Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2013, Article ID 290287, 2 pages
http://dx.doi.org/10.1155/2013/290287

Letter to the Editor

Periodic Solution of the Hematopoiesis Equation

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Received 4 December 2012; Accepted 30 December 2012

Wu and Liu (2012) presented some results for the existence and uniqueness of the periodic solutions for the hematopoiesis model. This paper gives a simple approach to find an approximate period of the model.

Wu and Liu studied the following hematopoiesis model [1]:

$$x'(t) = -ax(t) + \frac{\beta\theta^n}{\theta^n + x^n(t-\tau)}, \tag{1}$$

where x denotes the density of mature cells in blood circulation. The physical meaning of other parameters is referred to [1].

Equation (1) admits periodic solutions as revealed in [1]. Hereby we suggest a simple approach to the search for an approximate period of (1) using a simple amplitude-frequency formulation [2–5]. To this end, we rewrite (1) in the form

$$\begin{aligned} &x'(t)\theta^n + x'(t)x^n(t-\tau) + ax(t)\theta^n \\ &\quad + ax(t)x^n(t-\tau) - \beta\theta^n = 0. \end{aligned} \tag{2}$$

Assume that the periodic solution can be expressed in the form

$$x(t) = A\cos\omega t. \tag{3}$$

Submitting (3) into (2) results in the following residual:

$$\begin{aligned} R(\omega,t) = {} & -A\omega\theta^n \sin\omega t \\ & - A^{1+n}\omega\sin\omega t\cos^n\omega(t-\tau) + a\theta^n A\cos\omega t \\ & + aA^{1+n}\cos\omega t\cos^n\omega(t-\tau) - \beta\theta^n. \end{aligned} \tag{4}$$

In order to use the amplitude-frequency formulation [2–5], we choose two trial frequencies and locate them at $t = \pi/(4\omega)$.

Setting $\omega_1 = 1$, $\omega_1 t = \pi/4$, and $\omega_1 = 2$, $\omega_2 t = \pi/4$, respectively, we have

$$\begin{aligned} R_1 = {} & -\frac{\sqrt{2}}{2}A\theta^n - \frac{\sqrt{2}}{2}A^{1+n}\cos^n\left(\frac{\pi}{4}-\tau\right) \\ & + \frac{\sqrt{2}}{2}a\theta^n A + \frac{\sqrt{2}}{2}aA^{1+n}\cos^n\left(\frac{\pi}{4}-\tau\right) - \beta\theta^n, \\ R_2 = {} & -\sqrt{2}A\theta^n - \sqrt{2}A^{1+n}\cos^n\left(\frac{\pi}{4}-2\tau\right) \\ & + \frac{\sqrt{2}}{2}a\theta^n A + \frac{\sqrt{2}}{2}aA^{1+n}\cos^n\left(\frac{\pi}{4}-2\tau\right) - \beta\theta^n. \end{aligned} \tag{5}$$

The frequency can be then obtained approximately in the form [2–5]

$$\begin{aligned} \omega^2 &= \frac{R_1\omega_1^2 - R_2\omega_2^2}{R_1 - R_2} = \frac{R_1 - 4R_2}{R_1 - R_2} \\ &= \left(\frac{7\sqrt{2}}{2}A\theta^n + \frac{7\sqrt{2}}{2}A^{1+n}\cos^n\left(\frac{\pi}{4}-2\tau\right) - \frac{3\sqrt{2}}{2}a\theta^n A \right. \\ &\qquad \left. -\frac{3\sqrt{2}}{2}aA^{1+n}\cos^n\left(\frac{\pi}{4}-2\tau\right) + 3\beta\theta^n\right) \end{aligned}$$

$$\times \left(\left(\sqrt{2} - \frac{\sqrt{2}}{2} \right) A\theta^n - \left(\sqrt{2} - \frac{\sqrt{2}}{2} \right) A^{1+n} \cos^n \left(\frac{\pi}{4} - \tau \right) \right)^{-1}. \tag{6}$$

This formulation has been widely used to solve periodic solutions of various nonlinear oscillators [6–13], and it is often called as He's frequency formulation, He's amplitude-frequency formulation, or He's frequency-amplitude formulation. In case $\omega^2 < 0$, no period solution is admitted. A similar criterion is given for a nonlinear equation arising in electrospinning process [14].

Acknowledgments

The work is supported by PAPD (a project funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions), National Natural Science Foundation of China under Grant no. 10972053, and Project for Six Kinds of Top Talents in Jiangsu Province, China (Grant no. ZBZZ-035).

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