2} + \|u\|_{L^2}^{1/2}\right) + \|\nabla d\|_{L^2}^{1/2}\|\Delta d\|_{L^2}\left(\|\nabla\Delta d\|_{L^2}^{1/2} + \|d\|_{L^2}^{1/2}\right)\right] \\ &\leq C\|\sqrt{\rho}u_t\|_{L^2}\left(\|\nabla u\|_{L^2}\|\Delta u\|_{L^2}^{1/2} + \|\nabla u\|_{L^2} + \|\Delta d\|_{L^2} \times \|\nabla\Delta d\|_{L^2}^{1/2} + \|\Delta d\|_{L^2}\right).\end{aligned} \quad (29)$$

On the other hand, (3) can be rewritten as

$$-\Delta u + \nabla\pi = f := -\rho u_t - \rho u\cdot\nabla u - \nabla\cdot(\nabla d\odot\nabla d). \quad (30)$$

By the H^2-theory of Stokes system, we have

$$\begin{aligned}\|\Delta u\|_{L^2} &\leq C\|f\|_{L^2} \\ &\leq C\|\sqrt{\rho}u_t\|_{L^2} + C\|u\|_{L^4}\|\nabla u\|_{L^4} + C\|\nabla d\|_{L^4}\|\Delta d\|_{L^4} \\ &\leq C\|\sqrt{\rho}u_t\|_{L^2} + C\|\nabla u\|_{L^2}\|\Delta u\|_{L^2}^{1/2} + C\|\nabla u\|_{L^2} + C\|\Delta d\|_{L^2}\|\nabla\Delta d\|_{L^2}^{1/2} + C\|\Delta d\|_{L^2},\end{aligned} \quad (31)$$

which yields

$$\|\Delta u\|_{L^2} \leq C\|\sqrt{\rho}u_t\|_{L^2} + C\|\nabla u\|_{L^2}^2 + C + C\|\Delta d\|_{L^2}\|\nabla\Delta d\|_{L^2}^{1/2} + C\|\Delta d\|_{L^2}. \quad (32)$$

Inserting (32) into (29), we deduce that

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\int |\nabla u|^2 dx + \int \rho u_t^2 dx &\leq C\|\sqrt{\rho}u_t\|_{L^2}^{3/2}\|\nabla u\|_{L^2} + C\|\sqrt{\rho}u_t\|_{L^2}\left(\|\nabla u\|_{L^2}^2 + \|\nabla u\|_{L^2}\right) + C\|\sqrt{\rho}u_t\|_{L^2}\|\Delta d\|_{L^2}\|\nabla\Delta d\|_{L^2}^{1/2} + C\|\sqrt{\rho}u_t\|_{L^2}\|\Delta d\|_{L^2} \\ &\leq \frac{1}{8}\|\sqrt{\rho}u_t\|_{L^2}^2 + C\|\nabla u\|_{L^2}^4 + C + \frac{1}{8}\|\nabla\Delta d\|_{L^2}^2 + C\|\Delta d\|_{L^2}^4.\end{aligned} \quad (33)$$

Applying Δ to (4), testing by Δd, using $|d| = 1$, (21) and (22), and the Gagliardo-Nirenberg inequalities, we have

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\int |\Delta d|^2 dx + \int |\nabla\Delta d|^2 dx &\leq \int \left|\nabla\left(|\nabla d|^2 d\right)\right||\nabla\Delta d|\, dx + \int |\nabla(u\cdot\nabla d)||\nabla\Delta d|\, dx \\ &\leq C\left(\|\nabla d\|_{L^6}^3 + \|\nabla d\|_{L^4}\|\Delta d\|_{L^4} + \|u\|_{L^4}\|\Delta d\|_{L^4} + \|\nabla u\|_{L^2}\|\nabla d\|_{L^\infty}\right)\|\nabla\Delta d\|_{L^2} \\ &\leq C\left(\|\nabla d\|_{L^2}\|\Delta d\|_{L^2}^2 + \|\Delta d\|_{L^2}\|\nabla\Delta d\|_{L^2}^{1/2} + \|\Delta d\|_{L^2} + \|\nabla u\|_{L^2}^{1/2}\|\Delta d\|_{L^2}^{1/2}\|\nabla\Delta d\|_{L^2}^{1/2} + \|\nabla u\|_{L^2}^{1/2}\|\Delta d\|_{L^2}^{1/2} + \|\nabla u\|_{L^2} \times \|\nabla d\|_{L^2}^{1/2}\|\nabla\Delta d\|_{L^2}^{1/2}\right)\|\nabla\Delta d\|_{L^2} \\ &\leq \frac{1}{8}\|\nabla\Delta d\|_{L^2}^2 + C\|\Delta d\|_{L^2}^4 + C + C\|\nabla u\|_{L^2}^4.\end{aligned} \quad (34)$$

Here, we have used the Gagliardo-Nirenberg inequalities

$$\begin{aligned}\|\nabla d\|_{L^6}^3 &\leq C\|\nabla d\|_{L^2}\|\Delta d\|_{L^2}^2, \\ \|\nabla d\|_{L^\infty}^2 &\leq \|\nabla d\|_{L^2}\|\nabla\Delta d\|_{L^2}, \\ \|\Delta d\|_{L^4}^2 &\leq C\|\Delta d\|_{L^2}\|\nabla\Delta d\|_{L^2} + C\|\Delta d\|_{L^2}.\end{aligned} \quad (35)$$

Combining (33) and (34) and using the Gronwall inequality, we have

$$\|u\|_{L^\infty(0,T;H^1)} + \|u\|_{L^2(0,T;H^2)} \le C, \quad (36)$$

$$\left\|\sqrt{\rho}u_t\right\|_{L^2(0,T;L^2)} \le C, \quad (37)$$

$$\|d\|_{L^\infty(0,T;H^2)} + \|d\|_{L^2(0,T;H^3)} \le C. \quad (38)$$

Now, by the similar calculations as those in [17], we arrive at

$$\begin{gathered}\left\|(u_t, \nabla d_t)\right\|_{L^\infty(0,T;L^2)\cap L^2(0,T;H^1)} \le C, \\ \|(u, \nabla d)\|_{L^\infty(0,T;H^2)} \le C, \\ \|u\|_{L^2(0,T;W^{2,s})} \le C \quad \text{for some } s > 2, \\ \|\rho\|_{L^\infty(0,T;W^{1,r})} \le C, \qquad \|\rho_t\|_{L^\infty(0,T;L^r)} \le C.\end{gathered} \quad (39)$$

This completes the proof.

3. Proof of Theorem 3

This section is devoted to the proof of Theorem 3. By the results in [9], we only need to prove (9).

Similar to (21), we still have

$$\begin{aligned}&\int \left(u^2 + |\nabla d|^2\right) dx + 2\int_0^T \int \left(|\nabla u|^2 + \left|\Delta d + |\nabla d|^2 d\right|\right) dx\, dt \\ &\quad \le \int \left(u_0^2 + \left|\nabla d_0\right|^2\right) dx.\end{aligned} \quad (40)$$

We will use the following Gagliardo-Nirenberg inequalities:

$$\|u\|_{L^{2p/(p-2)}} \le C\|u\|_{L^2}^{1-(2/p)} \|\nabla u\|_{L^2}^{2/p}, \quad (41)$$

$$\|\nabla d\|_{L^{2p/(p-2)}} \le C\|\nabla d\|_{L^2}^{1-(2/p)} \|\Delta d\|_{L^2}^{2/p} + C\|\nabla d\|_{L^2}. \quad (42)$$

Testing (14) by $-\Delta d$, using $|d| = 1$, (40), (41), and (42), we have

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}\int |\nabla d|^2 dx + \int |\Delta d|^2 dx \\ &\quad = \int \left(u\cdot\nabla d - |\nabla d|^2 d\right)\Delta d\, dx \\ &\quad \le \left(\|u\|_{L^{2p/(p-2)}}\|\nabla d\|_{L^p} + \|\nabla d\|_{L^p}\|\nabla d\|_{L^{2p/(p-2)}}\right)\|\Delta d\|_{L^2} \\ &\quad \le C\|\nabla d\|_{L^p}\left(\|u\|_{L^2}^{1-(2/p)}\|\nabla u\|_{L^2}^{2/p} + \|\nabla d\|_{L^2}\right. \\ &\qquad\qquad \left. + \|\nabla d\|_{L^2}^{1-(2/p)}\|\Delta d\|_{L^2}^{2/p}\right)\|\Delta d\|_{L^2} \\ &\quad \le C\|\nabla d\|_{L^p}\left(\|\nabla u\|_{L^2}^{2/p} + 1 + \|\Delta d\|_{L^2}^{2/p}\right)\|\Delta d\|_{L^2} \\ &\quad \le \frac{1}{4}\|\Delta d\|_{L^2}^2 + C\|\nabla d\|_{L^p}^2\left(\|\nabla u\|_{L^2}^{4/p} + 1 + \|\Delta d\|_{L^2}^{4/p}\right) \\ &\quad \le \frac{1}{2}\|\Delta d\|_{L^2}^2 + \|\nabla u\|_{L^2}^2 + C\|\nabla d\|_{L^p}^{2p/(p-2)} + C,\end{aligned} \quad (43)$$

which gives (9).

This completes the proof.

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References

[1] S. Chandrasekhar, *Liquid Crystalsed*, Cambridge University Press, 2nd edition, 1992.

[2] J. L. Ericksen, "Hydrostatic theory of liquid crystals," *Archive for Rational Mechanics and Analysis*, vol. 9, pp. 371–378, 1962.

[3] F. M. Leslie, "Some constitutive equations for liquid crystals," *Archive for Rational Mechanics and Analysis*, vol. 28, no. 4, pp. 265–283, 1968.

[4] F.-H. Lin and C. Liu, "Nonparabolic dissipative systems modeling the flow of liquid crystals," *Communications on Pure and Applied Mathematics*, vol. 48, no. 5, pp. 501–537, 1995.

[5] R. Danchin, "Density-dependent incompressible fluids in bounded domains," *Journal of Mathematical Fluid Mechanics*, vol. 8, no. 3, pp. 333–381, 2006.

[6] H. Kim, "A blow-up criterion for the nonhomogeneous incompressible Navier-Stokes equations," *SIAM Journal on Mathematical Analysis*, vol. 37, no. 5, pp. 1417–1434, 2006.

[7] J. Fan and T. Ozawa, "Regularity criteria for the 3D density-dependent Boussinesq equations," *Nonlinearity*, vol. 22, no. 3, pp. 553–568, 2009.

[8] X. Xu and Z. Zhang, "Global regularity and uniqueness of weak solution for the 2-D liquid crystal flows," *Journal of Differential Equations*, vol. 252, no. 2, pp. 1169–1181, 2012.

[9] F. Lin, J. Lin, and C. Wang, "Liquid crystal flows in two dimensions," *Archive for Rational Mechanics and Analysis*, vol. 197, no. 1, pp. 297–336, 2010.

[10] F.-H. Lin and C. Liu, "Partial regularity of the dynamic system modeling the flow of liquid crystals," *Discrete and Continuous Dynamical Systems*, vol. 2, no. 1, pp. 1–22, 1996.

[11] F.-H. Lin and C. Liu, "Existence of solutions for the Ericksen-Leslie system," *Archive for Rational Mechanics and Analysis*, vol. 154, no. 2, pp. 135–156, 2000.

[12] F. Lin and C. Liu, "Static and dynamic theories of liquid crystals," *Journal of Partial Differential Equations*, vol. 14, no. 4, pp. 289–330, 2001.

[13] J. Fan and B. Guo, "Regularity criterion to some liquid crystal models and the Landau-Lifshitz equations in $\mathscr{R}^3$," *Science in China. Series A*, vol. 51, no. 10, pp. 1787–1797, 2008.

[14] J. Fan and T. Ozawa, "Regularity criteria for a simplified Ericksen-Leslie system modeling the flow of liquid crystals," *Discrete and Continuous Dynamical Systems. Series A*, vol. 25, no. 3, pp. 859–867, 2009.

[15] Y. Zhou and J. Fan, "A regularity criterion for the nematic liquid crystal flows," *Journal of Inequalities and Applications*, vol. 2010, Article ID 589697, 9 pages, 2010.

[16] H. Wen and S. Ding, "Solutions of incompressible hydrodynamic flow of liquid crystals," *Nonlinear Analysis: Real World Applications*, vol. 12, no. 3, pp. 1510–1531, 2011.

[17] J. Fan, H. Gao, and B. Guo, "Regularity criteria for the Navier-Stokes-Landau-Lifshitz system," *Journal of Mathematical Analysis and Applications*, vol. 363, no. 1, pp. 29–37, 2010.

[18] J. Li, “Global strong and weak solutions to nematic liquid crystal flow in two dimensions,” http://arxiv.org/abs/1211.0131.