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Research Article

Strongly Almost Lacunary I-Convergent Sequences

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We study some new strongly almost lacunary I-convergent generalized difference sequence spaces defined by an Orlicz function. We give also some inclusion relations related to these sequence spaces.

1. Introduction

The notion of ideal convergence was first introduced by Kostyrko et al. [1] as a generalization of statistical convergence which was later studied by many other authors.

By a lacunary sequence, we mean an increasing integer sequence $\theta = (k_r)$ such that $k_0 = 0$ and $h_r = k_r - k_{r-1} \to \infty$ as $r \to \infty$.

Throughout this paper, the intervals determined by θ will be denoted by $J_r = (k_{r-1}, k_r]$, and the ratio k_r/k_{r-1} will be defined by ϕ_r.

An Orlicz function is a function $M : [0,\infty) \to [0,\infty)$, which is continuous, nondecreasing, and convex with $M(0) = 0$, $M(x) > 0$, for $x > 0$ and $M(x) \to \infty$, as $x \to \infty$.

Let ℓ_∞, c, and c_0 be the Banach space of bounded, convergent, and null sequences $x = (x_k)$, respectively, with the usual norm $\|x\| = \sup_n |x_n|$.

A sequence $x \in \ell_\infty$ is said to be almost convergent if all of its Banach limits coincide. Let $\widehat{c}$ denote the space of all almost convergent sequences.

Lorentz [2] introduced the following sequence space

$$\widehat{c} = \left\{x \in \ell_\infty : \lim_m t_{m,n}(x) \text{ exists uniformly in } n\right\}, \quad (1)$$

where $t_{m,n}(x) = (x_n + x_{n+1} + \cdots + x_{m+n})/(m+1)$.

The following space of strongly almost convergent sequence was introduced by Maddox [3]:

$$[\widehat{c}] = \left\{x \in \ell_\infty : \lim_m t_{m,n}(|x - Le|) \text{ exists uniformly in } n \text{ for some } L\right\}, \quad (2)$$

where $e = (1, 1, \ldots)$.

Kızmaz [4] studied the difference sequence spaces $\ell_\infty(\Delta)$, $c(\Delta)$, and $c_0(\Delta)$ of crisp sets. The notion is defined as follows:

$$Z(\Delta) = \{x = (x_k) : (\Delta x_k) \in Z\}, \quad (3)$$

for $Z = \ell_\infty, c$, and c_0, where $\Delta x = (\Delta x_k) = (x_k - x_{k+1})$, for all $k \in N$.

The above spaces are Banach spaces, normed by

$$\|x\|_\Delta = |x_1| + \sup_k |\Delta x_k|. \quad (4)$$

Tripathy et al. [5] introduced the generalized difference sequence spaces which are defined as, for $m \geq 1$ and $n \geq 1$,

$$Z(\Delta_m^n) = \{x = (x_k) : (\Delta_m^n x_k) \in Z\}, \quad \text{for } Z = \ell_\infty, c, c_0. \quad (5)$$

This generalized difference has the following binomial representation:

$$\Delta_m^n x_k = \sum_{r=0}^{n} (-1)^r \binom{n}{r} x_{k+rm}. \quad (6)$$

2. Definitions and Preliminaries

Kostyrko et al. [1] introduced the following three definitions.

Let X be a nonempty set. Then a family of sets $I \subseteq 2^X$ (power sets of X) is said to be *ideal* if I is additive, that is, $A, B \in I \Rightarrow A \cup B \in I$, and hereditary, that is, $A \in I, B \subseteq A \Rightarrow B \in I$.

A sequence (x_k) in a normed space $(X, \|\cdot\|)$ is said to be *I-convergent* to $x_0 \in X$ if for each $\varepsilon > 0$, the set

$$E(\varepsilon) = \{k \in N : \|x_k - x_0\| \geq \varepsilon\} \text{ belongs to } I. \tag{7}$$

A sequence (x_k) in a normed space $(X, \|\cdot\|)$ is said to be *I-bounded* if there exists $M > 0$ such that the set $\{k \in N : \|x_k\| > M\}$ belongs to I.

Freedman et al. [6] defined the space N_θ. For any lacunary sequence $\theta = (k_r)$,

$$N_\theta = \left\{(x_k) : \lim_{r\to\infty} h_r^{-1} \sum_{k\in J_r} |x_k - L| = 0, \text{ for some } L\right\}. \tag{8}$$

The space N_θ is a BK space with the norm

$$\|(x_k)\|_\theta = \sup_r h_r^{-1} \sum_{k\in J_r} |x_k|. \tag{9}$$

The notion of lacunary ideal convergence of real sequences introduced by Tripathy et al. in [7, 8] and Hazarika [9, 10] introduced the lacunary ideal convergent sequences of fuzzy real numbers and studied some properties. In [5, 7], the lacunary ideal convergence is defined as follows.

Let $\theta = (k_r)$ be a lacunary sequence. Then a sequence (x_k) is said to be lacunary I-convergent if for every $\varepsilon > 0$, such that

$$\left\{r \in N : h_r^{-1} \sum_{k\in J_r} |x_k - x| \geq \varepsilon\right\} \in I, \tag{10}$$

we write $I_\theta - \lim x_k = x$.

Lindenstrauss and Tzafriri [11] used the idea of Orlicz function to construct the sequence space:

$$\ell_M = \left\{(x_k) \in w : \sum_{k=1}^{\infty} M\left(\frac{|x|}{\rho}\right) < \infty, \text{ for some } \rho > 0\right\}. \tag{11}$$

The space ℓ_M with the norm

$$\|x\| = \inf\left\{\rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \leq 1\right\} \tag{12}$$

becomes a Banach space which is called an Orlicz sequence space.

In this paper, we defined some new generalized difference lacunary I-convergent sequence spaces defined by Orlicz function. We also introduce and examine some new sequence spaces and study their different properties.

3. Main Results

Esi [12] introduced the strongly almost ideal convergent sequence spaces in 2-normed spaces. In this paper we introduced the strongly almost lacunary ideal convergent sequence spaces using generalized difference operator and Orlicz function.

Let I be an admissible ideal of N, M an Orlicz function, and $\theta = (k_r)$ a lacunary sequence. Further, let $s = (s_k)$ be a bounded sequence of positive real numbers and Δ_p^q a generalized difference operator.

For every $\varepsilon > 0$ and for some $\rho > 0$, we have introduced the following sequence spaces:

$$\begin{aligned}
&\widehat{w}^I\left(M, \Delta_p^q, s, \theta\right) \\
&= \Bigg\{ (x_k) : \\
&\qquad \left\{ r \in N : \frac{1}{h_r} \sum_{k\in J_r} \left\{ M\left(\frac{\left|\Delta_p^q t_{km}(x) - l\right|}{\rho}\right)\right\}^{s_k} \geq \varepsilon \right\} \in I, \\
&\qquad m \in N, \text{ for some } l \in R \Bigg\}, \\
&\widehat{w}_0^I\left(M, \Delta_p^q, s, \theta\right) \\
&= \Bigg\{ (x_k) : \\
&\qquad \left\{ r \in N : \frac{1}{h_r} \sum_{k\in J_r} \left\{ M\left(\frac{\left|\Delta_p^q t_{km}(x)\right|}{\rho}\right)\right\}^{s_k} \geq \varepsilon \right\} \in I, \\
&\qquad m \in N \Bigg\}, \\
&\widehat{w}_\infty^I\left(M, \Delta_p^q, s, \theta\right) \\
&= \Bigg\{ (x_k) : \\
&\qquad \left\{ r \in N : \exists K > 0 \text{ s.t. } \frac{1}{h_r} \sum_{k\in J_r} \left\{ M\left(\frac{\left|\Delta_p^q t_{km}(x)\right|}{\rho}\right)\right\}^{s_k} \geq K \right\} \\
&\qquad \in I, \ m \in N \Bigg\}.
\end{aligned} \tag{13}$$

Particular Cases. Consider the following.

(1) If $\theta = (2^r)$, we have $\widehat{w}^I(M, \Delta_p^q, s, \theta) = \widehat{w}^I(M, \Delta_p^q, s)$, $\widehat{w}_0^I(M, \Delta_p^q, s, \theta) = \widehat{w}_0^I(M, \Delta_p^q, s)$, and $\widehat{w}_\infty^I(M, \Delta_p^q, s, \theta) = \widehat{w}_\infty^I(M, \Delta_p^q, s)$.

(2) If $M(x) = x$, then $\widehat{w}^I(M,\Delta_p^q,s,\theta) = \widehat{w}^I(\Delta_p^q,s,\theta)$, $\widehat{w}_0^I(M,\Delta_p^q,s,\theta) = \widehat{w}_0^I(\Delta_p^q,s,\theta)$, and $\widehat{w}_\infty^I(M,\Delta_p^q,s,\theta) = \widehat{w}_\infty^I(\Delta_p^q,s,\theta)$.

(3) If $s_k = 1$ for all $k \in N$, $M(x) = x$, and $\theta = (2^r)$, then $\widehat{w}^I(M,\Delta_p^q,s,\theta) = \widehat{w}^I(\Delta_p^q)$, $\widehat{w}_0^I(M,\Delta_p^q,s,\theta) = \widehat{w}^I(\Delta_p^q)$, and $\widehat{w}_\infty^I(M,\Delta_p^q,s,\theta) = \widehat{w}^I(\Delta_p^q)$.

Theorem 1. *Let the sequence (s_k) be bounded; then $\widehat{w}_0^I(M,\Delta_p^q,s,\theta) \subset \widehat{w}^I(M,\Delta_p^q,s,\theta) \subset \widehat{w}_\infty^I(M,\Delta_p^q,s,\theta)$.*

Proof. Let $x \in \widehat{w}^I(M,\Delta_p^q,s,\theta)$. Then, for some $\rho > 0$, we have

$$\begin{aligned}
&\frac{1}{h_r}\sum_{k\in J_r}\left(M\left(\frac{|\Delta_p^q t_{km}(x)|}{2\rho}\right)\right)^{s_k} \\
&\quad\leq \frac{D}{h_r}\sum_{k\in J_r}\frac{1}{2^{p_k}}\left(M\left(\frac{|\Delta_p^q t_{km}(x)-l|}{\rho}\right)\right)^{s_k} \\
&\qquad+\frac{D}{h_r}\sum_{k\in J_r}\frac{1}{2^{p_k}}\left(M\left(\frac{|l|}{\rho}\right)\right)^{s_k} \\
&\quad\leq \frac{D}{h_r}\sum_{k\in J_r}\left(M\left(\frac{|\Delta_p^q t_{km}(x)-l|}{\rho}\right)\right)^{s_k} \\
&\qquad+D\max\left\{1,\sup\left(M\left(\frac{|l|}{\rho}\right)\right)^{H}\right\},
\end{aligned}\tag{14}$$

where $\sup_k s_k = H$ and $D = \max(1, 2^{H-1})$.

Hence, $x \in \widehat{w}_\infty^I(M,\Delta_p^q,s,\theta)$.

The inclusion $\widehat{w}_0^I(M,\Delta_p^q,s,\theta) \subset \widehat{w}^I(M,\Delta_p^q,s,\theta)$ is obvious. □

Theorem 2. *Let the sequence (s_k) be bounded; then $\widehat{w}^I(M,\Delta_p^q,s,\theta)$, $\widehat{w}_0^I(M,\Delta_p^q,s,\theta)$, and $\widehat{w}_\infty^I(M,\Delta_p^q,s,\theta)$ are closed under the operations of addition and scalar multiplication.*

Theorem 3. *Let M_1, M_2 be Orlicz functions; then we have*

(1) $\widehat{w}_0^I(M_1,\Delta_p^q,s,\theta)\cap\widehat{w}_0^I(M_2,\Delta_p^q,s,\theta) \subset \widehat{w}_0^I(M_1+M_2,\Delta_p^q,s,\theta)$,

(2) $\widehat{w}^I(M_1,\Delta_p^q,s,\theta)\cap\widehat{w}^I(M_2,\Delta_p^q,s,\theta) \subset \widehat{w}^I(M_1+M_2,\Delta_p^q,s,\theta)$,

(3) $\widehat{w}_\infty^I(M_1,\Delta_p^q,s,\theta)\cap\widehat{w}_\infty^I(M_2,\Delta_p^q,s,\theta) \subset \widehat{w}_\infty^I(M_1+M_2,\Delta_p^q,s,\theta)$.

Theorem 4. *Let $0 < s_k \leq u_k$ for all $k \in N$, and let (u_k/s_k) be bounded; then we have $\widehat{w}^I(M_1,\Delta_p^q,u,\theta) \subseteq \widehat{w}^I(M_1,\Delta_p^q,s,\theta)$.*

Theorem 5. *Let $\theta = (k_r)$ be a lacunary sequence with $1 < \liminf_r u_r \leq \sup_r u_r < \infty$. Then, for any Orlicz function M, $\widehat{w}^I(M,\Delta_p^q,s) = \widehat{w}^I(M,\Delta_p^q,s,\theta)$.*

Proof. Suppose $\liminf_r u_r > 1$ then there exists $\delta > 0$ such that $u_r = k_r/k_{r-1} \geq 1+\delta$ for all $r \geq 1$.

Then, for $x \in \widehat{w}^I(M,\Delta_p^q,s)$, we have

$$\left\{m \in N : \sum_{k\in J_r}\left(M\left(\frac{|\Delta_p^q t_{km}(x)-l|}{\rho}\right)\right)^{s_k} \geq \varepsilon\right\} \in I. \tag{15}$$

Let

$$\begin{aligned}
A_r &= \frac{1}{h_r}\sum_{k\in J_r}\left(M\left(\frac{|\Delta_p^q t_{km}(x)-l|}{\rho}\right)\right)^{s_k} \\
&= \frac{1}{h_r}\sum_{k=1}^{k_r}\left(M\left(\frac{|\Delta_p^q t_{km}(x)-l|}{\rho}\right)\right)^{s_k} \\
&\quad-\frac{1}{h_r}\sum_{k=1}^{k_{r-1}}\left(M\left(\frac{|\Delta_p^q t_{km}(x)-l|}{\rho}\right)\right)^{s_k} \\
&= \frac{k_r}{h_r}\left(\frac{1}{k_r}\sum_{k=1}^{k_r}\left(M\left(\frac{|\Delta_p^q t_{km}(x)-l|}{\rho}\right)\right)^{s_k}\right) \\
&\quad-\frac{k_{r-1}}{h_r}\left(\frac{1}{k_{r-1}}\sum_{k=1}^{k_{r-1}}\left(M\left(\frac{|\Delta_p^q t_{km}(x)-l|}{\rho}\right)\right)^{s_k}\right).
\end{aligned}\tag{16}$$

Since $h_r = k_r - k_{r-1}$, we have $k_r/h_r \leq (1+\delta)/\delta$ and $k_{r-1}/h_r \leq 1/\delta$.

So, for $\varepsilon > 0$ and for some $\rho > 0$,

$$\begin{gathered}
\left\{r \in N : \frac{k_r}{h_r}\left(\frac{1}{k_r}\sum_{k=1}^{k_r}\left(M\left(\frac{|\Delta_p^q t_{km}(x)-l|}{\rho}\right)\right)^{s_k}\right) \geq \varepsilon\right\} \in I, \\
m \in N, \text{ for some } l \in R, \\
\left\{r \in N : \frac{k_{r-1}}{h_r}\left(\frac{1}{k_{r-1}}\sum_{k=1}^{k_{r-1}}\left(M\left(\frac{|\Delta_p^q t_{km}(x)-l|}{\rho}\right)\right)^{s_k}\right) \geq \varepsilon\right\} \in I, \\
m \in N, \text{ for some } l \in R.
\end{gathered}\tag{17}$$

Hence, $\widehat{w}^I(M,\Delta_p^q,s) \subset \widehat{w}^I(M,\Delta_p^q,s,\theta)$.

Next, suppose that $\limsup_r q_r < \infty$. Then, there exists $\beta > 0$, such that, $q_r < \beta$ for all $r \geq 1$.

Let $x \in \widehat{w}^I(M,\Delta_p^q,s,\theta)$ and $\varepsilon > 0$. There exists $R > 0$ such that for every $j \geq R$,

$$A_j = \left\{r \in N : \frac{1}{h_r}\sum_{k\in J_j}\left(M\left(\frac{|\Delta_p^q t_{km}(x)-l|}{\rho}\right)\right)^{p_k} \geq \varepsilon\right\} \in I. \tag{18}$$

Let $K > 0$ such that $A_j \leq K$ for all $j = 1, 2, \ldots$. Now let n be any integer with $k_{r-1} < n \leq k_r$, where $r > R$. Then,

$$\begin{aligned}
\frac{1}{n}\sum_{k=1}^{n}\left(M\left(\frac{\left|\Delta_p^q t_{km}(x)-l\right|}{\rho}\right)\right)^{s_k}
&\leq \frac{1}{k_{r-1}}\sum_{k=1}^{k_r}\left(M\left(\frac{\left|\Delta_p^q t_{km}(x)-l\right|}{\rho}\right)\right)^{p_k} \\
&= \frac{1}{k_{r-1}}\left\{\sum_{k\in J_1}\left(M\left(\frac{\left|\Delta_p^q t_{km}(x)-l\right|}{\rho}\right)\right)^{p_k} + \sum_{k\in J_2}\left(M\left(\frac{\left|\Delta_p^q t_{km}(x)-l\right|}{\rho}\right)\right)^{p_k} + \cdots + \sum_{k\in J_r}\left(M\left(\frac{\left|\Delta_p^q t_{km}(x)-l\right|}{\rho}\right)\right)^{p_k}\right\} \\
&= \frac{k_1}{k_{r-1}}A_1 + \frac{k_2-k_1}{k_{r-1}}A_2 + \cdots + \frac{k_R-k_{R-1}}{k_{r-1}}A_R + \cdots + \frac{k_r-k_{r-1}}{k_{r-1}}A_r \\
&= \left(\sup_{j\geq 1} A_j\right)\frac{k_R}{k_{r-1}} + \sup_{j\geq R}\left(A_j\right)\frac{k_r-k_R}{k_{r-1}} \\
&< K\frac{k_R}{k_{r-1}} + \varepsilon\beta.
\end{aligned} \tag{19}$$

Since $k_{r-1} \to \infty$ as $r \to \infty$, it follows that

$$\left\{m \in N : \frac{1}{n}\sum_{k=1}^{n}\left(M\left(\frac{\left|\Delta_p^q t_{km}(x)-l\right|}{\rho}\right)\right)^{s_k} \geq \varepsilon\right\} \in I. \tag{20}$$

Hence, $\widehat{w}^I(M, \Delta_p^q, s, \theta) \subset \widehat{w}^I(M, \Delta_p^q, s)$. □

Theorem 6. *If* $\lim s_k > 0$ *and* x *is strongly almost lacunary convergent to* x_0*, with respect to the Orlicz function* M*, that is,* $(x_k) \to l(\widehat{w}^I(M, \Delta_p^q, s, \theta))$*, then* x_0 *is unique.*

Proof. Let $\lim s_k = s > 0$ and suppose that $x_k \to x_1(\widehat{w}^I(M, \Delta_p^q, s, \theta))$, $x_k \to x_0(\widehat{w}^I(M, \Delta_p^q, s, \theta))$.

Then there exist ρ_1 and ρ_2 such that

$$\left\{r \in N : \frac{1}{h_r}\sum_{k\in J_r}\left(M\left(\frac{\left|\Delta_p^q t_{km}(x)-x_0\right|}{\rho_1}\right)\right)^{s_k} \geq \varepsilon\right\} \in I, \quad m \in N,$$
$$\left\{r \in N : \frac{1}{h_r}\sum_{k\in J_r}\left(M\left(\frac{\left|\Delta_p^q t_{km}(x)-x_1\right|}{\rho_1}\right)\right)^{s_k} \geq \varepsilon\right\} \in I, \quad m \in N. \tag{21}$$

Let $\rho = \max(2\rho_1, 2\rho_2)$. Then we have

$$\begin{aligned}
\frac{1}{h_r}\sum_{k\in J_r}\left(M\left(\frac{\left|x_0-x_1\right|}{\rho}\right)\right)^{s_k}
&\leq \frac{D}{h_r}\sum_{k\in J_r}\left(M\left(\frac{\left|\Delta_p^q t_{km}(x)-x_0\right|}{\rho_1}\right)\right)^{s_k} \\
&+ \frac{D}{h_r}\sum_{k\in J_r}\left(M\left(\frac{\left|\Delta_p^q t_{km}(x)-x_1\right|}{\rho_2}\right)\right)^{s_k},
\end{aligned} \tag{22}$$

where $\sup_k s_k = H$ and $D = \max(1, 2^{H-1})$.

Thus, from (21), we get

$$\left\{r \in N : \frac{1}{h_r}\sum_{k\in J_r}\left(M\left(\frac{|x_0-x_1|}{\rho_1}\right)\right)^{s_k} \geq \varepsilon\right\} \in I. \tag{23}$$

Further, $M(|x_0 - x_1|/\rho)^{s_k} \to M(|x_0 - x_1|/\rho)^{s}$ as $k \to \infty$, and, therefore,

$$M\left(\frac{|x_0-x_1|}{\rho}\right)^{s_k} \longrightarrow M\left(\frac{|x_0-x_1|}{\rho}\right)^{s} = 0. \tag{24}$$

Hence, $x_0 = x_1$. □

4. Conclusion

The concept of lacunary I-convergence has been studied by various mathematicians. In this paper, we have introduced some fairly wide classes of strongly almost lacunary I-convergent sequences of real numbers using Orlicz function with the generalized difference operator. Giving particular values to the sequence $\theta = (k_r)$ and M, we obtain some new sequence spaces which are the special cases of the sequence spaces we have defined. There are lots more to be investigated in the future.

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References

[1] P. Kostyrko, T. Šalát, and W. Wilczyński, "I-convergence," *Real Analysis Exchange*, vol. 26, no. 2, pp. 669–685, 2001.

[2] G. G. Lorentz, "A contribution to the theory of divergent sequences," *Acta Mathematica*, vol. 80, pp. 167–190, 1948.

[3] I. J. Maddox, "Spaces of strongly summable sequences," *The Quarterly Journal of Mathematics*, vol. 18, pp. 345–355, 1967.

[4] H. Kızmaz, "On certain sequence spaces," *Canadian Mathematical Bulletin*, vol. 24, no. 2, pp. 169–176, 1981.

[5] B. C. Tripathy, A. Esi, and B. Tripathy, "On a new type of generalized difference Cesàro sequence spaces," *Soochow Journal of Mathematics*, vol. 31, no. 3, pp. 333–340, 2005.

[6] A. R. Freedman, J. J. Sember, and M. Raphael, "Some Cesàro-type summability spaces," *Proceedings of the London Mathematical Society*, vol. 37, no. 3, pp. 508–520, 1978.

[7] B. C. Tripathy, B. Hazarika, and B. Choudhary, "Lacunary I-convergent sequences," in *Proceedings of the Real Analysis Exchange Summer Symposium*, pp. 56–57, 2009.

[8] B. C. Tripathy, B. Hazarika, and B. Choudhary, "Lacunary I-convergent sequences," *Kyungpook Mathematical Journal*, vol. 52, no. 4, pp. 473–482, 2012.

[9] B. Hazarika, "Lacunary I-convergent sequence of fuzzy real numbers," *Pacific Journal of Science and Technology*, vol. 10, no. 2, pp. 203–206, 2009.

[10] B. Hazarika, "Fuzzy real valued lacunary I-convergent sequences," *Applied Mathematics Letters*, vol. 25, no. 3, pp. 466–470, 2012.

[11] J. Lindenstrauss and L. Tzafriri, "On Orlicz sequence spaces," *Israel Journal of Mathematics*, vol. 10, pp. 379–390, 1971.

[12] A. Esi, "Strongly almost summable sequence spaces in 2-normed spaces defined by ideal convergence and an Orlicz function," *Studia. Universitatis Babeş-Bolyai Mathematica*, vol. 57, no. 1, pp. 75–82, 2012.