

Research Article

MHD Forced Convection Laminar Boundary Layer Flow of Alumina-Water Nanofluid over a Moving Permeable Flat Plate with Convective Surface Boundary Condition

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This paper analyses a two-dimensional steady forced convection boundary layer viscous incompressible flow of alumina-water nanofluid over a moving permeable vertical flat plate under the effect of a magnetic field normal to the plate. Thermal convective surface boundary condition is applied. The nanofluid formulated in the present study is water dispersed with various volumetric fractions of the alumina (Al_2O_3) nanoparticles. The plate velocity and the free stream velocities are considered to be proportional to x^n , while the magnetic field and suction velocities are taken to be proportional to $(x)^{(n-1)/2}$. The similarity solution of the governing problem is obtained. Numerical studies are presented to show the effect of the nanoparticle volume fraction ϕ , the convective heat transfer parameter b , the power law exponent n , the wall velocity parameter A , and the suction parameter f_w on the velocity, temperature, skin-friction coefficient, and Nusselt number.

1. Introduction

It is known that nanofluids can tremendously enhance the heat transfer characteristics of the original (base) fluid. Thus, nanofluids have many applications in industry such as coolants, lubricants, heat exchangers, and microchannel heat sinks. Therefore, numerous methods have been taken to improve the thermal conductivity of these fluids by suspending nano-/micro-sized particle materials in liquids. Several recent studies on the modeling of free/mixed convection heat transfer in nanofluids have been published: Congedo et al. [1], Ghasemi and Aminossadati [2], and Ho et al. [3, 4]. These studies have used traditional finite difference and finite volume techniques with the tremendous call on computational resources that these techniques necessitate. Hamad et al. [5] used the application of a one-parameter group to present similarity reductions for problems of magnetic field effects on free-convection flow of a nanofluid past a semi-infinite vertical flat plate. Hamad and Pop [6] studied theoretically the similarity solution of the steady boundary layer flow near the stagnation-point flow on a permeable stretching

sheet in a porous medium saturated with a nanofluid and in the presence of internal heat generation/absorption. Hamad [7] found the analytical solutions of convective flow and heat transfer of an incompressible viscous nanofluid past a semi-infinite vertical stretching sheet in the presence of magnetic field. Abu-Nada and Chamkha [8] investigated the natural convection heat transfer characteristics in a differentially heated enclosure filled with a CuO-EG-water nanofluid for different published variable thermal conductivity and variable viscosity models. Teng et al. [9] studied the effect of particle size, temperature, and weight fraction on the thermal conductivity ratio of Al_2O_3 -water nanofluid. Ho et al. [10] presented an experimental study for natural convection heat transfer of a nanofluid in vertical square enclosure of different sizes. Bachok et al. [11] studied the steady boundary-layer flow of a nanofluid past a moving semi-infinite flat plate in a uniform free stream. Hady et al. [12] studied the natural convection boundary-layer flow over a downward-pointing vertical cone in a porous medium saturated with a non-Newtonian nanofluid in the presence of heat generation or absorption. The numerical modeling

of the conjugate heat transfer and fluid flow of Al_2O_3 /water nanofluid through the microchannel heat sink was presented by Lelea [13]. In this paper, the laminar flow regime was considered along with viscous dissipation effect. Also, the microchannel heat sink with square microchannels and $Dh = 50 \mu\text{m}$ was considered. Nandy and Mahapatra [14] analyzed the effects of velocity slip and heat generation/absorption on magnetohydrodynamic (MHD) stagnation-point flow and heat transfer over a stretching/shrinking surface, with convective boundary conditions, in the presence of nanoparticle fractions. They considered also the effects of Brownian motion and thermophoresis. Numerical study for the effects of thermal radiation and viscous dissipation on hydromagnetic mixed convection flow over a nonlinear stretching and shrinking sheets in nanofluids was presented by Pal et al. [15]. Rohni et al. [16] theoretically investigated the steady two-dimensional boundary-layer flow past a moving semi-infinite flat plate in a water-based nanofluid containing three different types of nanoparticles: copper (cuprum), alumina (aluminium oxide), and titania (titanium dioxide). For more information, see also Das et al. [17], Wang and Mujumdar [18–20], and Kakaç and Pramuanjaroenkij [21].

Heat transfer due to forced convection occurs in many areas. Examples include chemical process, biochemical process, and engineering. Its study is important due to the need to improve the efficiency of the heat transfer. Ali [22] studied the effect of temperature dependent viscosity on laminar mixed convection boundary layer flow and heat transfer on a continuously moving vertical isothermal surface and obtained local similarity solutions. According to previous studies, the forced convection flow over moving plate has many practical engineering applications such as liquid films in condensation process and in aerodynamics. Similarity solutions about moving plate were investigated by many authors. Among them, Ishak et al. [23] studied the boundary layer flow on a moving permeable plate parallel to a moving stream and concluded that dual solutions exist if the plate and the free stream move in the opposite directions. Magyari [24] studied the moving plate thermometer with uniform velocity. His approach was based on a series solution of the problem using the Merkin transformation method. Hoernel [25] investigated similarity solution for steady laminar boundary layer equations governing MHD flow near forward stagnation point of two dimensional moving axisymmetric bodies. Recently, Pal and Mondal [26] studied MHD forced convection and heat transfer of an incompressible Newtonian electrically conducting and heat generating/absorbing fluid having temperature-dependent viscosity over a nonisothermal permeable wedge in the presence of thermal radiation. In all of the above studies, convective thermal boundary conditions (either prescribed heat flux or prescribed surface temperature) were used.

According to previous scholars, the MHD flow has received the attention of many researchers due to its engineering applications. In metallurgy, for example, some processes involve the cooling of many continuous strips by drawing them through an electrically conducting fluid subject to a magnetic field (Kandasamy and Muhaimin [27]). This allows the rate of cooling to be controlled and final product with

the desired characteristics to be obtained. Another important application of hydromagnetic flow in metallurgy is in the purification of molten metals from nonmetallic inclusions through the application of a magnetic field. Research has also been carried out by previous researchers on the flow and heat transfer effects of electrically conducting fluids such as liquid metals, water mixed with a little acid and other equivalent substances in the presence of a magnetic field. The studies have involved different geometries and different boundary conditions. Herdricha et al. [28] studied MHD flow control for plasma technology applications. They identified potential applications for magnetically controlled plasmas in the fields of space technology as well as in plasma technology. Seddeek et al. [29] investigated the similarity solution in MHD flow and heat transfer over a wedge taking into account variable viscosity and thermal conductivities. Alam et al. [30] investigated the effects of variable suction and thermophoresis on steady MHD combined free-forced convective heat and mass transfer flow over a semi-infinite permeable inclined plate in the presence of thermal radiation. Aydin and Kaya [31] studied MHD mixed convection of a viscous dissipating fluid about a permeable vertical flat plate and found that the value of Richardson number determines the effect of the magnetic parameter on the momentum and heat transfer. Recently, Rahman and Salahuddin [32] investigated the effects of variable electric conductivity and viscosity on hydromagnetic heat and mass transfer flow along a radiated isothermal inclined permeable surface in a stationary fluid which includes internal heat generation.

The aim of the present study is to analyze steady, two-dimension, laminar MHD forced convection flow of alumina-water nanofluid on moving permeable flat plate with convective surface boundary condition. The similarity representation of the partial differential equations governing this problem is obtained. The motivation of this study is to explore the influence of nanofluid parameters with the effect of suction, convective heat transfer, and velocity parameter. The variation distribution of the shear stress and heat transfer rates (the physical quantities of interest in this study) with the parameters governing the problem are presented.

2. Formulation of the Problem

Consider a two-dimensional forced convective flow of water-based alumina nanoparticles over a moving permeable flat plate. A rectangular coordinate frame is selected in which the x -axis is vertical and y -axis is horizontal. A magnetic field of variable strength $B(x)$ is applied perpendicular to the plate. The temperature of the plate on the top is held at a uniform temperature T_w which is assumed greater than the ambient temperature T_∞ . It will be further assumed that the bottom surface of the plate is heated by convection from a hot fluid of temperature T_f , and this generates a heat transfer coefficient h_f (see Aziz [33]). For the thermophysical properties of the alumina, see Oztop and Abu-Nada [34]. The flow configuration and the coordinate system are shown in Figure 1. Kinematic viscosity and thermal diffusivity are assumed constants. Particle coagulation, the magnetic Reynolds number, the electric field owing to the

It is noticed that the continuity equation (1) is satisfied automatically and (3) and (7) yield

$$\frac{n+1}{(1-\phi)^{2.5}} f''' + \left(1 - \phi + \left(\frac{\rho_s}{\rho_f}\right)\phi\right) \times \left[\frac{n+1}{2} f f'' - n f'^2 + n\right] + M(f' - 1) = 0, \tag{10}$$

$$\frac{1}{Pr} \left(\frac{\kappa_{nf}}{\kappa_f}\right) \theta'' + \frac{1}{2} \left(1 - \phi + \frac{(\rho C_p)_s}{(\rho C_p)_f} \phi\right) f \theta' = 0, \tag{11}$$

where $Pr = (\mu C_p)_f / \kappa_f$ is the Prandtl number and $M = \sigma B_0^2 / (\rho_f u_\infty)$ is the magnetic parameter. Here, we assume that $T_f > T_w > T_\infty$.

The corresponding boundary conditions in (4) become

$$f'(0) = A, \quad f(0) = f_w, \quad \theta'(0) = -b \left(\frac{\kappa_f}{\kappa_{nf}}\right) [1 - \theta(0)], \quad f'(\infty) = 1, \quad \theta(\infty) = 0, \tag{12}$$

where $A = u_0 / U_\infty$ is a constant and is considered as a velocity parameter and $f_w = -2\nu_0 / \sqrt{\nu} U_\infty (n+1)$ is the suction parameter. The parameter $A > 0$ means that the plate is moving in the same direction as the free stream velocity, $A < 0$ means that the plate is moving in the opposite direction of the free steam, and $A = 0$ stands for static plate.

The parameter $b = (h_f / \kappa) \sqrt{\nu x} / U_e (n+1)$ is the convective heat transfer parameter, which should be constant. To fulfill this criterion, the coefficient of heat transfer h_f must be directly proportional to $1/\sqrt{x}$; that is, $h_f = c_1 / \sqrt{x}$.

2.1. Particular Cases

- (i) Here, we should mention that if $n = 0$ (the full stream velocity is constant), $f_w = 0$ (the plate is impermeable), $u_w = 0$ (the plate is stationary), and $M = 0$ (absence of magnetic field), our system (10)–(12) reduces to that in Aziz [33].
- (ii) If $f_w \neq 0$ (the plate is permeable), $u_w = 0$, and $M = 0$, our system (10)–(12) reduces to that in Ishak [37].

The parameters of engineering interest for the problem are the skin friction C_f and the local Nusselt number Nu , respectively. These physical quantities can be derived from the following definitions:

$$C_f = \frac{\mu_{nf}}{u_w^2 \rho_{nf}} \left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad Nu = \frac{-x}{(T_f - T_\infty)} \left(\frac{\kappa_{nf}}{\kappa_f}\right) \left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{13}$$

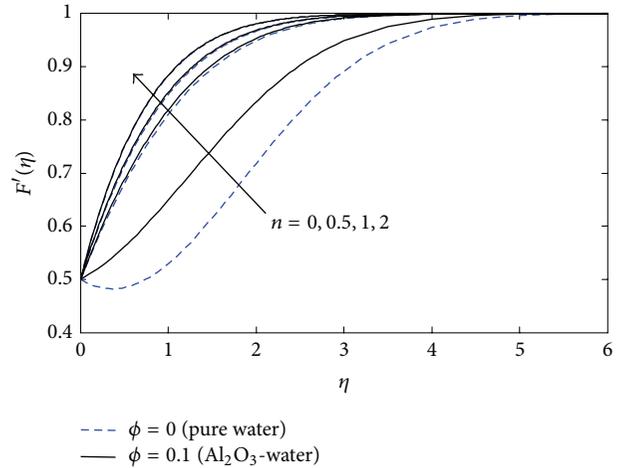


FIGURE 2: $b = 0.5, f_w = 0.5, Pr = 6.2, M = 0.5,$ and $A = 0.5$.

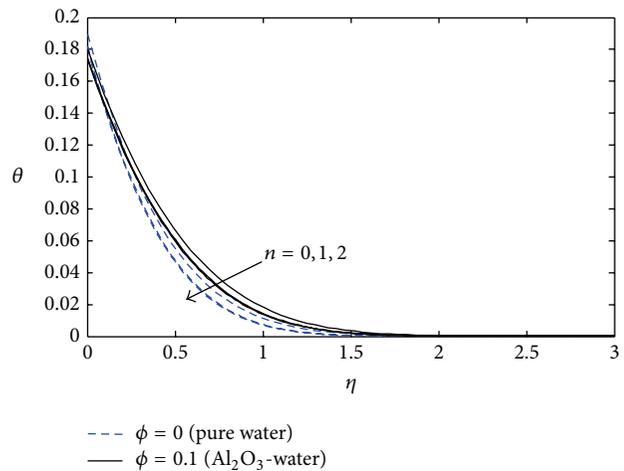


FIGURE 3: $b = 0.5, f_w = 0.5, Pr = 6.2, M = 0.5,$ and $A = 0.5$.

By substituting from (8) and (12) into (13), we get

$$Re_x^{1/2} C_f = \frac{\sqrt{n+1}}{A^2 (1-\phi)^{2.5}} f''(0), \tag{14}$$

$$Re^{-1/2} Nu = -\sqrt{n+1} \frac{\kappa_{nf}}{\kappa_f} \theta'(0).$$

3. Results and Discussion

The transformed system of nonlinear ordinary differential equations (10) and (11) together with boundary conditions (12) containing various parameters, namely, the power law parameter n , magnetic parameter M , the Prandtl number Pr , the nanoparticles volume fraction ϕ and convective heat transfer parameter b , suction parameter f_w and the velocity parameter A . Equations (10) and (11) together with boundary conditions given in (12) are solved numerically by an implicit finite difference method. To exhibit our results obtained by

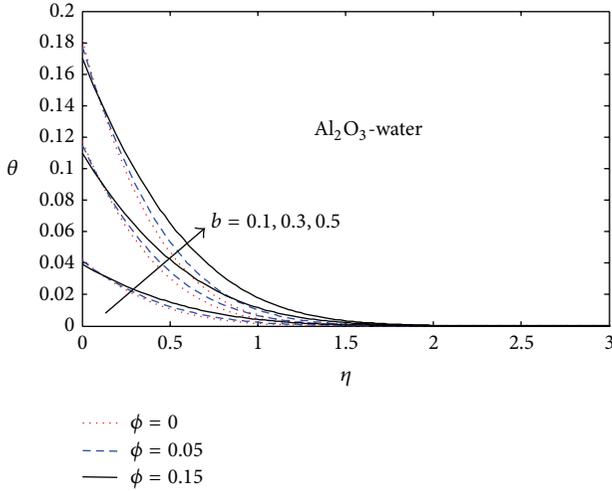


FIGURE 4: $n = 2$, $f_w = 0.5$, $Pr = 6.2$, $M = 0.5$, and $A = 0.5$.

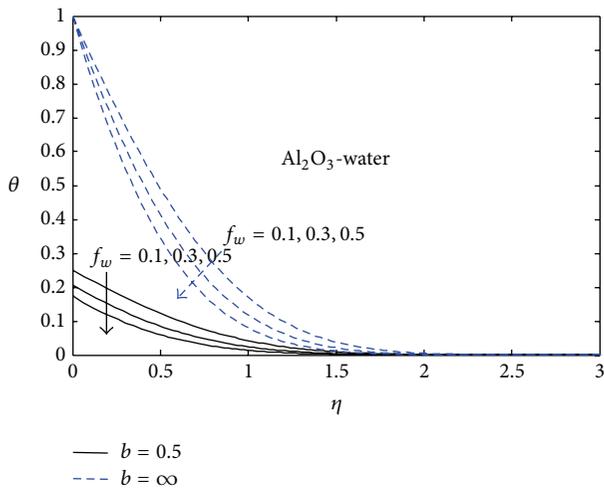


FIGURE 5: $n = 2$, $\phi = 0.1$, $Pr = 6.2$, $M = 0.5$, and $A = 0.5$.

numerical computations, we plot the graphs given in Figures 2 to 10, respectively.

To exhibit the effects of various values of the power law exponent n on velocity $F'(\eta)$ and temperature $\theta(\eta)$ for two values of the nanoparticles volume fraction ϕ , which are $\phi = 0$ (pure water) and $\phi \neq 0$ ($=0.1$) (Al_2O_3 -water), keeping the other parameters fixed ($Pr = 6.2$, $M = 0.5$, $b = 0.5$, $f_w = 0.5$, and $A = 0.5$), we plot Figures 2 and 3, respectively. It is clear from Figure 2 that the increased power law exponent increases the velocity. Also, it is clear that the presence of alumina nanoparticles reduces the momentum boundary layer thickness and increases the velocity. However, from Figure 3, it is seen that the increased power law exponent decreases the temperature and the existence of alumina nanoparticles increases the thermal boundary layer thickness. This is because the thermal conductivity of alumina is higher than that of water.

Figure 4 displays the influence of various values of the convective heat transfer parameter b and the nanoparticles volume fractions ϕ on temperature distribution. We have

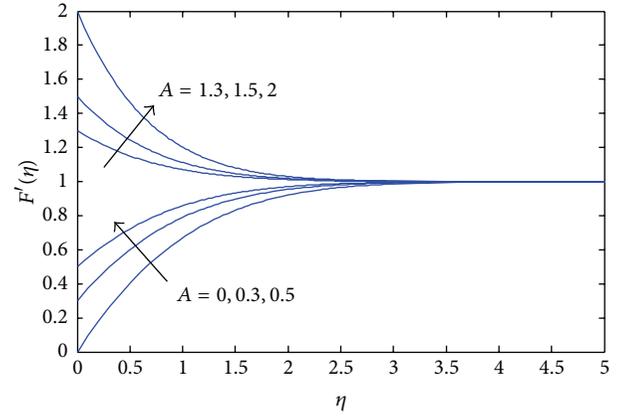


FIGURE 6: $b = 0.1$, $f_w = 0.1$, $Pr = 6.2$, $M = 0.5$, $n = 2$, and $\phi = 0.1$.

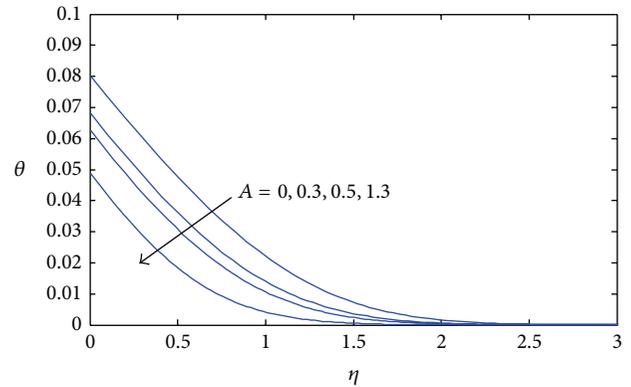


FIGURE 7: $b = 0.1$, $f_w = 0.1$, $Pr = 6.2$, $M = 0.5$, $n = 2$, and $\phi = 0.1$.

noted that the thermal boundary layer thickness increases when n or ϕ increases. This agrees with the physical behavior that, when the convective heat transfer increases, the hot fluid side convection resistance decreases, and, consequently, the surface temperature increases (see Ishak [37]); because of this, the thermal boundary layer thickness increases, and because also alumina has high thermal conductivity, then the thickness increases more for alumina-water nanofluid.

Figure 5 shows the effect of the variation of suction parameter f_w on temperature for two cases of convective heat transfer parameter b , first when $b = 0.5$ (convective boundary condition) and second when $b = \infty$ (neglect convective condition, i.e., back to normal condition $\theta(0) = 1$). All other parameters are nonzero. It can be seen that the thickness of the thermal boundary layer reduces with increasing values of f_w for the two cases of b .

Effects of the velocity parameter A on the velocity and temperature are shown in Figures 6 and 7, respectively. We have observed from Figure 6 that the velocity increases when A increases. We have noticed also that for each $A < 1$ ($u_0 < U_\infty$), the velocity increases till it becomes 1 and, for each $A > 1$ ($u_0 > U_\infty$) the velocity decreases till it becomes 1 while, from Figure 7, it is noted that the temperature decreases when A increases and the wall temperature also decreases when A increases.

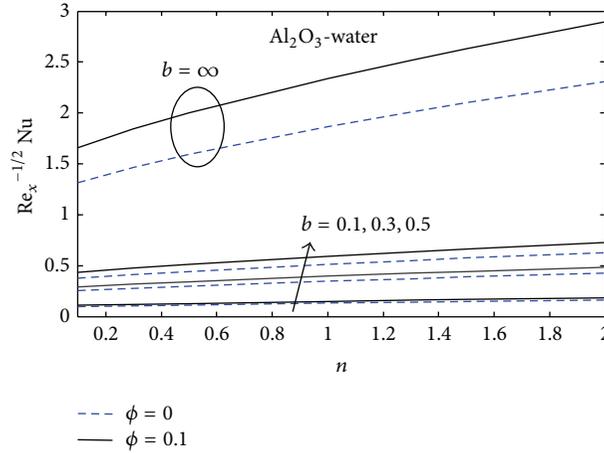


FIGURE 8: $Pr = 6.2$, $M = 0.5$, $A = 0.5$, and $f_w = 0.1$.

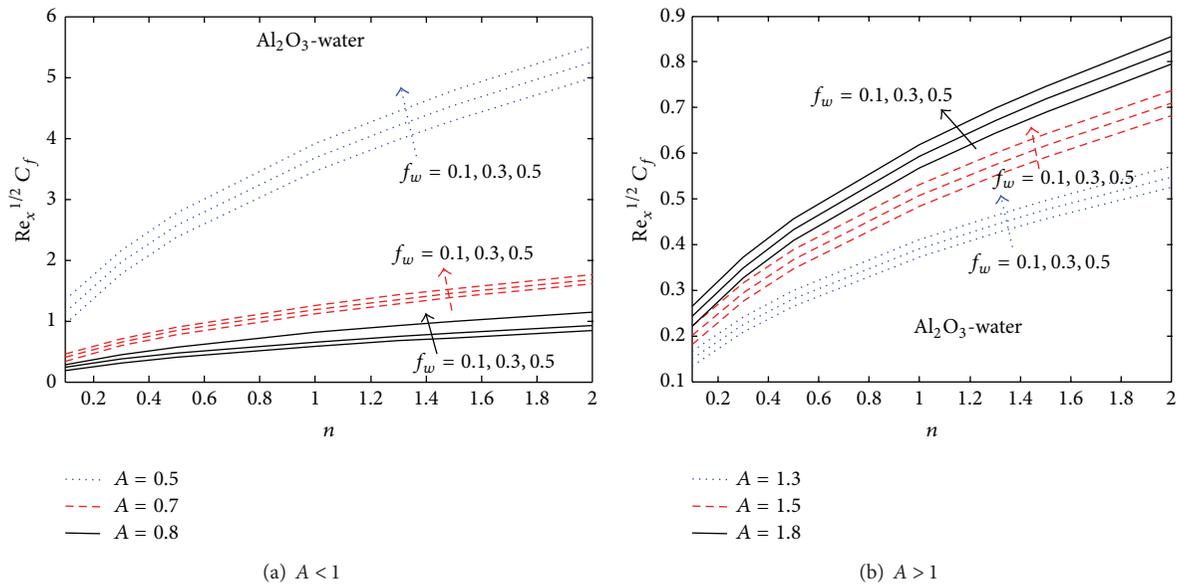


FIGURE 9: $Pr = 6.2$, $M = 0.5$, $b = 0.5$, and $\phi = 0.1$.

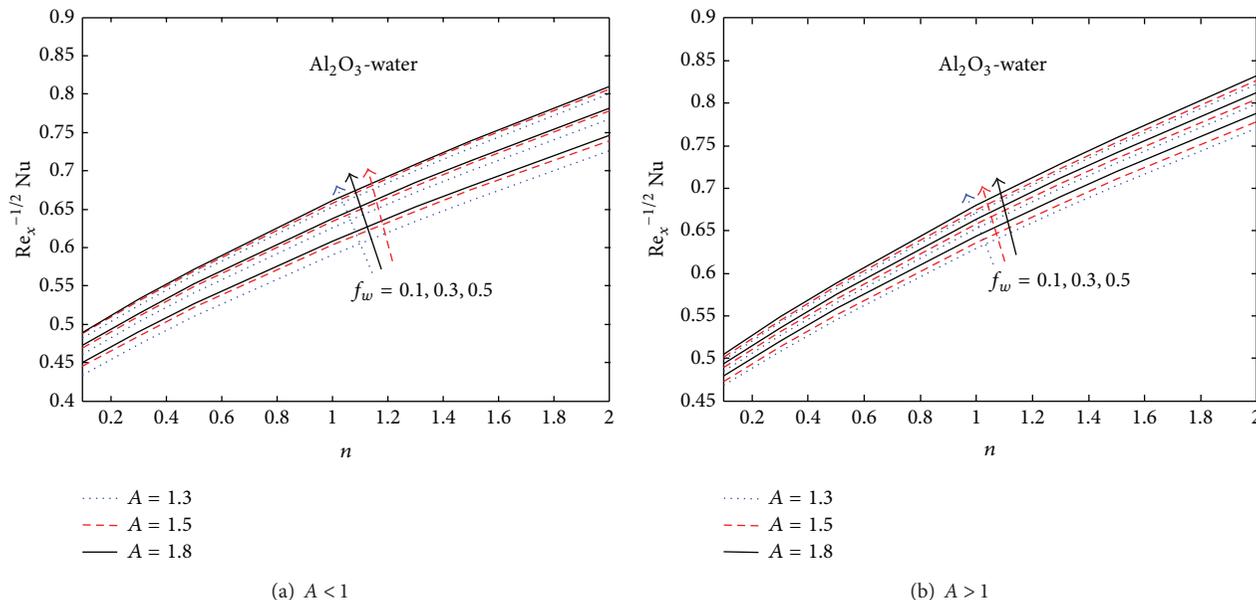
Figure 8 shows the effects of b on the shear stress versus the parameter n for pure water and alumina-water nanofluid ($\phi \neq 0$ ($=0.1$)). It is clear that the value of shear stress without considering the convective boundary condition ($b = \infty$) is greater than when considering the convective boundary condition for both $\phi = 0$ and $\phi \neq 0$, also for the nanofluid the shear stress is greater more than for pure water. It is also noticed that increasing parameter n leads to increasing the shear stress, as well as at each value of n , the shear stress be higher for higher value of b both $\phi = 0$ and $\phi \neq 0$.

Figures 9 and 10 show the effect of suction parameter f_w and the velocity parameter A on the shear stress and rate of heat transfer versus n , respectively. From Figure 9, we have seen that the shear stress increases when f_w increases. For $A < 1$ as in Figure 9(a), we have noted that the shear stress decreases when A increases, but, for $A > 1$ as in Figure 9(b), the shear stress increases with the increased A . However, from Figures 10(a) and 10(b), it is observed that

the rate of heat transfer increases with the increasing value of suction parameter f_w or the increasing value of the power law parameter n .

4. Conclusion

In this paper, we have studied the problem of heat transfer in MHD forced convection flow of alumina-water nanofluid over moving permeable vertical flat plate with convective surface boundary condition. New similarity transformations have been presented and solved numerically. It was found that alumina nanoparticles when suspended in a fluid are capable of increasing the heat transfer capability of the base fluid. The shear stress as well as heat transfer rate increases with nanoparticle volume fraction. Thermal boundary layer thickness as well as heat transfer rate increases when the convective heat transfer parameter b increases. Thermal boundary layer thickness decreases while both shear stress

FIGURE 10: $Pr = 6.2$, $M = 0.5$, $b = 0.5$, and $\phi = 0.1$.

and heat transfer rate increase with the increase of suction parameter. Temperature and wall temperature decrease while velocity and heat transfer rate increase when the velocity parameter A increases.

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