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Research Article

λ-Statistical Convergence in Paranormed Space

Mohammed A. Alghamdi[1] and Mohammad Mursaleen[2]

[1] *Department of Mathematics, King Abdulaziz University, Jeddah 21589, Saudi Arabia*
[2] *Department of Mathematics, Aligarh Muslim University, Aligarh 202002, India*

Correspondence should be addressed to Mohammad Mursaleen; mursaleenm@gmail.com

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The concept of λ-statistical convergence for sequences of real numbers was introduced in Mursaleen (2000). In this paper, we prove decomposition theorem for λ-statistical convergence. We also define and study λ-statistical convergence, λ-statistically Cauchy, and strongly λ_p-summability in Paranormed Space.

1. Introduction

The notion of statistical convergence was first introduced by Fast [1]. In the recent years, statistical summability became one of the most active areas of research in summability theory, which was further generalized as lacunary statistical convergence [2], λ-statical convergence [3], statistical A-summability [4], and statistical σ-convergence [5]. Maddox [6] studied this notion in locally convex Hausdorff topological spaces and Kolk [7] defined and studied this notion in Banach spaces while Çakalli [8] extended it to topological Hausdorff groups. The concept of statistical convergence is studied in probabilistic normed space and in intuitionistic fuzzy normed spaces in [9, 10]. Recently, the statistical convergence has been studied in Paranormed Space and locally solid Riesz spaces in [11, 12], respectively. Therefore, one can choose either some different setup to study these concepts or generalizing the existing concepts through different means. In this paper, we will study the concept of λ-statistical convergence, λ-statistical Cauchy, and strongly λ_p-summability in Paranormed Space.

A *paranorm* is a function $g : X \to \mathbb{R}$ defined on a linear space X such that for all $x, y, z \in X$

($P1$) $g(x) = 0$ if $x = \theta$,

($P2$) $g(-x) = g(x)$,

($P3$) $g(x + y) \le g(x) + g(y)$,

($P4$) if (α_n) is a sequence of scalars with $\alpha_n \to \alpha_0$ $(n \to \infty)$ and $x_n, a \in X$ with $x_n \to a$ $(n \to \infty)$ in the sense that $g(x_n - a) \to 0$ $(n \to \infty)$, then $\alpha_n x_n \to \alpha_0 a$ $(n \to \infty)$, in the sense that $g(\alpha_n x_n - \alpha_0 a) \to 0$ $(n \to \infty)$.

A paranorm g for which $g(x) = 0$ implies that $x = \theta$ is called a *total paranorm* on X, and the pair (X, g) is called a *total Paranormed Space*.

2. λ-Statistical Convergence

Let $\lambda = (\lambda_n)$ be a nondecreasing sequence of positive numbers tending to ∞ such that

$$\lambda_{n+1} \le \lambda_n + 1, \quad \lambda_1 = 0. \tag{1}$$

The generalized de la Vallée-Poussin mean is defined by

$$t_n(x) =: \frac{1}{\lambda_n} \sum_{j \in I_n} x_j, \tag{2}$$

where $I_n = [n - \lambda_n + 1, n]$.

A sequence $x = (x_j)$ is said to be (V, λ)-*summable* to a number L if

$$t_n(x) \longrightarrow L \quad \text{as } n \longrightarrow \infty. \tag{3}$$

Let K be a subset of the set of natural numbers $\mathbb{N}$. Then, the λ-*density* of K is defined as

$$\delta_\lambda(K) = \lim_n \frac{1}{\lambda_n} \left|\{n-\lambda_n+1 \le j \le n : j \in K\}\right|. \quad (4)$$

The number sequence $x=(x_j)$ is said to be λ-*statistically convergent* to the number L (c.f. [3, 13, 14]) if $\delta_\lambda(K(\epsilon)) = 0$; that is, if for each $\epsilon > 0$,

$$\lim_n \frac{1}{\lambda_n} \left|\{k \in I_n : |x_k - L| \ge \epsilon\}\right| = 0. \quad (5)$$

In this case we write $\text{st}_\lambda\text{-}\lim_k x_k = L$ and we denote the set of all λ-statistically convergent sequences by S_λ. In case $\lambda_n = n$, λ-density reduces to the natural density and λ-statistical convergence reduces to statistical convergence. This notion for double sequences has been studied in [15].

A sequence $x = (x_k)$ is said to be *strongly* λ_p-*summable* $(0 < p < \infty)$ to the *limit* L [14] if

$$\lim_n \frac{1}{\lambda_n} \sum_{k \in I_n} |x_k - L|^p = 0, \quad (6)$$

and we write it as $x_k \to L[V_\lambda]_p$. In this case L is called the $[V_\lambda]_p$-*limit* of x.

The following relation was established in [14].

Theorem 1. *If $0 < p < \infty$ and a sequence $x = (x_k)$ is strongly λ_p-summable to L, then it is λ-statistically convergent to L. If a bounded sequence is λ-statistically convergent to L, then it is strongly λ_p-summable to L.*

The following theorem is λ-statistical version of Connor's Decomposition Theorem [16].

Theorem 2. *If $x = (x_k)$ is strongly λ_p-summable or statistically λ-convergent to L, then there is a convergent sequence y and a λ-statistically null sequence z such that y is convergent to L, $x = y + z$ and*

$$\lim_n \frac{1}{\lambda_n} \left|\{k \in I_n : z_k \ne 0\}\right| = 0. \quad (7)$$

Moreover, if x is bounded, then y and z both are bounded.

Proof. By Theorem 1, it follows that x is λ-statistically convergent to L if x is strongly λ_p-summable to L. Set $N_0 = 0$ and choose a strictly increasing sequence of positive integers $N_1 < N_2 < N_3 < \cdots$ such that

$$\frac{1}{\lambda_n} \left|\left\{k \in I_n : |x_k - L| \ge j^{-1}\right\}\right| < j^{-1} \quad (8)$$

for $n > N_j$. Define y and z as follows.

If $N_0 < k < N_1$ set $z_k = 0$ and $y_k = x_k$. Let $j \ge 1$ and $N_j < k \le N_{j+1}$. Now we set

$$y_k = \begin{cases} x_k,\ z_k = 0, & \text{if } |x_k - L| < j^{-1}; \\ L,\ z_k = x_k - L, & \text{if } |x_k - L| \ge j^{-1}. \end{cases} \quad (9)$$

Clearly, $x = y + z$ and y and z are bounded, if x is bounded. Also, we observe that for $k > N_j$, we have

$$\begin{aligned} |y_k - L| < \epsilon \quad &\text{since } |y_k - L| = |x_k - L| < \epsilon \\ &\qquad \text{if } |x_k - L| < j^{-1}, \\ |y_k - L| = |L - L| = 0 \quad &\text{if } |x_k - L| > j^{-1}. \end{aligned} \quad (10)$$

Hence, $\lim_k y_k = L$, since ϵ was arbitrary.

Next we observe that

$$\left|\{k \in I_n : z_k \ne 0\}\right| \ge \left|\{k \in I_n : |z_k| \ge \epsilon\}\right| \quad (11)$$

for any natural number n and $\epsilon > 0$. Hence, $\lim_n (1/\lambda_n)|\{k \in I_n : z_k \ne 0\}| = 0$; that is, z is λ-statistically null.

We now show that if $\delta > 0$ and $j \in N$ such that $j^{-1} < \delta$, then $|\{k \in I_n : z_k \ne 0\}| < \delta$ for all $n > N_j$. Recall from the construction that if $N_j < k \le N_{j+1}$, then $z_k \ne 0$ only if $|x_k - L| > j^{-1}$. It follows that if $N_\ell < k \le N_{\ell+1}$, then

$$\{k \in I_n : z_k \ne 0\} \subseteq \left\{k \in I_n : |x_k - L| > \ell^{-1}\right\}. \quad (12)$$

Consequently,

$$\begin{aligned} \frac{1}{\lambda_n} \left|\{k \in I_n : z_k \ne 0\}\right| &\le \frac{1}{\lambda_n} \left|\left\{k \in I_n : |x_k - L| > \ell^{-1}\right\}\right| \\ &< \ell^{-1} < j^{-1} < \delta, \end{aligned} \quad (13)$$

if $N_\ell < n \le N_{\ell+1}$ and $\ell > j$. That is,

$$\lim_n \frac{1}{\lambda_n} \left|\{k \in I_n : z_k \ne 0\}\right| = 0. \quad (14)$$

This completes the proof of the theorem. □

3. Application to Fourier Series

Let $f : \mathbb{T} \to \mathbb{C}$ be a Lebesgue integrable function on the torus $\mathbb{T} := [-\pi, \pi)$; that is, $f \in L^1(\mathbb{T})$. The Fourier series of f is defined by

$$f(x) \sim \sum_{j \in \mathbb{Z}} \hat{f}(j) e^{ijx}, \quad x \in \mathbb{T}, \quad (15)$$

where the Fourier coefficients $\hat{f}(j)$ are defined by

$$\hat{f}(j) := \frac{1}{2\pi} \int_{\mathbb{T}} f(t) e^{-ijt} dt, \quad j \in \mathbb{Z}. \quad (16)$$

The symmetric partial sums of the series in (15) are defined by

$$s_k(f; x) := \sum_{|j| \le k} \hat{f}(j) e^{ijx}, \quad x \in \mathbb{T},\ k \in \mathbb{N}. \quad (17)$$

The conjugate series to the Fourier series in (15) is defined by [17, Vol. I, pp. 49]

$$\sum_{j \in \mathbb{Z}} (-i \ \text{sgn}\ j) \hat{f}(j) e^{ijx}. \quad (18)$$

Clearly, it follows from (15) and (18) that

$$\sum_{j\in\mathbb{Z}} \hat{f}(j) e^{ijx} + i \sum_{j\in\mathbb{Z}} (-i \text{ sgn } j) \hat{f}(j) e^{ijx} = 1 + 2\sum_{j=1}^{\infty} \hat{f}(j) e^{ijx}, \tag{19}$$

and the power series

$$1 + 2\sum_{j=1}^{\infty} \hat{f}(j) e^{ijx}, \quad \text{where } z := re^{ix},\ 0 \le r < 1, \tag{20}$$

is analytic on the open unit disk $|z| < 1$, due to the fact that

$$\left|\hat{f}(j)\right| \le \frac{1}{2\pi} \int_{\pi} |f(t)|\, dt, \quad j \in \mathbb{Z}. \tag{21}$$

The conjugate function $\hat{f}$ of a function $f \in L^1(\mathbb{T})$ is defined by

$$\begin{aligned} \hat{f}(x) &:= -\lim_{\varepsilon\downarrow 0} \frac{1}{\pi} \int_{\varepsilon\le|t|\le\pi} \frac{f(x+t)}{2\tan(t/2)} dt \\ &= \lim_{\varepsilon\downarrow 0} \frac{1}{\pi} \int_{\varepsilon}^{\pi} \frac{f(x-t) - f(x+t)}{2\tan(t/2)} dt \end{aligned} \tag{22}$$

in the "principal value" sense and that $\hat{f}(x)$ exists at almost every $x \in \mathbb{T}$.

The following is λ-statistical version of [18] (c.f. [19, Theorem 2.1 (ii)]).

Theorem 3. *If $f \in L^1(\mathbb{T})$, then for any $p > 0$ its Fourier series is strongly λ_p-summable to $f(x)$ at almost every $x \in \mathbb{T}$. Furthermore, its conjugate series* (18) *is strongly λ_p-summable for any $p > 0$ to the conjugate function $\hat{f}(x)$ defined in* (22) *at almost every $x \in \mathbb{T}$.*

From Theorems 1 and 3, we easily get the following useful result.

Theorem 4. *If $f \in L^1(\mathbb{T})$, then its Fourier series is λ-statistically convergent to $f(x)$ at almost every $x \in \mathbb{T}$. Furthermore, its conjugate series* (18) *is λ-statistically convergent to the conjugate function $\hat{f}(x)$ defined in* (22) *at almost every $x \in \mathbb{T}$.*

4. λ-Statistical Convergence in Paranormed Space

Recently, statistical convergence, statistical Cauchy, and strongly Cesàro summability have been studied in Paranormed Space by Alotaibi and Alroqi [11].

In this paper, we define and study the notion of λ-summable, λ-statistical convergence, λ-statistical Cauchy, and strongly λ_p-summability in Paranormed Space.

Let (X, g) be a Paranormed Space.

A sequence $x = (x_k)$ is said to be *convergent* to the number ξ in (X, g) if, for every $\varepsilon > 0$, there exists a positive integer k_0 such that $g(x_k - \xi) < \varepsilon$ whenever $k \ge k_0$. In this case, we write g-$\lim x = \xi$, and ξ is called the *g-limit* of x.

We define the following.

Definition 5. A sequence $x = (x_k)$ is said to be *λ-statistically convergent* to the number ξ in (X, g) if, for each $\epsilon > 0$,

$$\lim_{n} \frac{1}{\lambda_n} \left|\{k \in I_n : g(x_k - \xi) \ge \epsilon\}\right| = 0. \tag{23}$$

In this case we write $\text{st}_\lambda(g)$-$\lim x = \xi$.

Definition 6. A sequence $x = (x_k)$ is said to be *λ-statistically Cauchy* sequence in (X, g) if for every $\epsilon > 0$ there exists a number $N = N(\epsilon)$ such that

$$\lim_{n} \frac{1}{\lambda_n} \left|\left\{j \in I_n : g\left(x_j - x_N\right) \ge \epsilon\right\}\right| = 0. \tag{24}$$

Definition 7. A sequence $x = (x_k)$ is said to be *strongly λ_p-summable* $(0 < p < \infty)$ to the *limit* ξ in (X, g) if

$$\lim_{n} \frac{1}{\lambda_n} \sum_{k\in I_n} \left(g(x_k - \xi)\right)^p = 0, \tag{25}$$

and we write it as $x_k \to \xi[V_\lambda, g]_p$. In this case ξ is called the *$[V_\lambda, g]_p$-limit* of x.

Now we define another type of convergence in Paranormed Space.

Definition 8. A sequence (x_k) in a Paranormed Space (X, g) is said to *$\text{st}^*_\lambda(g)$-convergent* to $\xi \in X$ if there exists an index set $K = \{k_1 < k_2 < \cdots < k_n < \cdots\} \subseteq \mathbb{N}$, $n = 1, 2, \ldots$, with $\delta_\lambda(K) = 1$ such that $g(x_{k_n} - \xi) \to 0$ $(n \to \infty)$. In this case, we write $\xi = \text{st}^*_\lambda(g)$-$\lim x$.

First we prove the following results on λ-statistical convzergence in (X, g).

Theorem 9. *If g-$\lim x = \xi$, then $st_\lambda(g)$-$\lim x = \xi$ but converse need not be true in general.*

Proof. Let g-$\lim x = \xi$. Then, for every $\varepsilon > 0$, there is a positive integer N such that

$$g(x_n - \xi) < \varepsilon \tag{26}$$

for all $n \ge N$. Since the set $A(\epsilon) := \{k \in \mathbb{N} : g(x_k - \xi) \ge \varepsilon\}$ is finite, $\delta_\lambda(A(\epsilon)) = 0$. Hence, $\text{st}_\lambda(g)$-$\lim x = \xi$.

The following example shows that the converse need not be true.

Example 10. Let $X = \ell(1/k) := \{x = (x_k) : \sum_k |x_k|^{1/(k+1)} < \infty\}$ with the paranorm $g(x) = (\sum_k |x_k|^{1/(k+1)})$. Define a sequence $x = (x_k)$ by

$$x_k := \begin{cases} k, & \text{if } n - [\lambda_n] + 1 \le k \le n,\ n \in \mathbb{N}; \\ 0, & \text{otherwise,} \end{cases} \tag{27}$$

and write

$$K(\varepsilon) := \{k \le n : g(x_k) \ge \varepsilon\}, \quad 0 < \varepsilon < 1. \tag{28}$$

We see that

$$g(x_k) := \begin{cases} k^{1/(k+1)}, & \text{if } n - [\lambda_n] + 1 \le k \le n,\ n \in \mathbb{N}; \\ 0, & \text{otherwise,} \end{cases} \tag{29}$$

and hence

$$\lim_k g(x_k) := \begin{cases} 1, & \text{if } n - [\lambda_n] + 1 \le k \le n,\ n \in \mathbb{N}; \\ 0, & \text{otherwise.} \end{cases} \tag{30}$$

Therefore g-$\lim x$ does not exist. On the other hand $\delta_\lambda(K(\varepsilon)) = 0$; that is, $\mathrm{st}_\lambda(g)$-$\lim x = 0$.

This completes the proof of the theorem. □

We can easily prove the following results on λ-statistical convergence in (X, g) similar to those of [11].

Theorem 11. *If a sequence $x = (x_k)$ is λ-statistically convergent in (X, g), then $st_\lambda(g)$-limit is unique.*

Theorem 12. *Let $st_\lambda(g)$-$\lim x = \xi_1$ and $st_\lambda(g)$-$\lim y = \xi_2$. Then,*

(i) $st_\lambda(g)$-$\lim (x \pm y) = \xi_1 \pm \xi_2$,

(ii) $st_\lambda(g)$-$\lim \alpha x = \alpha\xi_1$, $\alpha \in \mathbb{R}$.

Theorem 13. *Let (X, g) be a complete Paranormed Space. Then a sequence $x = (x_k)$ of points in (X, g) is λ-statistically convergent if and only if it is λ-statistically Cauchy.*

Theorem 14. *(a) If $0 < p < \infty$ and $x_k \to \xi[V_\lambda, g]_p$, then $x = (x_k)$ is λ-statistically convergent to ξ in (X, g).*

(b) If $x = (x_k)$ is bounded and λ-statistically convergent to ξ in (X, g), then $x_k \to \xi[V_\lambda, g]_p$.

Theorem 15. *Let (X, g) be a complete Paranormed Space. Then a sequence $x = (x_k)$ of points in (X, g) is λ-statistically convergent if and only if it is λ-statistically Cauchy.*

Note that the proof of Theorem 2.4 [11] is incorrect and the correct proof is given in the following theorem which is generalization of Theorem 2.4 [11]. Another form of this result is given in [20] for ideal convergence.

Theorem 16. *A sequence $x = (x_k)$ in (X, g) is λ-statistically convergent to ξ if and only if it is $st^*_\lambda(g)$-convergent to ξ.*

Proof. Suppose that $x = (x_k)$ is λ-statistically convergent to ξ; that is, $st_\lambda(g)$-$\lim x = \xi$. Now, write for $r = 1, 2, \ldots$.

$$K_r := \left\{ n \in \mathbb{N} : g\left(x_{k_n} - \xi\right) \ge \frac{1}{r} \right\},$$
$$M_r := \left\{ n \in \mathbb{N} : g\left(x_{k_n} - \xi\right) < \frac{1}{r} \right\} \quad (r = 1, 2, \ldots). \tag{31}$$

Then $\delta_\lambda(K_r) = 0$,

$$M_1 \supset M_2 \supset \cdots \supset M_i \supset M_{i+1} \supset \cdots, \tag{32}$$

$$\delta_\lambda(M_r) = 1, \quad r = 1, 2, \ldots. \tag{33}$$

Now we have to show that, for $n \in M_r$, (x_{k_n}) is g-convergent to ξ. On contrary suppose that (x_{k_n}) is not g-convergent to ξ. Therefore, there is $\varepsilon > 0$ such that $g(x_{k_n} - \xi) \ge \varepsilon$ for infinitely many terms. Let $M_\varepsilon := \{n \in \mathbb{N} : g(x_{k_n} - \xi) < \varepsilon\}$ and $\varepsilon > 1/r$, $r \in \mathbb{N}$.

Then

$$\delta_\lambda(M_\varepsilon) = 0, \tag{34}$$

and by (32), $M_r \subset M_\varepsilon$. Hence $\delta_\lambda(M_r) = 0$, which contradicts (33) and we get that (x_{k_n}) is g-convergent to ξ. Hence, x is $\mathrm{st}^*_\lambda(g)$-convergent to ξ.

Conversely, suppose that x is $\mathrm{st}^*_\lambda(g)$-convergent to ξ. Then there exists a set $K = \{k_1 < k_2 < k_3 < \cdots < k_n < \cdots\}$ with $\delta_\lambda(K) = 1$ such that g-$\lim_{n\to\infty} x_{k_n} = \xi$. Therefore, there is a positive integer N such that $g(x_n - \xi) < \varepsilon$ for $n \ge N$. Put $K_\varepsilon := \{n \in \mathbb{N} : g(x_n - \xi) \ge \varepsilon\}$ and $K' := \{k_{N+1}, k_{N+2}, \ldots\}$. Then $\delta_\lambda(K') = 1$ and $K_\varepsilon \subseteq \mathbb{N} - K'$ which implies that $\delta_\lambda(K_\varepsilon) = 0$. Hence $x = (x_k)$ is λ-statistically convergent to ξ; that is $\mathrm{st}_\lambda(g)$-$\lim x = \xi$.

This completes the proof of the theorem. □

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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