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Research Article

New Exact Solutions of Some Nonlinear Systems of Partial Differential Equations Using the First Integral Method

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The first integral method introduced by Feng is adopted for solving some important nonlinear systems of partial differential equations, including classical Drinfel'd-Sokolov-Wilson system (DSWE), $(2+1)$-dimensional Davey-Stewartson system, and generalized Hirota-Satsuma coupled KdV system. This method provides polynomial first integrals for autonomous planar systems. Through the established first integrals, exact traveling wave solutions are formally derived in a concise manner. This method can also be applied to nonintegrable equations as well as integrable ones.

1. Introduction

Over the four decades or so, nonlinear partial differential equations (NPDEs) have been the subject of extensive studies in various branches of nonlinear sciences.

A special class of analytical solutions, the so-called traveling waves, for NPDEs is of fundamental importance because lots of mathematical-physical models are often described by such wave phenomena.

Therefore, the investigation of traveling wave solutions is becoming more and more attractive in nonlinear sciences nowadays. However, not all equations posed of these models are solvable. As a result, many new techniques have been successfully developed by diverse groups of mathematicians and physicists, such as the Exp-function method [1–3], the sine-cosine method [4–6], the extended tanh-function method [7, 8], the modified extended tanh-function method [9–11], the F-expansion method [12], and the first integral method (or the algebraic curve method) [13]. Of these, the first integral method, which is based on the ring theory of commutative algebra, was first established by Feng [14–23].

This method was further developed by some other mathematicians [11, 24–33]. The method is reliable, effective, precise, and does not require complicated and tedious computations. The main idea of the first integral method is to find first integrals of nonlinear differential equations in polynomial form. Taking the polynomials with unknown polynomial coefficients into account, the method provides exact and explicit solutions. The interest in the present work is to implement the first integral method to stress its power in handling nonlinear partial differential equations, so that we can apply it for solving various types of these equations.

In Section 2, we describe this method for finding exact travelling wave solutions of nonlinear evolution equations. In Section 3, we illustrate this method in detail with the classical Drinfel'd-Sokolov-Wilson system (DSWE), the (2+1)-dimensional Davey-Stewartson system, and the generalized Hirota-Satsuma coupled KdV system. In Section 4, we give some conclusions.

2. The First Integral Method

Hosseini et al. in [30] have summarized the first integral method in the following steps.

Step 1. Consider the following nonlinear system of partial differential equations with independent variables x and t and dependent variables u and v,

$$F_1(u, v, u_t, v_t, u_x, v_x, u_{tt}, v_{tt}, u_{xx}, v_{xx}, \ldots) = 0,$$
$$F_2(u, v, u_t, v_t, u_x, v_x, u_{tt}, v_{tt}, u_{xx}, v_{xx}, \ldots) = 0. \tag{1}$$

Applying the transformations $u(x,t) = u(\xi)$ and $v(x,t) = v(\xi)$, where $\xi = x - ct + \varsigma$, where ς is an arbitrary constant, converts (1) into a system of ordinary differential equations (ODEs)

$$G_1(u, v, u', v', \ldots) = 0,$$
$$G_2(u, v, u', v', \ldots) = 0, \tag{2}$$

where the prime denotes the derivatives with respect to the same variable ξ.

Step 2. Using some mathematical operations, the system (2) is converted into a second-order ODE

$$D(u, u', u'') = 0. \tag{3}$$

Step 3. By introducing new variables $X = u(\xi)$ and $Y = u'(\xi)$, (3) changes into a system of ODEs as the following system:

$$X' = Y, \tag{4a}$$

$$Y' = H(X, Y). \tag{4b}$$

Step 4. Now, the Division Theorem which is based on ring theory of commutative algebra is adopted to obtain one first integral to (4a) and (4b), which reduces (3) to a first-order integrable ordinary differential equation. Finally, an exact solution to (1) is then established, through solving the resulting first-order integrable differential equation.

Let us now recall the Division Theorem for two variables in the complex domain $C(w, z)$.

Theorem 1 (Division Theorem). *Suppose that $P(w,z)$, $Q(w,z)$ are polynomials in $C(w,z)$ and $P(w,z)$ is irreducible in (w,z). If $Q(w,z)$ vanishes at all zero points of (w,z), then there exists a polynomial $G(w,z)$ in $C(w,z)$ such that*

$$Q(w,z) = P(w,z)\,G(w,z). \tag{5}$$

The Division Theorem follows immediately from the Hilbert-Nullstellensatz Theorem [34], but it can also be proved by using the complex analysis [35].

Theorem 2 (Hilbert-Nullstellensatz Theorem). *Let k be a field and L an algebraic closure of k.*

(1) *Every ideal γ of $k[X_1, \ldots, X_n]$ not containing 1 admits at least one zero in L^n*

(2) *Let $x = (x_1, \ldots, x_n)$, $y = (y_1, \ldots, y_n)$ be two elements of L^n; for the set of polynomials of $k[X_1, \ldots, X_n]$ zero at x to be identical with the set of polynomials of $k[X_1, \ldots, X_n]$ zero at y, it is necessary and sufficient that there exists a k-automorphism s of L such that $y_i = s(x_i)$ for $1 \le i \le n$.*

(3) *For an ideal α of $k[X_1, \ldots, X_n]$ to be maximal, it is necessary and sufficient that there exists an x in L^n such that α is the set of polynomials of $k[X_1, \ldots, X_n]$ zero at x.*

(4) *For a polynomial Q of $k[X_1, \ldots, X_n]$ to be zero on the set of zeros in L^n of an ideal γ of $[X_1, \ldots, X_n]$, it is necessary and sufficient that there exists an integer $m > 0$ such that $Q^m \in \gamma$.*

3. Applications

In this section, we investigate three NPDEs by using the first integral method.

3.1. Classical Drinfel'd-Sokolov-Wilson System. Consider the classical Drinfel'd-Sokolov-Wilson system [36]

$$u_t + pvv_x = 0,$$
$$v_t + qv_{xxx} + ruv_x + su_xv = 0, \tag{6}$$

where p, q, r, s are some nonzero parameters.

Recently, DSWE and the coupled DSWE, a special case of the classical DSWE, have been studied by several authors [36] and the references therein.

Using a complex variation η defined as $\eta = k(x - ct) + \gamma$, we can convert (6) into ODEs, which read

$$-cu' + pvv' = 0, \tag{7}$$

$$-cv' + qk^2v''' + ruv' + su'v = 0, \tag{8}$$

where the prime denotes the derivative with respect to η.

Integrating (7), we obtain

$$u = \frac{pv^2}{2c} + c_1, \tag{9}$$

where c_1 is an arbitrary integration constant.

Substituting u into (8) yields

$$2cqk^2v''' + p(r + 2s)v^2v' + 2c(rc_1 - c)v' = 0. \tag{10}$$

Integrating (10), we get

$$2cqk^2v'' + p(r + 2s)\frac{v^3}{3} + 2c(rc_1 - c)v = c_2, \tag{11}$$

where c_2 is an arbitrary integration constant.

By introducing new variables $X = v(\xi)$ and $Y = v'(\xi)$, (11) changes into a system of ODEs

$$X' = Y, \tag{12a}$$

$$Y' = \left(-\frac{p(r + 2s)}{6cqk^2}\right)X^3 - \left(\frac{rc_1 - c}{qk^2}\right)X - \frac{c_2}{2cqk^2}. \tag{12b}$$

According to the first integral method, we suppose that $X(\xi)$ and $Y(\xi)$ are nontrivial solutions of (12a) and (12b), and

$P(X,Y) = \sum_{i=0}^{m} a_i(X)Y^i$ is an irreducible polynomial in the complex domain $C[X,Y]$ such that

$$P[X(\xi),Y(\xi)] = \sum_{i=0}^{m} a_i(X(\xi))Y^i(\xi) = 0, \quad (13)$$

where $a_i(X)$, $(i = 0,1,2,\ldots,m)$ are polynomials of X and $a_m(X) \neq 0$.

Equation (13) is called the first integral to (12a) and (12b). Due to the Division Theorem, there exists a polynomial $h(X) + g(X)Y$ in the complex domain $C[X,Y]$ such that

$$\begin{aligned}\frac{dP}{d\xi} &= \frac{\partial P}{\partial X}\frac{dX}{d\xi} + \frac{\partial P}{\partial Y}\frac{dY}{d\xi} \\ &= [h(X) + g(X)Y]\sum_{i=0}^{m} a_i(X)Y^i.\end{aligned} \quad (14)$$

Here, we have considered one case only, assuming that $m = 1$ in (13).

Suppose that $m = 1$, by equating the coefficients of Y^i $(i = 2,1,0)$ on both sides of (14), we have

$$a_1'(X) = g(X)a_1(X), \quad (15a)$$

$$a_0'(X) = h(X)a_1(X) + g(X)a_0(X), \quad (15b)$$

$$\begin{aligned}&a_1(X)\left(\left(-\frac{p(r+2s)}{6cqk^2}\right)X^3 - \left(\frac{rc_1 - c}{qk^2}\right)X - \frac{c_2}{2cqk^2}\right) \\ &\quad = h(X)a_0(X).\end{aligned} \quad (15c)$$

Since $a_i(X)$ $(i = 0,1)$ are polynomials, then from (15a) we have deduced that $a_1(X)$ is constant and $g(X) = 0$. For simplicity, take $a_1(X) = 1$.

Balancing the degrees of $h(X)$ and $a_0(X)$, we have concluded that $\deg(h(X)) = 1$ only. Suppose that $h(X) = AX + B$, and $A \neq 0$, then we find $a_0(X)$

$$a_0(X) = \frac{A}{2}X^2 + BX + D, \quad (16)$$

where D is an arbitrary integration constant.

Substituting $a_0(X)$, $a_1(X)$ and $h(X)$ for (15c) and setting all the coefficients of powers X to be zero, then we obtain a system of nonlinear algebraic equations and by solving it, we have obtained

$$\begin{aligned}&c_2 = 0, \qquad c_1 = \frac{3c - \left(\sqrt{3}kDq\sqrt{-p(r+2s)}\right)/\sqrt{cq}}{3r}, \\ &A = \frac{\sqrt{-p(r+2s)}}{\sqrt{3}k\sqrt{cq}}, \qquad B = 0,\end{aligned} \quad (17a)$$

$$\begin{aligned}&c_2 = 0, \qquad c_1 = \frac{3c + \left(kDq\sqrt{-p(r+2s)}\right)/\sqrt{3}\sqrt{cq}}{r}, \\ &A = -\frac{\sqrt{-p(r+2s)}}{\sqrt{3}k\sqrt{cq}}, \qquad B = 0.\end{aligned} \quad (17b)$$

Setting (17a) and (17b) in (13) leads to

$$\begin{aligned}&Y(\eta) + \left(\frac{\sqrt{-p(r+2s)}}{2\sqrt{3}k\sqrt{cq}}X^2(\eta) + D\right) = 0, \\ &Y(\eta) + \left(-\frac{\sqrt{-p(r+2s)}}{2\sqrt{3}k\sqrt{cq}}X^2(\eta) + D\right) = 0.\end{aligned} \quad (18)$$

Combining (18) with (12a), a first-order ordinary differential equation is derived, then by solving this derived equation and considering $X = v(\xi)$ and $v(x,t) = v(\xi)$, we have obtained

$$\begin{aligned}v_1(x,t) &= i(-3)^{1/4}c(q)^{1/4}\sqrt{k}\sqrt{D} \\ &\quad\times \tan\left[\left(\left(-\frac{1}{3}\right)^{1/4}\sqrt{D}(p)^{1/4}(r+2s)^{1/4}\right.\right. \\ &\qquad\qquad \left.\times\left[(k(x-ct)+\gamma) - 3\sqrt{cq}k\xi_0\right]\right) \\ &\qquad\qquad \left.\times\left(c(q)^{1/4}\sqrt{k}\right)^{-1}\right] \\ &\quad\times\left((p)^{3/4}(r+2s)^{1/4}\right)^{-1},\end{aligned} \quad (19)$$

$$\begin{aligned}v_2(x,t) &= (-3)^{1/4}c(q)^{1/4}\sqrt{k}\sqrt{D} \\ &\quad\times \tan\left[\left((-1)^{3/4}\sqrt{D}(p)^{1/4}(r+2s)^{1/4}\right.\right. \\ &\qquad\qquad \left.\times\left[(k(x-ct)+\gamma) - 3\sqrt{cq}k\xi_0\right]\right) \\ &\qquad\qquad \left.\times\left((3)^{1/4}c(q)^{1/4}\sqrt{k}\right)^{-1}\right] \\ &\quad\times\left((p)^{1/4}(r+2s)^{1/4}\right)^{-1},\end{aligned} \quad (20)$$

respectively, where ξ_0 is an arbitrary integration constant.

Also, by considering the solution u given by the relations (9), we have obtained

$$\begin{aligned}u_1(x,t) &= \left(\frac{p}{2c}\right) \\ &\quad\times\left[i(-3)^{1/4}c(q)^{1/4}\sqrt{k}\sqrt{D}\right. \\ &\qquad\times \tan\left[\left(\left(-\frac{1}{3}\right)^{1/4}\sqrt{D}(p)^{1/4}(r+2s)^{1/4}\right.\right. \\ &\qquad\qquad \left.\times\left[(k(x-ct)+\gamma) - 3\sqrt{cq}k\xi_0\right]\right) \\ &\qquad\qquad \left.\times\left(c(q)^{1/4}\sqrt{k}\right)^{-1}\right]\end{aligned}$$

$$\times\left(\left(p\right)^{1/4}\left(r+2s\right)^{1/4}\right)^{-1}\Big]^2 + \frac{3c-\left(\sqrt{3}kDq\sqrt{-p\left(r+2s\right)}\right)/\sqrt{cq}}{3r}, \tag{21}$$

$$\begin{aligned} u_2\left(x,t\right) = &\left(\frac{p}{2c}\right) \\ &\times\Bigg[\left(-3\right)^{1/4}c\left(q\right)^{1/4}\sqrt{k}\sqrt{D} \\ &\quad\times\tan\Bigg[\Bigg(\left(-1\right)^{3/4}\sqrt{D}\left(p\right)^{1/4}\left(r+2s\right)^{1/4} \\ &\qquad\times\left[\left(k\left(x-ct\right)+\gamma\right)-3\sqrt{cq}k\xi_0\right]\Bigg) \\ &\qquad\times\left(\left(3\right)^{1/4}c\left(q\right)^{1/4}\sqrt{k}\right)^{-1}\Bigg] \\ &\times\left(\left(p\right)^{1/4}\left(r+2s\right)^{1/4}\right)^{-1}\Bigg]^2 \\ &+\frac{3c+\left(kDq\sqrt{-p\left(r+2s\right)}\right)/\sqrt{3}\sqrt{cq}}{r}, \end{aligned} \tag{22}$$

respectively, where ξ_0 is an arbitrary integration constant.

Thus, two solutions (u_1, v_1) and (u_2, v_2) have been obtained for the system (6).

Comparing these results with the results obtained in [36], it can be seen that the solutions here are new.

3.2. (2 + 1)-*Dimensional Davey-Stewartson System.* The (2 + 1)-dimensional Davey-Stewartson system [37] reads

$$\begin{aligned} iu_t + u_{xx} - u_{yy} - 2|u|^2u - 2uv &= 0, \\ v_{xx} + v_{yy} + 2\left(|u|^2\right)_{xx} &= 0. \end{aligned} \tag{23}$$

This equation is completely integrable and used to describe the long-time evolution of a two-dimensional wave packet.

Using the wave variables

$$\begin{aligned} u &= e^{i\theta}u\left(\xi\right), \qquad v = v\left(\xi\right), \\ \theta &= px + qy + rt + \varepsilon, \qquad \xi = kx + cy + dt + \gamma, \end{aligned} \tag{24}$$

where p, q, r, k, c, and d are real constants, converts (23) into the ODE

$$\left(q^2 - p^2 - r\right)u + \left(k^2 - c^2\right)u'' - 2u^3 - 2uv = 0, \tag{25}$$

$$\left(k^2 + c^2\right)v'' + \left(u^2\right)'' = 0. \tag{26}$$

Integrating (26) in the system and neglecting constants of integration, we have found

$$v = -\frac{u^2}{k^2 + c^2}. \tag{27}$$

Substituting (27) into (25) of the system and integrating we find

$$\left(q^2 - p^2 - r\right)u + \left(k^2 - c^2\right)u'' - 2u^3 + \frac{2u^3}{k^2 + c^2} = 0. \tag{28}$$

By introducing new variables $X = u(\xi)$ and $Y = u'(\xi)$, (28) changes into a system of ODEs

$$X' = Y, \tag{29a}$$

$$Y' = \left(\frac{2 - 2k^2 - 2c^2}{c^4 - k^4}\right)X^3 + \left(\frac{q^2 - p^2 - r}{c^2 - k^2}\right)X. \tag{29b}$$

According to the first integral method, we suppose that $X(\xi)$ and $Y(\xi)$ are nontrivial solutions of (29a) and (29b), and $P(X,Y) = \sum_{i=0}^{m} a_i(X)Y^i$ is an irreducible polynomial in the complex domain $C[X,Y]$ such that

$$P\left[X\left(\xi\right), Y\left(\xi\right)\right] = \sum_{i=0}^{m} a_i\left(X\left(\xi\right)\right)Y^i\left(\xi\right) = 0, \tag{30}$$

where $a_i(X)$, $(i = 0, 1, 2, \ldots, m)$ are polynomials of X and $a_m(X) \neq 0$.

Equation (30) is called the first integral to (29a) and (29b). Due to the Division Theorem, there exists a polynomial $h(X) + g(X)Y$ in the complex domain $C[X,Y]$ such that

$$\begin{aligned} \frac{dP}{d\xi} &= \frac{\partial P}{\partial X}\frac{dX}{d\xi} + \frac{\partial P}{\partial Y}\frac{dY}{d\xi} \\ &= \left[h\left(X\right) + g\left(X\right)Y\right]\sum_{i=0}^{m} a_i\left(X\right)Y^i. \end{aligned} \tag{31}$$

Here, we have considered two different cases, assuming that $m = 1$ and $m = 2$ in (30).

Case 1. Suppose that $m = 1$, by equating the coefficients of Y^i $(i = 2, 1, 0)$ on both sides of (31), we have

$$a_1'\left(X\right) = g\left(X\right)a_1\left(X\right), \tag{32a}$$

$$a_0'\left(X\right) = h\left(X\right)a_1\left(X\right) + g\left(X\right)a_0\left(X\right), \tag{32b}$$

$$\begin{aligned} a_1\left(X\right)&\left(\left(\frac{2 - 2k^2 - 2c^2}{c^4 - k^4}\right)X^3 + \left(\frac{q^2 - p^2 - r}{c^2 - k^2}\right)X\right) \\ &= h\left(X\right)a_0\left(X\right). \end{aligned} \tag{32c}$$

Since $a_i(X)$ $(i = 0, 1)$ are polynomials, then from (32a) it can be deduced that $a_1(X)$ is constant and $g(X) = 0$. For simplicity, take $a_1(X) = 1$.

Balancing the degrees of $h(X)$ and $a_0(X)$, it can be concluded that $\deg(h(X)) = 1$ only. Suppose that $h(X) = AX + B$, and $A \neq 0$, then we find $a_0(X)$

$$a_0\left(X\right) = \frac{A}{2}X^2 + BX + D. \tag{33}$$

Substituting $a_0(X)$, $a_1(X)$, and $h(X)$ for (32c) and setting all the coefficients of powers X to be zero, then we obtain a system of nonlinear algebraic equations and by solving it, we obtain

$$D = \mp\frac{\sqrt{-\left(-1+c^2+k^2\right)/\left(c^2-k^2\right)}\sqrt{c^2+k^2}\left(p^2-q^2+r\right)}{2\left(-1+c^2+k^2\right)},$$

$$B=0, \qquad A=\mp\frac{2\sqrt{-\left(-1+c^2+k^2\right)/\left(c^2-k^2\right)}}{\sqrt{c^2+k^2}}. \tag{34}$$

Using the conditions (34) in (30), we obtain

$$Y(\xi) = \pm\frac{\sqrt{-\left(-1+c^2+k^2\right)/\left(c^2-k^2\right)}}{\sqrt{c^2+k^2}}X^2(\xi) \pm D, \tag{35}$$

respectively.

Combining (35) with (29a), we obtain the exact solutions to (28), and considering the solution v given by the relation (27), thus the exact traveling wave solutions to the (2 + 1)-dimensional Davey-Stewartson system (23) were obtained and can be written as

$$\begin{aligned} u_{1,2}(x,y,t) = &\pm\frac{i\sqrt{p^2-q^2+r}}{\sqrt{2-2/\left(c^2+k^2\right)}} \\ &\times\tanh\Big[\sqrt{p^2-q^2+r} \\ &\quad\times\Big(kx+cy+dt+\gamma \\ &\quad\quad\mp 2i\sqrt{c-k}\sqrt{c+k} \\ &\quad\quad\times\sqrt{-1+c^2+k^2}\sqrt{c^2+k^2}\xi_0\Big) \\ &\quad\times\left(\sqrt{2}\sqrt{c-k}\sqrt{c+k}\right)^{-1}\Big] \\ &\times\exp\left[i\left(px+qy+rt+\varepsilon\right)\right], \end{aligned} \tag{36}$$

$$\begin{aligned} v_{1,2}(x,y,t) = &\left(-\frac{1}{c^2+k^2}\right) \\ &\times\Bigg[\pm\frac{i\sqrt{p^2-q^2+r}}{\sqrt{2-2/\left(c^2+k^2\right)}} \\ &\quad\times\tanh\Big[\sqrt{p^2-q^2+r} \\ &\quad\quad\times\Big(kx+cy+dt+\gamma \\ &\quad\quad\quad\mp 2i\sqrt{c-k}\sqrt{c+k} \\ &\quad\quad\quad\times\sqrt{-1+c^2+k^2} \\ &\quad\quad\quad\times\sqrt{c^2+k^2}\xi_0\Big) \\ &\quad\quad\times\left(\sqrt{2}\sqrt{c-k}\sqrt{c+k}\right)^{-1}\Big] \\ &\quad\times\exp\left[i\left(px+qy+rt+\varepsilon\right)\right]\Bigg]^2, \end{aligned} \tag{37}$$

respectively, where, ξ_0 is an arbitrary integration constant.

Case 2. Suppose that $m = 2$, by equating the coefficients of Y^i $(i = 3, 2, 1, 0)$ on both sides of (31), we have

$$a_2'(X) = g(X)a_2(X), \tag{38a}$$

$$a_1'(X) = h(X)a_2(X) + g(X)a_1(X), \tag{38b}$$

$$\begin{aligned} a_0'(X) + 2a_2(X)&\left[\left(\frac{2-2k^2-2c^2}{c^4-k^4}\right)X^3 + \left(\frac{q^2-p^2-r}{c^2-k^2}\right)X\right] \\ &= h(X)a_1(X) + g(X)a_0(X), \end{aligned} \tag{38c}$$

$$\begin{aligned} a_1(X)&\left[\left(\frac{2-2k^2-2c^2}{c^4-k^4}\right)X^3 + \left(\frac{q^2-p^2-r}{c^2-k^2}\right)X\right] \\ &= h(X)a_0(X). \end{aligned} \tag{38d}$$

Since, $a_i(X)$ $(i = 0, 1, 2)$ are polynomials, then from (38a) it can be deduced that $a_2(X)$ is a constant and $g(X) = 0$. For simplicity, we take $a_2(X) = 1$. Balancing the degrees of $h(X)$ and $a_0(X)$ it can be concluded that $\deg(h(X)) = 1$ only.

In this case, it was assumed that $h(X) = AX + B$, and $A \neq 0$, then we find $a_1(X)$ and $a_0(X)$ as follows:

$$a_1(X) = \left(\frac{A}{2}\right)X^2 + BX + D, \tag{39a}$$

$$\begin{aligned} a_0(X) = &\left(\frac{A^2}{8} - \frac{1-k^2-c^2}{c^4-k^4}\right)X^4 + \frac{AB}{2}X^3 \\ &+ \left(\frac{AD+B^2}{2} - \frac{q^2-p^2-r}{c^2-k^2}\right)X^2 + BDX + F, \end{aligned} \tag{39b}$$

where A, B, D, and F are arbitrary constants.

Substituting $a_0(X)$, $a_1(X)$, $a_2(X)$, and $h(X)$ for (38d) and setting all the coefficients of powers X to be zero, a system of nonlinear algebraic equations was obtained and by solving it, we got

$$F = -\frac{\left(c^2+k^2\right)\left(p^2-q^2+r\right)^2}{4\left(-c^2+c^4+k^2-k^4\right)}, \qquad B = 0,$$

$$D=-\frac{p^2-q^2+r}{(c-k)\sqrt{c+k}\sqrt{-(-1+c^2+k^2)/(c-k)(c^2+k^2)}},$$
$$A=\frac{4\sqrt{-(-1+c^2+k^2)/(c-k)(c^2+k^2)}}{\sqrt{c+k}}, \quad (40a)$$

$$F=-\frac{(c^2+k^2)(p^2-q^2+r)^2}{4(-c^2+c^4+k^2-k^4)}, \qquad B=0,$$
$$D=\frac{p^2-q^2+r}{(c-k)\sqrt{c+k}\sqrt{-(-1+c^2+k^2)/(c-k)(c^2+k^2)}},$$
$$A=-\frac{4\sqrt{-(-1+c^2+k^2)/(c-k)(c^2+k^2)}}{\sqrt{c+k}}. \quad (40b)$$

Using the conditions (40a) and (40b) in (30), we obtain

$$\begin{aligned}Y(\xi) &= -\Bigg[\sqrt{-\frac{-1+c^2+k^2}{(c-k)(c^2+k^2)}}\\ &\times\Big(\mp 4\sqrt{(c+k)^2(-1+c^2+k^2)^3(c^2+k^2)(p^2-q^2+r)X^2(\xi)}\\ &+(c-k)(-1+c^2+k^2)\\ &\times(-2X^2(\xi)+(c^2+k^2)(p^2-q^2+r+2X^2(\xi)))\Big)\Bigg]\\ &\times\Big(2(c+k)^{3/2}\big((-1+c^2+k^2)^2\big)\Big)^{-1},\end{aligned} \quad (41a)$$

$$\begin{aligned}Y(\xi) &= -\Bigg[\sqrt{-\frac{-1+c^2+k^2}{(c-k)(c^2+k^2)}}\\ &\times\Big(\mp 4\sqrt{(c+k)^2(-1+c^2+k^2)^3(c^2+k^2)(p^2-q^2+r)X^2(\xi)}\\ &-(c-k)(-1+c^2+k^2)\\ &\times(-2X^2(\xi)+(c^2+k^2)(p^2-q^2+r+2X^2(\xi)))\Big)\Bigg]\\ &\times\Big(2(c+k)^{3/2}\big((-1+c^2+k^2)^2\big)\Big)^{-1}.\end{aligned} \quad (41b)$$

Combining (41a) and (41b) with (29a) we have obtained the exact solutions to (28), and considering the solution v given by the relation (27), thus the exact traveling wave solutions to the (2+1)-dimensional Davey-Stewartson system (23) can be written as

$$\begin{aligned}u_{3,4}(x,y,t) &= \frac{\sqrt{p^2-q^2+r}}{2\sqrt{1-1/(c^2+k^2)}}\\ &\times\Bigg(\pm 2\mp\sqrt{2}\\ &\times\tanh\Bigg[\Bigg(\sqrt{p^2-q^2+r}\\ &\times\Big(\mp i\sqrt{-1+c^2+k^2}\sqrt{c^2+k^2}\\ &\times(kx+cy+dt+\gamma)\\ &\pm 2\sqrt{c-k}\sqrt{c+k}\\ &\times(-1+c^2+k^2)(c^2+k^2)\xi_0\Big)\Bigg)\\ &\times\Bigg(\Big(\sqrt{2}\sqrt{c-k}\sqrt{c+k}\Big)\sqrt{-1+c^2+k^2}\\ &\times\sqrt{c^2+k^2}\Bigg)^{-1}\Bigg)\Bigg]\\ &\times\exp[i(px+qy+rt+\varepsilon)],\end{aligned} \quad (42)$$

$$\begin{aligned}v_{3,4}(x,y,t) &= -\left(\frac{1}{c^2+k^2}\right)\\ &\times\Bigg[\frac{\sqrt{p^2-q^2+r}}{2\sqrt{1-1/(c^2+k^2)}}\\ &\times\Bigg(\pm 2\mp\sqrt{2}\\ &\times\tanh\Bigg[\Bigg(\sqrt{p^2-q^2+r}\\ &\times\Big(\mp i\sqrt{-1+c^2+k^2}\sqrt{c^2+k^2}\\ &\times(kx+cy+dt+\gamma)\\ &\pm 2\sqrt{c-k}\sqrt{c+k}\\ &\times(-1+c^2+k^2)(c^2+k^2)\xi_0\Big)\end{aligned}$$

$$\times\left(\left(\sqrt{2}\sqrt{c-k}\sqrt{c+k}\right)\sqrt{-1+c^2+k^2}\times\sqrt{c^2+k^2}\right)^{-1}\Bigg)\Bigg]\Bigg]^2\times\exp\left[i\left(px+qy+rt+\varepsilon\right)\right], \quad (43)$$

$$u_{5,6}(x,y,t) = \frac{\sqrt{p^2-q^2+r}}{2\sqrt{1-1/(c^2+k^2)}}\times\Bigg(\mp 2\pm\sqrt{2}\times\tanh\Bigg[\Bigg(\sqrt{p^2-q^2+r}\times\Bigg(\mp i\sqrt{-1+c^2+k^2}\sqrt{c^2+k^2}\times(kx+cy+dt+\gamma)\mp 2\sqrt{c-k}\sqrt{c+k}\times\left(-1+c^2+k^2\right)\left(c^2+k^2\right)\xi_0\Bigg)\Bigg)\times\left(\left(\sqrt{2}\sqrt{c-k}\sqrt{c+k}\right)\sqrt{-1+c^2+k^2}\times\sqrt{c^2+k^2}\right)^{-1}\Bigg)\Bigg]\times\exp\left[i\left(px+qy+rt+\varepsilon\right)\right], \quad (44)$$

$$v_{5,6}(x,y,t) = -\left(\frac{1}{c^2+k^2}\right)\times\Bigg[\frac{\sqrt{p^2-q^2+r}}{2\sqrt{1-1/(c^2+k^2)}}\times\Bigg(\mp 2\pm\sqrt{2}\times\tanh\Bigg[\Bigg(\sqrt{p^2-q^2+r}\times\Bigg(\mp i\sqrt{-1+c^2+k^2}\sqrt{c^2+k^2}\times(kx+cy+dt+\gamma)\mp 2\sqrt{c-k}\sqrt{c+k}\times\left(-1+c^2+k^2\right)\left(c^2+k^2\right)\xi_0\Bigg)\Bigg)\times\left(\left(\sqrt{2}\sqrt{c-k}\sqrt{c+k}\right)\sqrt{-1+c^2+k^2}\times\sqrt{c^2+k^2}\right)^{-1}\Bigg)\Bigg]\Bigg]^2\times\exp\left[i\left(px+qy+rt+\varepsilon\right)\right], \quad (45)$$

respectively, where ξ_0 is an arbitrary integration constant.

Equations (36)-(37) and (42)–(45) are new types of exact traveling wave solutions to the (2+1)-dimensional Davey-Stewartson system (23). It could not be obtained by the methods presented in [37].

3.3. *Generalized Hirota-Satsuma Coupled KdV System*

Consider the generalized Hirota-Satsuma coupled KdV system [38]

$$u_t = \frac{1}{4}u_{xxx} + 3uu_x + 3\left(w - v^2\right)_x, \quad (46)$$

$$v_t = -\frac{1}{2}v_{xxx} - 3uv_x, \quad (47)$$

$$w_t = -\frac{1}{2}w_{xxx} - 3uw_x. \quad (48)$$

When $w = 0$, (46)–(48) reduce to be the well-known Hirota-Satsuma coupled KdV system. We seek traveling wave solutions for (46)–(48) in the form

$$u(x,t) = u(\xi), \qquad v(x,t) = v(\xi), \qquad w(x,t) = w(\xi), \qquad \xi = k(x-ct)+\varsigma, \quad (49)$$

where ς is an arbitrary constant.

Substituting (49) into (46)-(47) yields an ODE

$$-cku' = \frac{1}{4}k^3u''' + 3kuu' + 3k\left(w - v^2\right)', \quad (50)$$

$$-ckv' = -\frac{1}{2}k^3v''' - 3kuv', \quad (51)$$

$$-ckw' = -\frac{1}{2}k^3w''' - 3kuw'. \quad (52)$$

Let

$$u = \alpha v^2 + \beta v + \gamma, \qquad w = A_0 v + B_0, \quad (53)$$

where α, γ, β, A_0, and B_0 are constants [38]. Inserting (53) into (50) and (51) integrating once we know that (50) and (51) give rise to the same equation

$$k^2 v'' = -2\alpha v^3 - 3\beta v^2 + 2(c - 3\gamma) v + c_1, \tag{54}$$

where c_1 is an integration constant. Integrating (54) once again we have

$$k^2 v'^2 = -\alpha v^4 - 2\beta v^3 + 2(c - 3\gamma) v^2 + 2c_1 v + c_2, \tag{55}$$

where c_2 is an integration constant. By means of (53)–(55) we get

$$\begin{aligned} k^2 u'' &= 2\alpha k^2 v'^2 + k^2 (2\alpha v + \beta) v'' \\ &= 2\alpha \left[-\alpha v^4 - 2\beta v^3 + 2(c - 3\gamma) v^2 + 2c_1 v + c_2\right] \\ &\quad + (2\alpha v + \beta)\left[-2\alpha v^3 - 3\beta v^2 + 2(c - 3\gamma) v + c_1\right]. \end{aligned} \tag{56}$$

Integrating (50) once we have

$$\frac{1}{4}k^2 u'' + \frac{3}{2}u^2 + cu + 3\left(w - v^2\right) + c_3 = 0, \tag{57}$$

where c_3 is an integration constant. Inserting (53) and (56) into (57) gives

$$\begin{gathered} 3\alpha c - 3\alpha\gamma + \frac{3}{4}\beta^2 - 3 = 0, \\ \frac{1}{2}\left[\alpha c_1 + \beta c + \gamma\beta\right] + A_0 = 0, \\ \frac{1}{4}\left[2\alpha c_2 + \beta c_1\right] + \frac{3}{2}\gamma^2 + c\gamma + 3B_0 + c_3 = 0. \end{gathered} \tag{58}$$

Let

$$\begin{gathered} c_1 = \frac{1}{2\alpha^2}\left[\beta^3 + 2c\alpha\beta - 6\alpha\beta\gamma\right], \\ v(\xi) = aP(\xi) - \frac{\beta}{2\alpha}. \end{gathered} \tag{59}$$

Therefore from (58), we have

$$k^2 P''(\xi) - a\left(\frac{3\beta^2}{2\alpha} + 2c - 6\gamma\right) P(\xi) + 2\alpha a^3 P^3(\xi) = 0. \tag{60}$$

By introducing new variables $X = P(\xi)$ and $Y = P'(\xi)$, (60) changes into a system of ODEs

$$X' = Y, \tag{61a}$$

$$Y' = -\left(\frac{2\alpha a^3}{k^2}\right) X^3 + \frac{a}{k^2}\left(\frac{3\beta^2}{2\alpha} + 2c - 6\gamma\right) X. \tag{61b}$$

According to the first integral method, we suppose that $X(\xi)$ and $Y(\xi)$ are nontrivial solutions of (61a) and (61b), and $P(X, Y) = \sum_{i=0}^{m} a_i(X) Y^i$ is an irreducible polynomial in the complex domain $C[X, Y]$ such that

$$P[X(\xi), Y(\xi)] = \sum_{i=0}^{m} a_i(X(\xi)) Y^i(\xi) = 0, \tag{62}$$

where $a_i(X)$, $(i = 0, 1, 2, \ldots, m)$ are polynomials of X and $a_m(X) \neq 0$.

Equation (62) is called the first integral to (61a) and (61b). Due to the Division Theorem, there exists a polynomial $h(X) + g(X)Y$ in the complex domain $C[X, Y]$ such that

$$\begin{aligned} \frac{dP}{d\xi} &= \frac{\partial P}{\partial X}\frac{dX}{d\xi} + \frac{\partial P}{\partial Y}\frac{dY}{d\xi} \\ &= \left[h(X) + g(X) Y\right] \sum_{i=0}^{m} a_i(X) Y^i. \end{aligned} \tag{63}$$

Here, we have considered two different cases, assuming that $m = 1$ and $m = 2$ in (62).

Case 1. Suppose that $m = 1$, by equating the coefficients of Y^i $(i = 2, 1, 0)$ on both sides of (63), we have

$$a_1'(X) = g(X) a_1(X), \tag{64a}$$

$$a_0'(X) = h(X) a_1(X) + g(X) a_0(X), \tag{64b}$$

$$\begin{aligned} a_1(X)&\left[-\left(\frac{2\alpha a^3}{k^2}\right) X^3 + \frac{a}{k^2}\left(\frac{3\beta^2}{2\alpha} + 2c - 6\gamma\right) X\right] \\ &= h(X) a_0(X). \end{aligned} \tag{64c}$$

Since $a_i(X)$ $(i = 0, 1)$ are polynomials, then from (64a) it was deduced that $a_1(X)$ is constant and $g(X) = 0$. For simplicity, take $a_1(X) = 1$.

Balancing the degrees of $h(X)$ and $a_0(X)$, it was concluded that $\deg(h(X)) = 1$ only. Suppose that $h(X) = AX + B$, and $A \neq 0$, then we find

$$a_0(X) = \frac{A}{2}X^2 + BX + D, \tag{65}$$

where D is an arbitrary integration constant.

Substituting $a_0(X)$, $a_1(X)$, and $h(X)$ for (64c) and setting all the coefficients of powers X to be zero, then we have obtained a system of nonlinear algebraic equations and by solving it, we obtain

$$\begin{gathered} A = \mp\frac{2ia^{3/2}\sqrt{\alpha}}{k}, \qquad B = 0, \\ c = \frac{\mp 4i\sqrt{a}Dk\alpha^{3/2} - 3\beta^2 + 12\alpha\gamma}{4\alpha}. \end{gathered} \tag{66}$$

Using the conditions (66) in (62), we obtain

$$Y(\xi) = \left(\pm\frac{ia^{3/2}\sqrt{\alpha}}{k}\right) X^2(\xi) - D, \tag{67}$$

respectively. Combining (67) with (61a), the exact solutions to (60) were obtained, and considering the solutions given by the relation (53), then the exact traveling wave solutions to the generalized Hirota-Satsuma coupled KdN system (46)–(48) are obtained and can be written as

$$u_1(x,t) = \alpha\left(a\left[(1-i)\sqrt{D}\sqrt{k} \times \tanh\left(\left(\left(\frac{1}{2}+\frac{i}{2}\right)a^{3/2}\sqrt{D}\alpha^{1/4} \times \left[-(k(x-ct)+\varsigma)+2k\xi_0\right]\right) \times \left(\sqrt{k}\right)^{-1}\right) \times \left(a^{3/2}\alpha^{1/4}\right)^{-1}\right] - \frac{\beta}{2\alpha}\right)^2 + \beta\left(a\left[(1-i)\sqrt{D}\sqrt{k} \times \tanh\left(\left(\left(\frac{1}{2}+\frac{i}{2}\right)a^{3/2}\sqrt{D}\alpha^{1/4} \times \left[-(k(x-ct)+\varsigma)+2k\xi_0\right]\right)\left(\sqrt{k}\right)^{-1}\right) \times \left(a^{3/2}\alpha^{1/4}\right)^{-1}\right] - \frac{\beta}{2\alpha}\right) + \gamma, \tag{68}$$

$$v_1(x,t) = a\left[(1-i)\sqrt{D}\sqrt{k} \times \tanh\left(\left(\left(\frac{1}{2}+\frac{i}{2}\right)a^{3/2}\sqrt{D}\alpha^{1/4} \times \left[-(k(x-ct)+\varsigma)+2k\xi_0\right]\right) \times \left(\sqrt{k}\right)^{-1}\right)\left(a^{3/2}\alpha^{1/4}\right)^{-1}\right] - \frac{\beta}{2\alpha}, \tag{69}$$

$$w_1(x,t) = A_0\left(a\left[(1-i)\sqrt{D}\sqrt{k} \times \tanh\left(\left(\left(\frac{1}{2}+\frac{i}{2}\right)a^{3/2}\sqrt{D}\alpha^{1/4} \times \left[-(k(x-ct)+\varsigma)+2k\xi_0\right]\right) \times \left(\sqrt{k}\right)^{-1}\right) \times \left(a^{3/2}\alpha^{1/4}\right)^{-1}\right] - \frac{\beta}{2\alpha}\right) + B_0, \tag{70}$$

$$u_2(x,t) = \alpha\left(a\left[(1-i)\sqrt{D}\sqrt{k} \times \tan\left(\left(\left(\frac{1}{2}+\frac{i}{2}\right)a^{3/2}\sqrt{D}\alpha^{1/4} \times \left[-(k(x-ct)+\varsigma)+2k\xi_0\right]\right) \times \left(\sqrt{k}\right)^{-1}\right) \times \left(a^{3/2}\alpha^{1/4}\right)^{-1}\right] - \frac{\beta}{2\alpha}\right)^2 + \beta\left(a\left[(1-i)\sqrt{D}\sqrt{k} \times \tanh\left(\left(\left(\frac{1}{2}+\frac{i}{2}\right)a^{3/2}\sqrt{D}\alpha^{1/4} \times \left[-(k(x-ct)+\varsigma)+2k\xi_0\right]\right) \times \left(\sqrt{k}\right)^{-1}\right) \times \left(a^{3/2}\alpha^{1/4}\right)^{-1}\right] - \frac{\beta}{2\alpha}\right) + \gamma, \tag{71}$$

$$v_2(x,t) = a\left[(1-i)\sqrt{D}\sqrt{k} \times \tan\left(\left(\left(\frac{1}{2}+\frac{i}{2}\right)a^{3/2}\sqrt{D}\alpha^{1/4} \times \left[-(k(x-ct)+\varsigma)+2k\xi_0\right]\right) \times \left(\sqrt{k}\right)^{-1}\right) \times \left(a^{3/2}\alpha^{1/4}\right)^{-1}\right] - \frac{\beta}{2\alpha}, \tag{72}$$

$$w_2(x,t) = A_0\left(a\left[(1-i)\sqrt{D}\sqrt{k} \times \tan\left(\left(\left(\frac{1}{2}+\frac{i}{2}\right)a^{3/2}\sqrt{D}\alpha^{1/4} \times \left[-(k(x-ct)+\varsigma)+2k\xi_0\right]\right) \times \left(\sqrt{k}\right)^{-1}\right) \times \left(a^{3/2}\alpha^{1/4}\right)^{-1}\right] - \frac{\beta}{2\alpha}\right) + B_0, \tag{73}$$

respectively, where ξ_0 is an arbitrary constant.

Case 2. Suppose that $m = 2$, by equating the coefficients of Y^i $(i = 3, 2, 1, 0)$ on both sides of (63), we have

$$a_2'(X) = g(X)\, a_2(X), \tag{74a}$$

$$a_1'(X) = h(X)\, a_2(X) + g(X)\, a_1(X), \tag{74b}$$

$$a_0'(X) + 2a_2(X)\left[-\left(\frac{2\alpha a^3}{k^2}\right)X^3 + \frac{a}{k^2}\left(\frac{3\beta^2}{2\alpha} + 2c - 6\gamma\right)X\right] = h(X)\, a_1(X) + g(X)\, a_0(X), \tag{74c}$$

$$a_1(X)\left[-\left(\frac{2\alpha a^3}{k^2}\right)X^3 + \frac{a}{k^2}\left(\frac{3\beta^2}{2\alpha} + 2c - 6\gamma\right)X\right] = h(X)\, a_0(X). \tag{74d}$$

Since $a_i(X)$ $(i = 0, 1, 2)$ are polynomials, then from (74a) it can be deduced that $a_2(X)$ is a constant and $g(X) = 0$. For simplicity, we take $a_2(X) = 1$. Balancing the degrees of $h(X)$ and $a_0(X)$ it can be concluded that $\deg(h(X)) = 1$, only.

In this case, it was assumed that $h(X) = AX + B$, and $A \neq 0$, then we find $a_1(X)$ and $a_0(X)$ as follows:

$$a_1(X) = \left(\frac{A}{2}\right)X^2 + BX + D, \tag{75a}$$

$$a_0(X) = \left(\frac{A^2}{8} + \frac{\alpha a^3}{k^2}\right)X^4 + \frac{1}{2}(AB)X^3 + \left(\frac{AD + B^2}{2} + \frac{-a}{k^2}\left(\frac{3\beta^2}{2\alpha}\right) + 2c - 6\gamma\right)X^2 + BDX + F, \tag{75b}$$

where A, B, D, and F are arbitrary constants.

Substituting $a_0(X)$, $a_1(X)$, $a_2(X)$, and $h(X)$ for (74d) and setting all the coefficients of powers X to be zero, a system of nonlinear algebraic equations was obtained and by solving it, we got

$$F = 0, \quad c = \frac{3\left(-\beta^2 + 4\alpha\gamma\right)}{4\alpha}, \quad B = 0, \quad D = 0, \quad A = -\frac{4ia^{3/2}\sqrt{\alpha}}{k}, \tag{76a}$$

$$F = 0, \quad c = \frac{3\left(-\beta^2 + 4\alpha\gamma\right)}{4\alpha}, \quad B = 0, \quad D = 0, \quad A = \frac{4ia^{3/2}\sqrt{\alpha}}{k}. \tag{76b}$$

Using the conditions (76a) in (62), we obtain

$$Y(\xi) = \frac{ia^3kX^2(\xi)\sqrt{\alpha} \mp \sqrt{-a^3\left(-2 + a^3\right)k^2X^4(\xi)\,\alpha}}{2k^2}, \tag{77}$$

respectively. Combining (77) with (61a), the exact solutions to (60) were obtained, and considering the solutions given by the relation (53), then the exact traveling wave solutions to the generalized Hirota-Satsuma coupled KdN system (46)–(48) are obtained and can be written as

$$u_3(x,t) = \alpha\left(a\left[-(2k) \times \left(ia^{3/2}\left(a^{3/2} - \sqrt{-2 + a^3}\right) \times \sqrt{\alpha}[k(x - ct) + \varsigma] + 2k\xi_0\right)^{-1}\right] - \frac{\beta}{2\alpha}\right)^2 + \beta\left(a\left[-(2k) \times \left(ia^{3/2}\left(a^{3/2} - \sqrt{-2 + a^3}\right) \times \sqrt{\alpha}\,[k(x - ct) + \varsigma] + 2k\xi_0\right)^{-1}\right] - \frac{\beta}{2\alpha}\right) + \gamma, \tag{78}$$

$$v_3(x,t) = a\left[-(2k) \times \left(ia^{3/2}\left(a^{3/2} - \sqrt{-2 + a^3}\right) \times \sqrt{\alpha}[k(x - ct) + \varsigma] + 2k\xi_0\right)^{-1}\right] - \frac{\beta}{2\alpha}, \tag{79}$$

$$w_3(x,t) = A_0\left(a\left[-(2k) \times \left(ia^{3/2}\left(a^{3/2} - \sqrt{-2 + a^3}\right) \times \sqrt{\alpha}[k(x - ct) + \varsigma] + 2k\xi_0\right)^{-1}\right] - \frac{\beta}{2\alpha}\right) + B_0, \tag{80}$$

$$u_4(x,t) = \alpha\left(a\left[-(2k) \times \left(ia^{3/2}\left(a^{3/2} + \sqrt{-2 + a^3}\right) \times \sqrt{\alpha}[k(x - ct) + \varsigma] + 2k\xi_0\right)^{-1}\right] - \frac{\beta}{2\alpha}\right)^2$$

$$+\beta\left(a\left[-(2k)\times\left(ia^{3/2}\left(a^{3/2}+\sqrt{-2+a^3}\right)\times\sqrt{\alpha}\left[k(x-ct)+\varsigma\right]+2k\xi_0\right)^{-1}\right]-\frac{\beta}{2\alpha}\right)^2+\gamma, \tag{81}$$

$$v_4(x,t)=a\left[-(2k)\times\left(ia^{3/2}\left(a^{3/2}+\sqrt{-2+a^3}\right)\times\sqrt{\alpha}[k(x-ct)+\varsigma]+2k\xi_0\right)^{-1}\right]-\frac{\beta}{2\alpha}, \tag{82}$$

$$w_4(x,t)=A_0\left(a\left[-(2k)\times\left(ia^{3/2}\left(a^{3/2}+\sqrt{-2+a^3}\right)\times\sqrt{\alpha}[k(x-ct)+\varsigma]+2k\xi_0\right)^{-1}\right]-\frac{\beta}{2\alpha}\right)+B_0. \tag{83}$$

Similarly, as for the case of (76b) from (62) we obtain

$$Y(\xi)=\mp\frac{\pm ia^3kX^2(\xi)\sqrt{\alpha}+\sqrt{-a^3(-2+a^3)k^2X^4(\xi)\alpha}}{2k^2}, \tag{84}$$

respectively. Combining (84) with (61a), the exact solutions to (60) were obtained, and considering the solutions given by the relation (53), then the exact traveling wave solutions to the generalized Hirota-Satsuma coupled KdN system (46)–(48) are obtainred and can be written as

$$u_5(x,t)=\alpha\left(a\left[-(2k)\times\left(-ia^{3/2}\left(a^{3/2}+\sqrt{-2+a^3}\right)\times\sqrt{\alpha}[k(x-ct)+\varsigma]+2k\xi_0\right)^{-1}\right]-\frac{\beta}{2\alpha}\right)^2,$$
$$+\beta\left(a\left[-(2k)\times\left(-ia^{3/2}\left(a^{3/2}+\sqrt{-2+a^3}\right)\times\sqrt{\alpha}\left[k(x-ct)+\varsigma\right]+2k\xi_0\right)^{-1}\right]-\frac{\beta}{2\alpha}\right)+\gamma, \tag{85}$$

$$v_5(x,t)=a\left[-(2k)\times\left(-ia^{3/2}\left(a^{3/2}+\sqrt{-2+a^3}\right)\times\sqrt{\alpha}\left[k(x-ct)+\varsigma\right]+2k\xi_0\right)^{-1}\right]-\frac{\beta}{2\alpha}, \tag{86}$$

$$w_5(x,t)=A_0\left(a\left[-(2k)\times\left(-ia^{3/2}\left(a^{3/2}+\sqrt{-2+a^3}\right)\times\sqrt{\alpha}[k(x-ct)+\varsigma]+2k\xi_0\right)^{-1}\right]-\frac{\beta}{2\alpha}\right)+B_0, \tag{87}$$

$$u_6(x,t)=\alpha\left(a\left[-(2k)\times\left(ia^{3/2}\left(-a^{3/2}+\sqrt{-2+a^3}\right)\times\sqrt{\alpha}[k(x-ct)+\varsigma]+2k\xi_0\right)^{-1}\right]-\frac{\beta}{2\alpha}\right)^2$$
$$+\beta\left(a\left[-(2k)\times\left(ia^{3/2}\left(-a^{3/2}+\sqrt{-2+a^3}\right)\times\sqrt{\alpha}\left[k(x-ct)+\varsigma\right]+2k\xi_0\right)^{-1}\right]-\frac{\beta}{2\alpha}\right)+\gamma, \tag{88}$$

$$v_6(x,t)=a\left[-(2k)\times\left(ia^{3/2}\left(-a^{3/2}+\sqrt{-2+a^3}\right)\times\sqrt{\alpha}\left[k(x-ct)+\varsigma\right]+2k\xi_0\right)^{-1}\right]-\frac{\beta}{2\alpha}, \tag{89}$$

$$w_6(x,t) = A_0\Bigg(a\Bigg[-(2k) \times \Big(ia^{3/2}\Big(-a^{3/2}+\sqrt{-2+a^3}\Big) \times \sqrt{\alpha}[k(x-ct)+\varsigma]+2k\xi_0\Big)^{-1}\Bigg] - \frac{\beta}{2\alpha}\Bigg)+B_0, \tag{90}$$

respectively, where ξ_0 is an arbitrary integration constant.

Comparing the results (68)–(73), (78)–(83), and (85)–(90) with the results in [38, 39], it can be seen that the solutions here are new.

Remark 3. We note that our results were based on the assumptions $m = 1$ and $m = 2$. The discussion becomes more complicated for $m = 3$ and $m = 4$ because the hyper-elliptic integrals, the irregular singular point theory, and the elliptic integrals of the second kind are involved. Also, we do not need to consider the case $m \geq 5$ because an algebraic equation with its degree greater than or equal to 5 is generally not solvable.

4. Conclusion

In this paper, the first integral method was applied successfully to obtain solutions of some important nonlinear systems, namely, the classical Drinfel'd-Sokolov-Wilson system (DSWE), the (2 + 1)-dimensional Davey-Stewartson system, and the generalized Hirota-Satsuma coupled KdV system. Also, we conclude that the proposed method is powerful, effective, and can be extended to solve more other nonlinear evolution equations as well as linear ones, and this will be done in a future work.

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