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Research Article

Fixed Point of a New Three-Step Iteration Algorithm under Contractive-Like Operators over Normed Spaces

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We introduce a new three-step iteration scheme and prove that this new iteration scheme is convergent to fixed points of contractive-like operators. Also, by providing an example, we show that our new iteration method is faster than another iteration method due to Suantai (2005). Furthermore, it is shown that this new iteration method is equivalent to some other iteration methods in the sense of convergence. Finally, it is proved that this new iteration method is T-stable.

1. Introduction and Preliminaries

Most of nonlinear equations $f(x) = y$ appearing in physical formulations can similarly be transformed into a fixed point equation of the form $x = Tx$. To obtain results on existence and uniqueness of such equations' solution, an approximate fixed point theorem is applied. That is, this application will bring us to the solution of the original equation via help of a particular fixed point iteration method. For this reason, it is crucial to define a new iteration method. To decide whether an iteration method is useful for application, it is of paramount importance to answer the following questions.

(i) Does it converge to fixed point of an operator?

(ii) Is it faster than the iterations defined in the existing literature?

(iii) Is its convergence equivalent to the convergence of the other iteration methods?

(iv) Is it T-stable? and so forth.

Throughout this paper we examine four essential concepts based on the above questions for a new three-step iteration method when applied to contractive-like mapping.

As a background to our exposition, we now give some information about literature of those concepts.

The first concept of this work is about convergence of fixed point iteration methods. Fixed point iteration methods may exhibit radically different behaviors for various classes of mappings. While a particular fixed point iteration method is convergent for an appropriate class of mappings, it could not be convergent for others. Therefore, it is important to determine whether an iteration method converges to fixed point of a mapping. In this field, there are numerous works regarding convergence of various iterative methods, as one can see in [1–14].

In this work, the second concept is the rate of convergence of iteration methods. After examining convergence of an iteration method, it is important to check whether this iteration method is faster than some well-known iteration methods or not. If it is faster than some current iteration methods, then it could be more useful than the others. More details about the rate of convergence can be found in [10, 15–17].

The third concept for this work is equivalence among convergences of iteration methods. Rhoades and Soltuz [13, 18–20] showed that the convergence of Mann iteration is equivalent to Ishikawa iteration for different classes of operators. They also showed in [21] that the convergence of modified Mann iteration is equivalent to modified Ishikawa iteration under certain mappings. Afterward, Rhoades and Soltuz [14] studied that Mann and Ishikawa iteration sequences are equivalent to a multistep iteration scheme for various classes of the operators. In addition, Soltuz [22] proved that the convergence of Ishikawa iteration is equivalent to that of Mann and Picard iterations for quasicontractive operators. One can find detailed literature concerning this topic in the following list [3–5, 23–25].

The final concept in this work is stability of fixed point iteration methods. The topic of stability, as an application of the theory of fixed point, has been studied by many authors including Harder and Hicks [26, 27], Rhoades [28, 29], Osilike [30, 31], Ostrowski [32], Berinde [33], Olantiwo [9] and Singh and Prasad [34]. First stability result in metric spaces is due to Ostrowski [32], where he established the stability of Picard iteration by employing Banach's contraction condition. Afterward, several authors studied this concept in different ways.

Throughout this paper, we denote the set of natural numbers by $\mathbb{N}$. Let E be a normed space, C a nonempty convex subset of a normed space E, and T a self map of C. Let (a_n), (b_n), (c_n), (α_n), $(\beta_n) \subset [0,1]$ be real sequences satisfying certain conditions. Let $(x_n) \subset C$ be a sequence generated by a particular iteration process including the operator T. That is,

$$x_{n+1} = f(T, x_n), \tag{1}$$

where f is suitable function and $x_0 \in C$ is the initial approximation. Suppose that (x_n) converges to a fixed point x^* of T. Let $(y_n) \subset C$ be an arbitrary sequence and set

$$\epsilon_n = \| y_{n+1} - f(T, y_n) \|, \tag{2}$$

for all $n \in \mathbb{N}$. Then, the iteration algorithm (1) is said to be T-stable or stable with respect to T if and only if $\lim_{n\to\infty} \epsilon_n = 0$ implies that $\lim_{n\to\infty} y_n = p$. If, in (1),

$$\begin{gathered} x_1 = x \in C, \\ f(T, x_n) = T x_n, \quad n \in \mathbb{N}, \end{gathered} \tag{3}$$

then it is called the Picard iteration process [35].

The Mann iteration procedure given in [7] is defined by

$$\begin{gathered} u_1 = u \in C \\ f(T, u_n) = (1-\alpha_n) u_n + \alpha_n T u_n, \quad n \in \mathbb{N}. \end{gathered} \tag{4}$$

The sequence (x_n) defined by

$$\begin{gathered} x_1 = x \in C, \\ f(T, x_n) = (1-\alpha_n) x_n + \alpha_n T y_n, \\ y_n = (1-\beta_n) x_n + \beta_n T x_n, \quad n \in \mathbb{N}, \end{gathered} \tag{5}$$

is known as the Ishikawa iteration process [6].

The Noor iteration method [8] is defined by

$$\begin{gathered} x_1 = x \in C, \\ f(T, x_n) = (1-\alpha_n) x_n + \alpha_n T y_n, \\ y_n = (1-\beta_n) x_n + \beta_n T z_n, \\ z_n = (1-c_n) x_n + c_n T x_n, \quad n \in \mathbb{N}. \end{gathered} \tag{6}$$

Suantai [11] proposed an iterative scheme as follows:

$$\begin{gathered} x_1 = x \in C, \\ f(T, x_n) = (1-\alpha_n-\beta_n) x_n + \alpha_n T y_n + \beta_n T z_n, \\ y_n = (1-a_n-b_n) x_n + a_n T z_n + b_n T x_n, \\ z_n = (1-c_n) x_n + c_n T x_n, \quad n \in \mathbb{N}. \end{gathered} \tag{7}$$

Agarwal et al. established an S-iteration method in [1] as follows:

$$\begin{gathered} x_1 = x \in C, \\ f(T, x_n) = (1-\alpha_n) T x_n + \alpha_n T y_n, \\ y_n = (1-\beta_n) x_n + \beta_n T x_n, \quad n \in \mathbb{N}. \end{gathered} \tag{8}$$

Thianwan [12] introduced a two-step Mann iteration by

$$\begin{gathered} x_1 = x \in C, \\ f(T, x_n) = (1-\alpha_n) y_n + \alpha_n T y_n, \\ y_n = (1-\beta_n) x_n + \beta_n T x_n, \quad n \in \mathbb{N}. \end{gathered} \tag{9}$$

Recently, Phuengrattana and Suantai [10] defined an SP iteration process as follows:

$$\begin{gathered} x_1 = x \in C, \\ f(T, x_n) = (1-\alpha_n) y_n + \alpha_n T y_n, \\ y_n = (1-\beta_n) z_n + \beta_n T z_n, \\ z_n = (1-c_n) x_n + c_n T x_n, \quad n \in \mathbb{N}. \end{gathered} \tag{10}$$

Inspired by the above iteration process, we will introduce the following new iterative algorithm:

$$\begin{gathered} x_1 = x \in C, \\ f(T, x_n) = (1-\alpha_n-\beta_n) y_n + \alpha_n T y_n + \beta_n T z_n, \\ y_n = (1-a_n-b_n) z_n + a_n T z_n + b_n T x_n, \\ z_n = (1-c_n) x_n + c_n T x_n, \quad n \in \mathbb{N}, \end{gathered} \tag{11}$$

where (a_n), (b_n), (c_n), (α_n), and (β_n) are real sequences in $[0,1]$ satisfying

$$\begin{gathered} (\alpha_n+\beta_n)_{n=0}^{\infty}, \quad (a_n+b_n)_{n=0}^{\infty} \in [0,1], \quad \forall n \in \mathbb{N}, \\ \sum_{n=0}^{\infty} (\alpha_n+\beta_n) = \infty. \end{gathered} \tag{12}$$

Some special cases of the new iteration process given in (11) are as follows.

(i) If $c_n = 1$ and $\beta_n = \alpha_n = a_n = b_n = 0$ for all $n \in \mathbb{N}$, then (11) reduces to Picard iteration (3).

(ii) If $c_n = \beta_n = a_n = b_n = 0$ for all $n \in \mathbb{N}$, then (11) reduces to Mann iteration (4).

(iii) If $c_n = \beta_n = a_n = 0$ for all $n \in \mathbb{N}$, then (11) reduces to Ishikawa iteration (5).

(iv) If $c_n = b_n = 0$ and $\alpha_n + \beta_n = 1$ for all $n \in \mathbb{N}$, then (11) reduces to S-iteration (8).

(v) If $\beta_n = b_n = c_n = 0$ for all $n \in \mathbb{N}$, then (11) reduces to two-step Mann iteration (9).

(vi) If $\beta_n = b_n = 0$ for all $n \in \mathbb{N}$, then (11) reduces to SP iteration (10).

Quite recently, Imoru and Olatinwo [36] introduced a class of operators called contractive-like mappings by

$$\|Tx - Ty\| \leq \varphi(\|x - Tx\|) + \delta\|x - y\| \quad \forall x, y \in E, \tag{13}$$

where $\delta \in [0, 1)$ and $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ is a monotone increasing function with $\varphi(0) = 0$.

In inequality (13), if we take $\varphi(t) = Lt$, then it is reduced to the contractive definition due to Osilike and Udomene [31]. Also, by putting $L = 2\delta$ in (13), the class of quasicontractive operators reduces to class of operators due to Berinde [2].

In [2] it was shown that the class of these operators is wider than class of Zamfirescu operators given in [37], where $\delta := \max\{a, b/(1-b), c/(1-c)\}$, $\delta \in [0, 1)$, and a, b, and c are real numbers satisfying $0 < a < 1$, $0 < b$, and $c < 1/2$. Besides, it is easy to see that special case of Zamfirescu operator gives Kannans' and Chatterjeas' results given in [38] and [39], respectively.

In this paper, we will prove that the new iteration method (11) is convergent to fixed point of contractive-like mappings satisfying (13). Also, by using a counterexample given in [17], we compare the rates of convergence between the new iteration method (11) and the iteration method (7) for the same class of mappings satisfying (13). Moreover, we establish an equivalence among convergences of some iteration methods including the new iteration method (11). Finally, we prove that the new iteration method (11) is T-stable.

We end this section with the following definition and lemma which will be useful in proving our main results.

Definition 1 (see [17]). Assume that $(a_n)_{n\in\mathbb{N}}$ and $(b_n)_{n\in\mathbb{N}}$ are two real convergent sequences with limits a and b, respectively. Then $(a_n)_{n\in\mathbb{N}}$ is said to converge faster than $(b_n)_{n\in\mathbb{N}}$ if

$$\lim_{n\to\infty} \left| \frac{a_n - a}{b_n - b} \right| = 0. \tag{14}$$

Lemma 2 (see [33]). *If ρ is a real number satisfying $0 \leq \rho < 1$ and $(\xi_n)_{n\in\mathbb{N}}$ is a sequence of positive numbers such that $\lim_{n\to\infty}\xi_n = 0$, then for any sequence of positive numbers $(\xi_n)_{n\in\mathbb{N}}$ satisfying*

$$a_{n+1} \leq \rho a_n + \xi_n, \quad n = 1, 2, \ldots, \tag{15}$$

one has

$$\lim_{n\to\infty} a_n = 0. \tag{16}$$

2. Main Results

Theorem 3. *Let C be a nonempty closed convex subset of an arbitrary Banach space E and $T : C \to C$ be a mapping satisfying (13) with $F_T \neq \varnothing$. Let (x_n) a sequence defined by (11) with real sequences (a_n), (b_n), (c_n), (α_n), $(\beta_n) \subset [0, 1]$ satisfying $(\alpha_n + \beta_n)_{n=0}^{\infty}$, $(a_n + b_n)_{n=0}^{\infty} \subset [0, 1]$, and $\sum_{n=0}^{\infty}(\alpha_n + \beta_n) = \infty$. Then the iterative sequence (x_n) converges strongly to the fixed point of T.*

Proof. Let x^* be the fixed point of T. It can be seen easily from (13) that x^* is the unique fixed point of T. To show that (x_n) converges to the fixed point $x^* = Tx^*$, we use condition (13). Hence, we have

$$\begin{aligned}
\|x_{n+1} - x^*\| &\leq \|(1 - \alpha_n - \beta_n) y_n + \alpha_n T y_n + \beta_n T z_n - x^*\| \\
&\leq (1 - \alpha_n - \beta_n)\|y_n - x^*\| + \alpha_n\|Ty_n - x^*\| \\
&\quad + \beta_n\|Tz_n - x^*\| \\
&\leq [(1 - \alpha_n - \beta_n) + \alpha_n\delta]\|y_n - x^*\| \\
&\quad + \beta_n\|Tz_n - x^*\| + \alpha_n\varphi(\|x^* - Tx^*\|) \\
&\leq [(1 - \alpha_n - \beta_n) + \alpha_n\delta]\|y_n - x^*\| \\
&\quad + \beta_n\delta\|z_n - x^*\| \\
&\quad + [\beta_n + \alpha_n]\varphi(\|x^* - Tx^*\|),
\end{aligned} \tag{17}$$

$$\begin{aligned}
\|y_n - x^*\| &\leq \|(1 - a_n - b_n) z_n + a_n T z_n + b_n T x_n - x^*\| \\
&\leq (1 - a_n - b_n)\|z_n - x^*\| + a_n\|Tz_n - x^*\| \\
&\quad + b_n\|Tx_n - x^*\| \\
&\leq [(1 - a_n - b_n) + a_n\delta]\|z_n - x^*\| \\
&\quad + a_n\varphi(\|x^* - Tx^*\|) + b_n\|Tx_n - x^*\| \\
&\leq [(1 - a_n - b_n) + a_n\delta]\|z_n - x^*\| \\
&\quad + b_n\delta\|x_n - x^*\| + (b_n + a_n)\varphi(\|x^* - Tx^*\|),
\end{aligned} \tag{18}$$

$$\begin{aligned}
\|z_n - x^*\| &\leq \|(1 - c_n) x_n + c_n T x_n - x^*\| \\
&\leq (1 - c_n)\|x_n - x^*\| + c_n\|Tx_n - x^*\| \\
&\leq [1 - c_n(1 - \delta)]\|x_n - x^*\| \\
&\quad + c_n\varphi(\|x^* - Tx^*\|).
\end{aligned} \tag{19}$$

By combining (17)–(19), we derive

$$\begin{aligned}
&\|x_{n+1} - x^*\| \\
&\quad \leq [(1 - \alpha_n - \beta_n) + \alpha_n\delta] \\
&\qquad \times [[(1 - a_n - b_n) + a_n\delta] \\
&\qquad\quad \times [[1 - c_n(1 - \delta)]\|x_n - x^*\| + c_n\varphi(\|x^* - Tx^*\|)]
\end{aligned}$$

$$+ b_n\delta \|x_n - x^*\| + (b_n + a_n)\varphi(\|x^* - Tx^*\|)]$$
$$+ \beta_n\delta [[1 - c_n(1-\delta)]\|x_n - x^*\| + c_n\varphi(\|x^* - Tx^*\|)]$$
$$+ [\beta_n + \alpha_n]\varphi(\|x^* - Tx^*\|). \tag{20}$$

Since $\varphi(\|x^* - Tx^*\|) = 0$, (20) becomes

$$\begin{aligned}\|x_{n+1} - x^*\| &\le [[1 - \alpha_n - \beta_n + \alpha_n\delta] \\ &\times [[1 - a_n - b_n + a_n\delta][1 - c_n(1-\delta)] + b_n\delta] \\ &+ \beta_n\delta[1 - c_n(1-\delta)]] \\ &\times \|x_n - x^*\| \\ &\le ([1 - \alpha_n - \beta_n + \alpha_n\delta] \\ &\times [1 - (a_n + b_n)(1-\delta)] + \beta_n\delta)\|x_n - x^*\| \\ &\le (1 - (\alpha_n + \beta_n)(1-\delta))\|x_n - x^*\|.\end{aligned} \tag{21}$$

By continuing the above processes, we obtain the following estimates

$$\begin{aligned}\|x_{n+1} - x^*\| &\le (1 - (\alpha_n + \beta_n)(1-\delta))\|x_n - x^*\| \\ \|x_n - x^*\| &\le (1 - (\alpha_{n-1} + \beta_{n-1})(1-\delta))\|x_{n-1} - x^*\| \\ &\vdots \\ \|x_1 - x^*\| &\le (1 - (\alpha_0 + \beta_0)(1-\delta))\|x_0 - x^*\|,\end{aligned} \tag{22}$$

$$\begin{aligned}\|x_{n+1} - x^*\| &\le \prod_{i=0}^{n}[1 - (\alpha_i + \beta_i)(1-\delta)]\|x_0 - x^*\| \\ &\le \|x_0 - x^*\| e^{(-(1-\delta)\sum_{i=0}^{n}(\alpha_i+\beta_i))},\end{aligned} \tag{23}$$

for all $n \in \mathbb{N}$. Since $0 < \delta < 1$, $\alpha_n, \beta_n \in [0,1]$ and $\sum_{n=0}^{\infty}(\alpha_n + \beta_n) = \infty$, we have

$$\begin{aligned}&\lim_{n\to\infty}\sup\|x_{n+1} - x^*\| \\ &\le \lim_{n\to\infty}\sup\left(\|x_0 - x^*\| e^{(-(1-\delta)\sum_{i=0}^{n}(\alpha_i+\beta_i))}\right) = 0.\end{aligned} \tag{24}$$

Therefore $\lim_{n\to\infty}\|x_n - x^*\| = 0$; that is, $x_n \to x^* \in F_T$ for all $n \in \mathbb{N}$. □

Theorem 3 allows us to give the following example which compares the rates of convergence between the new iteration method (11) and the iteration method (7) for contractive-like operators. In the following example, for convenience, we use sequences (v_n) and (s_n) associated with the iterative methods (11) and (7), respectively.

Example 4 (see [17]). Define a mapping $T : [0,1] \to [0,1]$ as $Tx = x/2$. Let $\alpha_n = \beta_n = a_n = b_n = c_n = 0$, for $n = 1, 2, \ldots, 24$, and $\alpha_n = \beta_n = a_n = b_n = 2/\sqrt{n}$, $c_n = 4/\sqrt{n}$, for all $n \ge 24$.

It can be seen easily that the mapping T satisfies condition (13) with the unique fixed point $0 \in F_T$. Furthermore, it is easy to see that Example 4 satisfies all the conditions of Theorem 3.

Indeed, let $x_0 \neq 0$ be initial point for iterative methods (11) and (7). By using iterative methods (11) and (7), we have

$$\begin{aligned}v_n &= \left(1 - \frac{2}{\sqrt{n}} - \frac{2}{\sqrt{n}}\right) \\ &\times \left(\left(1 - \frac{2}{\sqrt{n}} - \frac{2}{\sqrt{n}}\right)\left(\left(1 - \frac{4}{\sqrt{n}}\right)x_n + \frac{4}{\sqrt{n}}Tx_n\right)\right. \\ &\quad \left. + \frac{2}{\sqrt{n}}T\left(\left(1 - \frac{4}{\sqrt{n}}\right)x_n + \frac{4}{\sqrt{n}}Tx_n\right) + \frac{2}{\sqrt{n}}Tx_n\right) \\ &+ \frac{2}{\sqrt{n}}T\left(\left(1 - \frac{2}{\sqrt{n}} - \frac{2}{\sqrt{n}}\right)\left(\left(1 - \frac{4}{\sqrt{n}}\right)x_n + \frac{4}{\sqrt{n}}Tx_n\right)\right. \\ &\quad \left. + \frac{2}{\sqrt{n}}T\left(\left(1 - \frac{4}{\sqrt{n}}\right)x_n + \frac{4}{\sqrt{n}}Tx_n\right) + \frac{2}{\sqrt{n}}Tx_n\right) \\ &+ \frac{2}{\sqrt{n}}T\left(\left(1 - \frac{4}{\sqrt{n}}\right)x_n + \frac{4}{\sqrt{n}}Tx_n\right) \\ &= \left(1 - \frac{2}{\sqrt{n}} - \frac{2}{\sqrt{n}}\right) \\ &\times \left(\left(1 - \frac{2}{\sqrt{n}} - \frac{2}{\sqrt{n}}\right)\left(\left(1 - \frac{4}{\sqrt{n}}\right)x_n + \frac{4}{\sqrt{n}}\frac{1}{2}x_n\right)\right. \\ &\quad \left. + \frac{2}{\sqrt{n}}\frac{1}{2}\left(\left(1 - \frac{4}{\sqrt{n}}\right)x_n + \frac{4}{\sqrt{n}}\frac{1}{2}x_n\right) + \frac{2}{\sqrt{n}}\frac{1}{2}x_n\right) \\ &+ \frac{2}{\sqrt{n}}\frac{1}{2} \\ &\times \left(\left(1 - \frac{2}{\sqrt{n}} - \frac{2}{\sqrt{n}}\right)\left(\left(1 - \frac{4}{\sqrt{n}}\right)x_n + \frac{4}{\sqrt{n}}\frac{1}{2}x_n\right)\right. \\ &\quad \left. + \frac{2}{\sqrt{n}}\frac{1}{2}\left(\left(1 - \frac{4}{\sqrt{n}}\right)x_n + \frac{4}{\sqrt{n}}\frac{1}{2}x_n\right) + \frac{2}{\sqrt{n}}\frac{1}{2}x_n\right) \\ &+ \frac{2}{\sqrt{n}}\frac{1}{2}\left(\left(1 - \frac{4}{\sqrt{n}}\right)x_n + \frac{4}{\sqrt{n}}\frac{1}{2}x_n\right)\end{aligned}$$

$$= \left(1 - \frac{6}{\sqrt{n}} + \frac{16}{n} - \frac{18}{n\sqrt{n}}\right) x_n$$
$$\vdots$$
$$= \prod_{i=25}^{n} \left(1 - \frac{6}{\sqrt{i}} + \frac{16}{i} - \frac{18}{i\sqrt{i}}\right) x_0, \tag{25}$$

$$\begin{aligned} s_n &= \left(1 - \frac{2}{\sqrt{n}} - \frac{2}{\sqrt{n}}\right) x_n \\ &\quad + \frac{2}{\sqrt{n}} \frac{1}{2} \\ &\quad \times \left(\left(1 - \frac{2}{\sqrt{n}} - \frac{2}{\sqrt{n}}\right) x_n \right. \\ &\qquad + \frac{2}{\sqrt{n}} \frac{1}{2} \left(\left(1 - \frac{4}{\sqrt{n}}\right) x_n + \frac{4}{\sqrt{n}} \frac{1}{2} x_n \right) + \frac{2}{\sqrt{n}} \frac{1}{2} x_n \Big) \\ &\quad + \frac{2}{\sqrt{n}} \frac{1}{2} \left(\left(1 - \frac{4}{\sqrt{n}}\right) x_n + \frac{4}{\sqrt{n}} \frac{1}{2} x_n \right) \\ &= \left(1 - \frac{2}{\sqrt{n}} - \frac{4}{n} - \frac{2}{n\sqrt{n}}\right) x_n \\ &\;\vdots \\ &= \prod_{i=25}^{n} \left(1 - \frac{2}{\sqrt{i}} - \frac{4}{i} - \frac{2}{i\sqrt{i}}\right) x_0. \end{aligned} \tag{26}$$

Now, let us compare these results as follows:

$$\begin{aligned} \left| \frac{v_n - 0}{s_n - 0} \right| &= \left| \prod_{i=25}^{n} \left(1 - \frac{4i - 20\sqrt{i} + 16}{i\sqrt{i} - 2i - 4\sqrt{i} - 2}\right) \right| \\ &= \left| \prod_{i=25}^{n} \left(1 - \frac{4\left(\sqrt{i} - 1\right)\left(\sqrt{i} - 4\right)}{\left(\sqrt{i} - 4\right)\left(i + 2\sqrt{i} + 4\right) + 14}\right) \right|. \end{aligned} \tag{27}$$

Since

$$\begin{aligned} 0 &\leq \lim_{n\to\infty} \prod_{i=25}^{n} \left(1 - \frac{4\left(\sqrt{i} - 1\right)\left(\sqrt{i} - 4\right)}{\left(\sqrt{i} - 4\right)\left(i + 2\sqrt{i} + 4\right) + 14}\right) \\ &\leq \lim_{n\to\infty} \prod_{i=25}^{n} \left(1 - \frac{1}{i}\right) \\ &= \lim_{n\to\infty} \frac{24}{n} = 0, \end{aligned} \tag{28}$$

finally we get

$$\lim_{n\to\infty} \left| \frac{v_n - 0}{s_n - 0} \right| = 0. \tag{29}$$

Thus, from Definition 1, we conclude that the iteration method (11) is faster than the iteration method (7).

Theorem 5. *Let C be a nonempty closed convex subset of an arbitrary Banach space E and $T : C \to C$ a mapping satisfying (13) with $F_T \neq \varnothing$. If $u_1 = x_1 \in C$ and $\alpha_n + \beta_n \geq A > 0$ for all $n \in \mathbb{N}$, then the following statements are equivalent.*

(i) *Mann iteration (4) converges to fixed point x^*.*

(ii) *The new iteration (11) converges to fixed point x^*.*

Proof. (i) ⇒ (ii): Suppose that Mann iteration (4) converges to fixed point x^*; that is, $u_n \to x^*$ as $n \to \infty$. We will show that the new iteration (11) converges to the fixed point x^*; that is, $x_n \to x^*$ as $n \to \infty$. Using (4), (11), and (13), we have

$$\begin{aligned} \|u_{n+1} - x_{n+1}\| &= \|(1 - \alpha_n - \beta_n) u_n + \alpha_n T u_n + \beta_n T u_n \\ &\qquad - (1 - \alpha_n - \beta_n) y_n - \alpha_n T y_n - \beta_n T z_n\| \\ &\leq (1 - \alpha_n - \beta_n) \|u_n - y_n\| + \alpha_n \|T u_n - T y_n\| \\ &\quad + \beta_n \|T u_n - T z_n\| \\ &\leq (1 - \alpha_n - \beta_n + \alpha_n \delta) \|u_n - y_n\| \\ &\quad + \beta_n \delta \|u_n - z_n\| \\ &\quad + (\alpha_n + \beta_n) \varphi (\|u_n - T u_n\|), \end{aligned} \tag{30}$$

$$\begin{aligned} \|u_n - y_n\| &= \|u_n - (1 - a_n - b_n) z_n - a_n T z_n - b_n T x_n\| \\ &\leq (1 - a_n - b_n) \|u_n - z_n\| \\ &\quad + a_n \|u_n - T u_n + T u_n - T z_n\| \\ &\quad + b_n \|u_n - T u_n + T u_n - T x_n\| \\ &\leq (1 - a_n - b_n) \|u_n - z_n\| + a_n \|u_n - T u_n\| \\ &\quad + a_n \|T u_n - T z_n\| \\ &\quad + b_n \|u_n - T u_n\| + b_n \|T u_n - T x_n\| \\ &\leq (1 - a_n - b_n + a_n \delta) \|u_n - z_n\| + b_n \delta \|u_n - x_n\| \\ &\quad + (a_n + b_n) \|u_n - T u_n\| \\ &\quad + (a_n + b_n) \varphi (\|u_n - T u_n\|), \end{aligned} \tag{31}$$

$$\begin{aligned} \|u_n - z_n\| &= \|u_n - (1 - c_n) x_n - c_n T x_n\| \\ &\leq (1 - c_n) \|u_n - x_n\| \\ &\quad + c_n \|u_n - T u_n + T u_n - T x_n\| \\ &\leq (1 - c_n) \|u_n - x_n\| + c_n \|u_n - T u_n\| \\ &\quad + c_n \|T u_n - T x_n\| \\ &\leq (1 - c_n (1 - \delta)) \|u_n - x_n\| + c_n \|u_n - T u_n\| \\ &\quad + c_n \varphi (\|u_n - T u_n\|). \end{aligned} \tag{32}$$

Substituting (32) in (31), we get

$$\begin{aligned}\|u_n - y_n\| \leq\ & [(1 - a_n - b_n + a_n\delta)(1 - c_n(1 - \delta)) + b_n\delta] \\ & \times \|u_n - x_n\| \\ & + [(1 - a_n - b_n + a_n\delta)c_n + (a_n + b_n)] \\ & \times \|u_n - Tu_n\| \\ & + [(1 - a_n - b_n + a_n\delta)c_n + (a_n + b_n)] \\ & \times \varphi(\|u_n - Tu_n\|).\end{aligned} \tag{33}$$

By combining (30), (32), and (33) and using the assumption $\alpha_n + \beta_n \geq A$, we have

$$\begin{aligned}\|u_{n+1} - x_{n+1}\| \leq\ & [1 - A(1 - \delta)]\|u_n - x_n\| \\ & + [(1 - \alpha_n - \beta_n + \alpha_n\delta) \\ & \quad \times [(1 - a_n - b_n + a_n\delta)c_n + (a_n + b_n)] \\ & \quad + \beta_n\delta c_n] \\ & \times \|u_n - Tu_n\| \\ & + [(1 - \alpha_n - \beta_n + \alpha_n\delta) \\ & \quad \times [(1 - a_n - b_n + a_n\delta)c_n + (a_n + b_n)] \\ & \quad + \beta_n\delta c_n + (\alpha_n + \beta_n)] \\ & \times \varphi(\|u_n - Tu_n\|).\end{aligned} \tag{34}$$

Denote that

$$\begin{aligned} a_n &= \|u_n - x_n\|, \\ \rho &= [1 - A(1 - \delta)] \in (0, 1), \\ \xi_n &= [(1 - \alpha_n - \beta_n + \alpha_n\delta) \\ & \quad \times [(1 - a_n - b_n + a_n\delta)c_n + (a_n + b_n)] \\ & \quad + \beta_n\delta c_n] \\ & \times \|u_n - Tu_n\| \\ & + [(1 - \alpha_n - \beta_n + \alpha_n\delta) \\ & \quad \times [(1 - a_n - b_n + a_n\delta)c_n + (a_n + b_n)] \\ & \quad + \beta_n\delta c_n + (\alpha_n + \beta_n)] \\ & \times \varphi(\|u_n - Tu_n\|).\end{aligned} \tag{35}$$

Since $\lim_{n\to\infty}\|u_n - x^*\| = 0$ and $Tx^* = x^* \in F_T \neq \varnothing$, it follows from (13) that $\lim_{n\to\infty}\|u_n - Tu_n\| = 0$. Hence by Lemma 2 we see that

$$\|u_n - x_n\| \longrightarrow 0 \quad \text{as } n \longrightarrow \infty. \tag{36}$$

Also, from triangle inequality we have

$$\|x_n - x^*\| \leq \|u_n - x_n\| + \|x^* - u_n\| \tag{37}$$

and this leads to $x_n \to x^*$ as $n \to \infty$.

(ii) $\Rightarrow$ (i) : Now, suppose that $x_n \to x^*$ as $n \to \infty$. We will show that $u_n \to x^*$ as $n \to \infty$. Using (4), (11), and (13), the following estimates can be obtained:

$$\begin{aligned}\|x_{n+1} - u_{n+1}\| &= \|(1 - \alpha_n - \beta_n)y_n + \alpha_n Ty_n + \beta_n Tz_n \\ & \qquad - (1 - \alpha_n - \beta_n)u_n - \alpha_n Tu_n - \beta_n Tu_n\| \\ & \leq (1 - \alpha_n - \beta_n)\|y_n - u_n\| + \alpha_n\|Ty_n - Tu_n\| \\ & \quad + \beta_n\|Tz_n - Tu_n\| \\ & \leq (1 - \alpha_n - \beta_n + \alpha_n\delta)\|y_n - u_n\| \\ & \quad + \alpha_n\varphi(\|y_n - Ty_n\|) \\ & \quad + \beta_n\delta\|z_n - u_n\| + \beta_n\varphi(\|z_n - Tz_n\|),\end{aligned} \tag{38}$$

$$\begin{aligned}\|y_n - u_n\| &\leq (1 - a_n - b_n)\|z_n - u_n\| \\ & \quad + a_n\|Tz_n - z_n\| + a_n\|z_n - u_n\| \\ & \quad + b_n\|Tx_n - x_n\| + b_n\|x_n - u_n\| \\ &= (1 - b_n)\|z_n - u_n\| + b_n\|x_n - u_n\| \\ & \quad + a_n\|Tz_n - z_n\| + b_n\|Tx_n - x_n\|,\end{aligned} \tag{39}$$

$$\begin{aligned}\|z_n - u_n\| &= \|(1 - c_n)x_n + c_n Tx_n - u_n\| \\ & \leq (1 - c_n)\|x_n - u_n\| + c_n\|Tx_n - x_n\| \\ & \quad + c_n\|x_n - u_n\| \\ &= \|x_n - u_n\| + c_n\|Tx_n - x_n\|.\end{aligned} \tag{40}$$

By substituting (40) in (39), we obtain

$$\begin{aligned}\|y_n - u_n\| \leq\ & \|x_n - u_n\| \\ & + [(1 - b_n)c_n + b_n]\|Tx_n - x_n\| \\ & + a_n\|Tz_n - z_n\|.\end{aligned} \tag{41}$$

Again by substituting (40) and (41) in (38) and using the assumption $\alpha_n + \beta_n \geq A$, we have

$$\begin{aligned}&\|x_{n+1} - u_{n+1}\| \\ &\quad \leq (1 - A(1 - \delta))\|x_n - u_n\| \\ &\qquad + [(1 - \alpha_n - \beta_n + \alpha_n\delta)[(1 - b_n)c_n + b_n] + \beta_n c_n\delta] \\ &\qquad \times \|Tx_n - x_n\| \\ &\qquad + (1 - \alpha_n - \beta_n + \alpha_n\delta)a_n\|Tz_n - z_n\| \\ &\qquad + \alpha_n\varphi(\|y_n - Ty_n\|) + \beta_n\varphi(\|z_n - Tz_n\|).\end{aligned} \tag{42}$$

Now define

$$a_n = \|u_n - x_n\|,$$
$$\rho = [1 - A(1-\delta)] \in (0,1),$$
$$\begin{aligned}\xi_n = {} & [(1-\alpha_n-\beta_n+\alpha_n\delta)[(1-b_n)c_n+b_n]+\beta_n c_n\delta] \\ & \times \|Tx_n - x_n\| \\ & + (1-\alpha_n-\beta_n+\alpha_n\delta) a_n \|Tz_n - z_n\| \\ & + \alpha_n\varphi(\|y_n - Ty_n\|) + \beta_n\varphi(\|z_n - Tz_n\|).\end{aligned} \tag{43}$$

Since $x_n \to x^*$ as $n \to \infty$ and $Tx^* = x^* \in F_T$, it follows from (13) that

$$\begin{aligned}&\|y_n - Ty_n\| \\ &\quad \le \|y_n - x^*\| + \|x^* - Ty_n\| \\ &\quad \le \|y_n - x^*\| + \delta\|x^* - y_n\| + \varphi(\|x^* - Tx^*\|) \\ &\quad \le (1+\delta)(1-a_n-b_n)\|z_n - x^*\| \\ &\qquad + (1+\delta)a_n\|Tz_n - x^*\| \\ &\qquad + (1+\delta)b_n\|Tx_n - x^*\| + \varphi(\|x^* - Tx^*\|) \\ &\quad \le (1+\delta)(1-a_n-b_n)\|(1-c_n)x_n + c_nTx_n - x^*\| \\ &\qquad + (1+\delta)a_n\delta\|z_n - x^*\| + (1+\delta)a_n\varphi(\|x^* - Tx^*\|) \\ &\qquad + (1+\delta)b_n\delta\|x_n - x^*\| + (1+\delta)b_n\varphi(\|x^* - Tx^*\|) \\ &\qquad + \varphi(\|x^* - Tx^*\|) \\ &\quad \le \big[(1+\delta)(1-a_n-b_n)(1-c_n) \\ &\qquad + (1+\delta)(1-a_n-b_n)c_n\delta + (1+\delta)a_n\delta(1-c_n) \\ &\qquad + (1+\delta)a_n\delta^2c_n + (1+\delta)b_n\delta\big] \\ &\qquad \times \|x_n - x^*\| \longrightarrow 0 \quad \text{as } n \longrightarrow \infty.\end{aligned} \tag{44}$$

Since the function φ is continuous, we get

$$\begin{aligned}\lim_{n\to\infty}\|y_n - Ty_n\| &= \lim_{n\to\infty}\|x_n - Tx_n\| \\ &= \lim_{n\to\infty}\|z_n - Tz_n\| \\ &= \lim_{n\to\infty}\varphi(\|y_n - Ty_n\|) \\ &= \lim_{n\to\infty}\varphi(\|z_n - Tz_n\|) = 0.\end{aligned} \tag{45}$$

Thus Lemma 2 and (42) give $\|x_n - u_n\| \to 0$ as $n \to \infty$.

Also, from triangle inequality we have

$$\|u_n - x^*\| \le \|u_n - x_n\| + \|x_n - x^*\| \longrightarrow 0 \tag{46}$$

and this yields $u_n \to x^*$ as $n \to \infty$. □

With regard to ([5], Corollary 2) and Theorem 5, we can without hesitation give the following corollary.

Corollary 6. *Let C be a nonempty closed convex subset of an arbitrary Banach space E and $T : C \to C$ a mapping satisfying (13) with $F_T \neq \varnothing$. If the initial point is the same for all iterations and $\alpha_n + \beta_n \ge A > 0$, for all $n \in \mathbb{N}$, then the following expressions are equivalent.*

(1) *The Picard iteration (1) converges to the fixed point x^* of T.*

(2) *The Krasnoselskij iteration [40] converges to the fixed point x^* of T.*

(3) *The Mann iteration (4) converges to the fixed point x^* of T.*

(4) *The Ishikawa iteration (5) converges to the fixed point x^* of T.*

(5) *The Noor iteration (6) converges to the fixed point x^* of T.*

(6) *The S-iteration (8) converges to the fixed point x^* of T.*

(7) *The two-step Mann iteration (9) converges to the fixed point x^* of T.*

(8) *The SP iteration (10) converges to the fixed point x^* of T.*

(9) *The multistep iteration [14] converges to the fixed point x^* of T.*

(10) *The new multistep iteration [41] converges to the fixed point x^* of T.*

(11) *The new iteration (11) converges to the fixed point x^* of T.*

Theorem 7. *Let $(E, \|\cdot\|)$ be an arbitrary Banach space, $T : E \to E$ a self-map of E satisfying (13) with $F_T \neq \varnothing$, and x^* the unique fixed point of T. For $x_0 \in E$, let (x_n) be the new iteration method defined by (11) with real sequences (a_n), (b_n), (c_n), (α_n), $(\beta_n) \subset [0,1]$ satisfying $0 < A \le \alpha_n + \beta_n$, for all $n \in \mathbb{N}$. Then the new iteration method (11) is T-stable.*

Proof. Let (y_n) be an arbitrary sequence in E. Define

$$\varepsilon_n = \|y_{n+1} - (1-\alpha_n-\beta_n)u_n - \alpha_nTu_n - \beta_nTv_n\|, \tag{47}$$

for all $n \in \mathbb{N}$, where $u_n = (1-a_n-b_n)v_n + a_nTv_n + b_nTy_n$ and $v_n = (1-c_n)y_n + c_nTy_n$. Suppose that $x_n \to x^*$ as $n \to \infty$

and $\lim_{n\to\infty}\epsilon_n = 0$. Then, we prove that $\lim_{n\to\infty} y_n = x^*$. From condition (13), we have the following estimates:

$$\begin{aligned}\|y_{n+1}-x^*\| &\le \|y_{n+1}-(1-\alpha_n-\beta_n)u_n-\alpha_nTu_n-\beta_nTv_n\| \\ &\quad+\|(1-\alpha_n-\beta_n)u_n+\alpha_nTu_n+\beta_nTv_n-x^*\| \\ &\le \varepsilon_n+\|(1-\alpha_n-\beta_n)u_n+\alpha_nTu_n+\beta_nTv_n-x^*\| \\ &\le \varepsilon_n+(1-\alpha_n-\beta_n)\|u_n-x^*\| \\ &\quad+\alpha_n\delta\|u_n-x^*\|+\beta_n\delta\|v_n-x^*\| \\ &\quad+\alpha_n\varphi(\|x^*-Tx^*\|)+\beta_n\varphi(\|x^*-Tx^*\|) \\ &= \varepsilon_n+[1-\alpha_n-\beta_n+\alpha_n\delta]\|u_n-x^*\| \\ &\quad+\beta_n\delta\|v_n-x^*\|,\end{aligned} \tag{48}$$

$$\begin{aligned}\|u_n-x^*\| &\le \|(1-a_n-b_n)v_n+a_nTv_n+b_nTy_n-x^*\| \\ &\le (1-a_n-b_n)\|v_n-x^*\|+a_n\|Tv_n-x^*\| \\ &\quad+b_n\|Ty_n-x^*\| \\ &\le [1-a_n-b_n+a_n\delta]\|v_n-x^*\| \\ &\quad+a_n\varphi(\|x^*-Tx^*\|)+b_n\|Ty_n-x^*\| \\ &\le [1-a_n-b_n+a_n\delta]\|v_n-x^*\| \\ &\quad+b_n\delta\|y_n-x^*\|+(b_n+a_n)\varphi(\|x^*-Tx^*\|) \\ &= [1-a_n-b_n+a_n\delta]\|v_n-x^*\|+b_n\delta\|y_n-x^*\|,\end{aligned} \tag{49}$$

$$\begin{aligned}\|v_n-x^*\| &\le \|(1-c_n)y_n+c_nTy_n-x^*\| \\ &\le (1-c_n)\|y_n-x^*\|+c_n\|Ty_n-x^*\| \\ &\le [1-c_n(1-\delta)]\|y_n-x^*\|.\end{aligned} \tag{50}$$

Substituting (49) and (50) in (48) and using the assumption $\alpha_n+\beta_n \ge A > 0$, for all $n\in\mathbb{N}$, we obtain

$$\begin{aligned}&\|y_{n+1}-x^*\| \\ &\le \varepsilon_n+[[1-\alpha_n-\beta_n+\alpha_n\delta] \\ &\qquad\times[[1-a_n-b_n+a_n\delta][1-c_n(1-\delta)]+b_n\delta] \\ &\qquad+\beta_n\delta[1-c_n(1-\delta)]] \\ &\quad\times\|y_n-x^*\| \\ &\le \varepsilon_n+([1-\alpha_n-\beta_n+\alpha_n\delta][1-(a_n+b_n)(1-\delta)]+\beta_n\delta) \\ &\quad\times\|y_n-x^*\| \\ &\le \varepsilon_n+(1-(\alpha_n+\beta_n)(1-\delta))\|y_n-x^*\| \\ &\le \varepsilon_n+(1-A(1-\delta))\|y_n-x^*\|.\end{aligned} \tag{51}$$

Thus an application of Lemma 2 to (51) yields $\lim_{n\to\infty} y_n = x^*$.

Conversely, assume that $\lim_{n\to\infty} y_n = x^*$. We now show that $\lim_{n\to\infty}\varepsilon_n = 0$. From condition (13) and triangle inequality we have

$$\begin{aligned}\varepsilon_n &= \|y_{n+1}-(1-\alpha_n-\beta_n)u_n-\alpha_nTu_n-\beta_nTv_n\| \\ &\le \|y_{n+1}-x^*\|+\|x^*-(1-\alpha_n-\beta_n)u_n-\alpha_nTu_n-\beta_nTv_n\| \\ &\le \|y_{n+1}-x^*\|+(1-\alpha_n-\beta_n)\|u_n-x^*\| \\ &\quad+\alpha_n\|Tu_n-x^*\|+\beta_n\|Tv_n-x^*\| \\ &\le \|y_{n+1}-x^*\|+[1-\alpha_n-\beta_n+\alpha_n\delta]\|u_n-x^*\| \\ &\quad+\beta_n\delta\|v_n-x^*\| \\ &\le \|y_{n+1}-x^*\|+(1-(\alpha_n+\beta_n)(1-\delta))\|y_n-x^*\|.\end{aligned} \tag{52}$$

Since $\delta\in[0,1)$ and $\alpha_n+\beta_n\in[0,1]$, for all $n\in\mathbb{N}$,

$$0 < 1-(\alpha_n+\beta_n)(1-\delta) < 1. \tag{53}$$

By taking the limit as $n\to\infty$ of both sides of (52) and using the assumption $\lim_{n\to\infty}\|y_n-x^*\| = 0$, we have $\lim_{n\to\infty}\varepsilon_n = 0$. □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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