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Research Article

Local Fractional Series Expansion Method for Solving Wave and Diffusion Equations on Cantor Sets

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We proposed a local fractional series expansion method to solve the wave and diffusion equations on Cantor sets. Some examples are given to illustrate the efficiency and accuracy of the proposed method to obtain analytical solutions to differential equations within the local fractional derivatives.

1. Introduction

Fractional calculus theory [1–3] has been applied to a wide class of complex problems encompassing physics, biology, mechanics, and interdisciplinary areas [4–9]. Various methods, for example, the Adomian decomposition method [10], the Rach-Adomian-Meyers modified decomposition method [11], the variational iteration method [12, 13], the homotopy perturbation method [13, 14], the fractal Laplace and Fourier transforms [15], the homotopy analysis method [16], the heat-balance integral method [17–19], the fractional variational iteration method [20–22], the fractional subequation method [23, 24], and the generalized Exp-function method [25], have been utilized to solve fractional differential equations [3, 15].

The characteristics of fractal materials have local and fractal behaviors well described by nondifferential functions. However, the classic fractional calculus is not valid for differential equation on Cantor sets due to its no-local nature. In contrast, the local fractional calculus is one of the best candidates for dealing with such problems [26–44]. The local fractional calculus theory has played crucial applications in several fields, such as theoretical physics, transport problems in fractal media described by nondifferential functions. There are some versions of the local fractional calculus where different approaches in definition of the local fractional derivative exist, among them the local fractional derivative of Kolwankar et al. [32–38], the fractal derivative of Chen et al. [39, 40], the fractal derivative of Parvate et al. [41, 42], the modified Riemann-Liouville of Jumarie [43, 44], and versions described in [45–52].

In order to deal with local fractional ordinary and partial differential equations, there are some developed technologies, for example, the local fractional variational iteration method [45, 46], the local fractional Fourier series method [47, 48], the Cantor-type cylindrical-coordinate method [49], the Yang-Fourier transform [50, 51], and the Yang-Laplace transform [52].

The local fractional derivative is defined as follows [26–31, 45–52]:

$$f^{(\alpha)}(x_0) = \left.\frac{d^\alpha f(x)}{dx^\alpha}\right|_{x=x_0} = \lim_{x \to x_0} \frac{\Delta^\alpha (f(x) - f(x_0))}{(x-x_0)^\alpha}, \quad (1)$$

where $\Delta^\alpha(f(x) - f(x_0)) \cong \Gamma(1+\alpha)\Delta(f(x) - f(x_0))$, and $f(x)$ is satisfied with the condition [26, 47]

$$|f(x) - f(x_0)| \le \tau^\alpha |x - x_0|^\alpha \quad (2)$$

so that [26–31]

$$\left|f(x)-f(x_0)\right|<\varepsilon^{\alpha} \tag{3}$$

with $U:|x-x_0|<\delta$, for $\varepsilon,\delta>0$ and $\varepsilon,\delta\in R$.

The main idea of this paper is to present the local fractional series expansion method for effective solutions of wave and diffusion equations on Cantor sets involving local fractional derivatives. The paper has been organized as follows. Section 2 gives a local fractional series expansion method. Some illustrative examples are shown in Section 3. The conclusions are presented in Section 4.

2. Analysis of the Method

Let us consider the local fractional differential equation

$$u_t^{n\alpha}=L_\alpha u, \tag{4}$$

where L is a linear local operator with respect to x, $n\in\{1,2\}$.

In accordance with the results in [28, 47], there are multiterm separated functions of independent variables t and x, namely,

$$u(x,t)=\sum_{i=0}^{\infty}T_i(t)\,X_i(x), \tag{5}$$

where $T_i(t)$ and $X_i(x)$ are local fractional continuous functions.

Moreover, there is a nondifferential series term

$$T_i(t)=p_i\frac{t^{i\alpha}}{\Gamma(1+i\alpha)}, \tag{6}$$

where p_i is a coefficient.

In view of (6), we may present the solution in the form

$$u(x,t)=\sum_{i=0}^{\infty}p_i\frac{t^{i\alpha}}{\Gamma(1+i\alpha)}X_i(x). \tag{7}$$

Then, following (7), we have

$$u(x,t)=\sum_{i=0}^{\infty}\frac{t^{i\alpha}}{\Gamma(1+i\alpha)}X_i(x). \tag{8}$$

Hence,

$$\begin{gathered}u_t^{n\alpha}=\sum_{i=0}^{\infty}\frac{1}{\Gamma(1+i\alpha)}t^{i\alpha}X_{i+1}(x)=\sum_{i=0}^{\infty}\frac{1}{\Gamma(1+i\alpha)}t^{i\alpha}X_{i+n}(x),\\ L_\alpha u=L_\alpha\left[\sum_{i=0}^{\infty}\frac{t^{i\alpha}}{\Gamma(1+i\alpha)}X_i(x)\right]=\sum_{i=0}^{\infty}\frac{t^{i\alpha}}{\Gamma(1+i\alpha)}\left(L_\alpha X_i\right)(x).\end{gathered} \tag{9}$$

In view of (9), we have

$$\sum_{i=0}^{\infty}\frac{1}{\Gamma(1+i\alpha)}t^{i\alpha}X_{i+n}(x)=\sum_{i=0}^{\infty}\frac{t^{i\alpha}}{\Gamma(1+i\alpha)}\left(L_\alpha X_i\right)(x). \tag{10}$$

Hence, from (10) we can obtain a recursion; namely,

$$X_{i+n}(x)=\left(L_\alpha X_i\right)(x), \tag{11}$$

with $n=1$; we arrive at the following relation:

$$X_{i+1}(x)=\left(L_\alpha X_i\right)(x), \tag{12}$$

with $n=2$; we may rewrite (11) as

$$X_{i+2}(x)=\left(L_\alpha X_i\right)(x). \tag{13}$$

By the recursion formulas, we can obtain the solution of (4) as

$$u(x,t)=\sum_{i=0}^{\infty}\frac{t^{i\alpha}}{\Gamma(1+i\alpha)}X_i(x). \tag{14}$$

The convergent condition is

$$\lim_{n\to\infty}\left[\frac{t^{i\alpha}}{\Gamma(1+i\alpha)}X_i(x)\right]=0. \tag{15}$$

This approach is termed *the local fractional series expansion method* (LFSEM)

3. Applications to Wave and Diffusion Equations on Cantor Sets

In this section, four examples for wave and diffusion equations on Cantor sets will demonstrate the efficiency of LFSEM.

Example 1. Let us consider the diffusion equation on Cantor set

$$u_t^{\alpha}(x,t)-u_x^{2\alpha}(x,t)=0,\quad 0<\alpha\le 1 \tag{16}$$

with the initial condition

$$u(x,0)=\frac{x^\alpha}{\Gamma(1+\alpha)}. \tag{17}$$

Following (12), we have recursive formula

$$\begin{gathered}X_{i+1}(\mathrm{x})=\frac{\partial^{2\alpha}X_i(x)}{\partial x^{2\alpha}},\\ X_0(x)=\frac{x^\alpha}{\Gamma(1+\alpha)}.\end{gathered} \tag{18}$$

Hence, we get

$$\begin{gathered}X_0(x)=\frac{x^\alpha}{\Gamma(1+\alpha)},\\ X_1(x)=0,\\ X_2(x)=0,\\ \vdots\end{gathered} \tag{19}$$

and so on.

Therefore, through (19) we get the solution

$$u(x,t)=\frac{x^\alpha}{\Gamma(1+\alpha)}. \tag{20}$$

Example 2. Let us consider the diffusion equation on Cantor set

$$u_t^{\alpha}(x,t) - \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \cdot u_x^{2\alpha}(x,t) = 0, \quad 0 < \alpha \le 1 \tag{21}$$

with the initial condition

$$u(x,0) = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}. \tag{22}$$

Following (12), we get

$$\begin{aligned} X_{i+1}(x) &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \frac{\partial^{2\alpha} X_i(x)}{\partial x^{2\alpha}}, \\ X_0(x) &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}. \end{aligned} \tag{23}$$

By using the recursive formula (23), we get consequently

$$\begin{aligned} X_0(x) &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}, \\ X_1(x) &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}, \\ X_2(x) &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}, \\ &\vdots \end{aligned} \tag{24}$$

As a direct result of these recursive calculations, we arrive at

$$u(x,t) = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \sum_{i=0}^{\infty} \frac{t^{i\alpha}}{\Gamma(1+i\alpha)} = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} E_{\alpha}(t^{\alpha}). \tag{25}$$

Example 3. Let us consider the following wave equation on Cantor sets:

$$u_t^{2\alpha}(x,t) - \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \cdot u_x^{2\alpha}(x,t) = 0, \quad 0 < \alpha \le 1 \tag{26}$$

with the initial condition

$$u(x,0) = \frac{x^{\alpha}}{\Gamma(1+\alpha)}. \tag{27}$$

In view of (14), we obtain

$$\begin{aligned} X_{i+2}(x) &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \frac{\partial^{2\alpha} X_i(x)}{\partial x^{2\alpha}}, \\ X_0(x) &= u(x,0) = \frac{x^{\alpha}}{\Gamma(1+\alpha)}, \\ X_{i+2}(x) &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} \frac{\partial^{2\alpha} X_i(x)}{\partial x^{2\alpha}}, \\ X_1(x) &= u_x^{(\alpha)}(x,0) = 1. \end{aligned} \tag{28}$$

Hence, using the relations (29), the recursive calculations yield

$$\begin{aligned} X_0(x) &= \frac{x^{\alpha}}{\Gamma(1+\alpha)}, \\ X_1(x) &= 1, \end{aligned} \tag{29}$$

$$\begin{aligned} X_2(x) &= 0, \\ X_3(x) &= 0, \\ X_4(x) &= 0, \\ &\vdots \end{aligned} \tag{30}$$

and so on.

Finally, we obtain

$$u(x,t) = \frac{x^{\alpha}}{\Gamma(1+\alpha)} + \frac{t^{2\alpha}}{\Gamma(1+2\alpha)}. \tag{31}$$

Example 4. Let us consider the wave equation on Cantor sets [26, 30]

$$u_t^{2\alpha}(x,t) - \mathrm{c}u_x^{2\alpha}(x,t) = 0, \quad 0 < \alpha \le 1, \tag{32}$$

where c is a constant.

The initial condition is

$$u(x,0) = E_{\alpha}(x^{\alpha}). \tag{33}$$

By using (14) we have

$$\begin{aligned} X_{i+2}(x) &= c\frac{\partial^{2\alpha} X_i(x)}{\partial x^{2\alpha}}, \\ X_0(x) &= u(x,0) = E_{\alpha}(x^{\alpha}), \\ X_{i+2}(x) &= c\frac{\partial^{2\alpha} X_i(x)}{\partial x^{2\alpha}}, \\ X_0(x) &= u_x^{(\alpha)}(x,0) = E_{\alpha}(x^{\alpha}). \end{aligned} \tag{34}$$

Then, through the iterative relations (35), we have

$$\begin{aligned} X_0(x) &= E_{\alpha}(x^{\alpha}), \\ X_1(x) &= E_{\alpha}(x^{\alpha}), \end{aligned} \tag{35}$$

$$\begin{aligned} X_2(x) &= cE_{\alpha}(x^{\alpha}), \\ X_3(x) &= cE_{\alpha}(x^{\alpha}), \\ X_4(x) &= c^2E_{\alpha}(x^{\alpha}), \\ &\vdots \end{aligned} \tag{36}$$

Therefore, we obtain

$$
\begin{aligned}
u(x,t) &= E_\alpha(x^\alpha)\sum_{i=0}^{\infty}c^i\frac{t^{2i\alpha}}{\Gamma(1+2i\alpha)} \\
&\quad + E_\alpha(x^\alpha)\sum_{i=0}^{\infty}c^i\frac{t^{(2i+1)\alpha}}{\Gamma(1+(2i+1)\alpha)} \\
&= E_\alpha(x^\alpha)\left[\cosh_\alpha(ct^\alpha)+\sin_\alpha(ct^\alpha)\right],
\end{aligned}
\tag{37}
$$

where

$$
\begin{aligned}
\cosh_\alpha(t^\alpha) &= \sum_{i=0}^{\infty}\frac{t^{2i\alpha}}{\Gamma(1+2i\alpha)}, \\
\sinh_\alpha(t^\alpha) &= \sum_{i=0}^{\infty}\frac{t^{(2i+1)\alpha}}{\Gamma(1+(2i+1)\alpha)}.
\end{aligned}
\tag{38}
$$

For more details concerning (38), we refer to [26–28].

4. Conclusions

In this work, the local fractional series expansion method is demonstrated as an effective method for solutions of a wide class of problems. Analytical solutions of the wave and diffusion equations on Cantor sets involving local fractional derivatives are successfully developed by recurrence relations resulting in convergent series solutions. In this context, the suggested method is a potential tool for development of approximate solutions of local fractional differential equations with fractal initial value conditions, which, of course, draws new problems beyond the scope of the present work.

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References

[1] A. A. Kilbas, H. M. Srivastava, and J. J. Trujillo, *Theory and Applications of Fractional Differential Equations*, Elsevier, Amsterdam, The Netherlands, 2006.

[2] F. Mainardi, *Fractional Calculus and Waves in Linear Viscoelasticity*, Imperial College Press, London, UK, 2010.

[3] I. Podlubny, *Fractional Differential Equations*, Academic Press, San Diego, Calif, USA, 1999.

[4] R. L. Magin, *Fractional Calculus in Bioengineering,*, Begerll House, West Redding, Conn, USA, 2006.

[5] J. Klafter, S. C. Lim, and R. Metzler, *Fractional Dynamics in Physics: Recent Advances*, World Scientific, Singapore, 2011.

[6] G. M. Zaslavsky, *Hamiltonian Chaos and Fractional Dynamics*, Oxford University Press, Oxford, UK, 2008.

[7] B. J. West, M. Bologna, and P. Grigolini, *Physics of Fractal Operators*, Springer, New York, NY, USA, 2003.

[8] V. E. Tarasov, *Fractional Dynamics: Applications of Fractional Calculus to Dynamics of Particles, Fields and Media*, Springer, Berlin, Germany, 2011.

[9] J. A. Tenreiro Machado, A. C. J. Luo, and D. Baleanu, *Nonlinear Dynamics of Complex Systems: Applications in Physical, Biological and Financial Systems*, Springer, New York, NY, USA, 2011.

[10] J. S. Duan, T. Chaolu, R. Rach, and L. Lu, "The Adomian decomposition method with convergence acceleration techniques for nonlinear fractional differential equations," *Computers & Mathematics With Applications*, 2013.

[11] J. S. Duan, T. Chaolu, and R. Rach, "Solutions of the initial value problem for nonlinear fractional ordinary differential equations by the Rach-Adomian-Meyers modified decomposition method," *Applied Mathematics and Computation*, vol. 218, no. 17, pp. 8370–8392, 2012.

[12] S. Das, "Analytical solution of a fractional diffusion equation by variational iteration method," *Computers & Mathematics with Applications*, vol. 57, no. 3, pp. 483–487, 2009.

[13] S. Momani and Z. Odibat, "Comparison between the homotopy perturbation method and the variational iteration method for linear fractional partial differential equations," *Computers & Mathematics with Applications*, vol. 54, no. 7-8, pp. 910–919, 2007.

[14] S. Momani and Z. Odibat, "Homotopy perturbation method for nonlinear partial differential equations of fractional order," *Physics Letters A*, vol. 365, no. 5-6, pp. 345–350, 2007.

[15] D. Baleanu, K. Diethelm, E. Scalas, and J. J. Trujillo, *Fractional Calculus Models and Numerical Methods*, Series on Complexity, Nonlinearity and Chaos, World Scientific, Boston, Mass, USA, 2012.

[16] H. Jafari and S. Seifi, "Homotopy analysis method for solving linear and nonlinear fractional diffusion-wave equation," *Communications in Nonlinear Science and Numerical Simulation*, vol. 14, no. 5, pp. 2006–2012, 2009.

[17] J. Hristov, "Heat-balance integral to fractional (half-time) heat diffusion sub-model," *Thermal Science*, vol. 14, no. 2, pp. 291–316, 2010.

[18] J. Hristov, "Integral-balance solution to the stokes' first problem of a viscoelastic generalized second grade fluid," *Thermal Science*, vol. 16, no. 2, pp. 395–410, 2012.

[19] J. Hristov, "Transient flow of a generalized second grade fluid due to a constant surface shear stress: an approximate integral-balance solution," *International Review of Chemical Engineering*, vol. 3, no. 6, pp. 802–809, 2011.

[20] G. C. Wu and E. W. M. Lee, "Fractional variational iteration method and its application," *Physics Letters A*, vol. 374, no. 25, pp. 2506–2509, 2010.

[21] Y. Khan, N. Faraz, A. Yildirim, and Q. Wu, "Fractional variational iteration method for fractional initial-boundary value problems arising in the application of nonlinear science," *Computers & Mathematics with Applications*, vol. 62, no. 5, pp. 2273–2278, 2011.

[22] G. C. Wu, "A fractional variational iteration method for solving fractional nonlinear differential equations," *Computers & Mathematics with Applications*, vol. 61, no. 8, pp. 2186–2190, 2011.

[23] S. Zhang and H.-Q. Zhang, "Fractional sub-equation method and its applications to nonlinear fractional PDEs," *Physics Letters A*, vol. 375, no. 7, pp. 1069–1073, 2011.

[24] H. Jafari, H. Tajadodi, N. Kadkhoda, and D. Baleanu, "Fractional subequation method for Cahn-Hilliard and Klein-Gordon equations," *Abstract and Applied Analysis*, vol. 2013, Article ID 587179, 5 pages, 2013.

[25] S. Zhang, Q. A. Zong, D. Liu, and Q. Gao, "A generalized exp-function method for fractional Riccati differential equations," *Communications in Fractional Calculus*, vol. 1, pp. 48–52, 2010.

[26] X. J. Yang, *Advanced Local Fractional Calculus and Its Applications*, World Science, New York, NY, USA, 2012.

[27] X. J. Yang, "Local fractional integral transforms," *Progress in Nonlinear Science*, vol. 4, pp. 1–225, 2011.

[28] X. J. Yang, *Local Fractional Functional Analysis and Its Applications*, Asian Academic, Hong Kong, 2011.

[29] W. P. Zhong, X. J. Yang, and F. Gao, "A Cauchy problem for some local fractional abstract differential equation with fractal conditions," *Journal of Applied Functional Analysis*, vol. 8, no. 1, pp. 92–99, 2013.

[30] W. H. Su, X. J. Yang, H. Jafari, and D. Baleanu, "Fractional complex transform method for wave equations on Cantor sets within local fractional differential operator," *Advances in Difference Equations*, vol. 2013, no. 1, pp. 97–107, 2013.

[31] M. S. Hu, D. Baleanu, and X. J. Yang, "One-phase problems for discontinuous heat transfer in fractal media," *Mathematical Problems in Engineering*, vol. 2013, Article ID 358473, 3 pages, 2013.

[32] K. M. Kolwankar and A. D. Gangal, "Local fractional Fokker-Planck equation," *Physical Review Letters*, vol. 80, no. 2, pp. 214–217, 1998.

[33] A. Carpinteri and A. Sapora, "Diffusion problems in fractal media defined on Cantor sets," *ZAMM Journal of Applied Mathematics and Mechanics*, vol. 90, no. 3, pp. 203–210, 2010.

[34] A. Carpinteri and P. Cornetti, "A fractional calculus approach to the description of stress and strain localization in fractal media," *Chaos, Solitons and Fractals*, vol. 13, no. 1, pp. 85–94, 2002.

[35] A. Carpinteri, B. Chiaia, and P. Cornetti, "The elastic problem for fractal media: basic theory and finite element formulation," *Computers and Structures*, vol. 82, no. 6, pp. 499–508, 2004.

[36] A. Babakhani and V. Daftardar-Gejji, "On calculus of local fractional derivatives," *Journal of Mathematical Analysis and Applications*, vol. 270, no. 1, pp. 66–79, 2002.

[37] F. Ben Adda and J. Cresson, "About non-differentiable functions," *Journal of Mathematical Analysis and Applications*, vol. 263, no. 2, pp. 721–737, 2001.

[38] Y. Chen, Y. Yan, and K. Zhang, "On the local fractional derivative," *Journal of Mathematical Analysis and Applications*, vol. 362, no. 1, pp. 17–33, 2010.

[39] W. Chen, "Time-space fabric underlying anomalous diffusion," *Chaos, Solitons and Fractals*, vol. 28, no. 4, pp. 923–925, 2006.

[40] W. Chen, H. Sun, X. Zhang, and D. Korošak, "Anomalous diffusion modeling by fractal and fractional derivatives," *Computers & Mathematics with Applications*, vol. 59, no. 5, pp. 1754–1758, 2010.

[41] A. K. Golmankhaneh, V. Fazlollahi, and D. Baleanu, "Newtonian mechanics on fractals subset of real-line," *Romania Reports in Physics*, vol. 65, pp. 84–93, 2013.

[42] A. Parvate and A. D. Gangal, "Fractal differential equations and fractal-time dynamical systems," *Pramana*, vol. 64, no. 3, pp. 389–409, 2005.

[43] G. Jumarie, "Probability calculus of fractional order and fractional Taylor's series application to Fokker-Planck equation and information of non-random functions," *Chaos, Solitons and Fractals*, vol. 40, no. 3, pp. 1428–1448, 2009.

[44] G. Jumarie, "Laplace's transform of fractional order via the Mittag-Leffler function and modified Riemann-Liouville derivative," *Applied Mathematics Letters*, vol. 22, no. 11, pp. 1659–1664, 2009.

[45] X. J. Yang and D. Baleanu, "Fractal heat conduction problem solved by local fractional variation iteration method," *Thermal Science*, vol. 17, no. 2, pp. 625–628, 2013.

[46] W. H. Su, D. Baleanu, X.-J. Yang, and H. Jafari, "Damped wave equation and dissipative wave equation in fractal strings within the local fractional variational iteration method," *Fixed Point Theory and Applications*, vol. 2013, no. 1, pp. 89–102, 2013.

[47] M. S. Hu, R. P. Agarwal, and X.-J. Yang, "Local fractional Fourier series with application to wave equation in fractal vibrating string," *Abstract and Applied Analysis*, vol. 2012, Article ID 567401, 15 pages, 2012.

[48] G. A. Anastassiou and O. Duman, *Advances in Applied Mathematics and Approximation Theory*, Springer, New York, NY, USA, 2013.

[49] X. J. Yang, H. M. Srivastava, J. H. He, and D. Baleanu, "Cantor-type cylindrical-coordinate method for differential equations with local fractional derivatives," *Physics Letters A*, vol. 377, no. 28–30, pp. 1696–1700, 2013.

[50] X. Yang, M. Liao, and J. Chen, "A novel approach to processing fractal signals using the Yang-Fourier transforms," in *Proceedings of the International Workshop on Information and Electronics Engineering (IWIEE '12)*, vol. 29, pp. 2950–2954, March 2012.

[51] W. Zhong, F. Gao, and X. Shen, "Applications of Yang-Fourier transform to local fractional equations with local fractional derivative and local fractional integral," *Advanced Materials Research*, vol. 461, pp. 306–310, 2012.

[52] W. P. Zhong and F. Gao, "Application of the Yang-Laplace transforms to solution to nonlinear fractional wave equation with fractional derivative," in *Proceedings of the 3rd International Conference on Computer Technology and Development*, pp. 209–213, ASME, 2011.