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Research Article

Some New Classes of Extended General Mixed Quasi-Variational Inequalities

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We consider and study a new class of variational inequality, which is called the extended general mixed quas-variational inequality. We use the auxiliary principle technique to study the existence of a solution of the extended general mixed quasi-variational inequality. Several special cases are also discussed. Results proved in this paper may stimulate further research in this area.

1. Introduction

Mixed quasi-variational inequalities are very important and significant generalization of variational inequalities involving the nonlinear bifunction. It is well known that a large class of problems arising in various branches of pure and applied sciences can be studied in the general framework of mixed quasi-variational inequalities. In recent years, Noor [1–4] has shown that the optimality conditions of the differentiable nonconvex functions involving two arbitrary functions can be characterized by a class of variational inequalities, which is called the extended general variational inequalities. For the numerical results, existence theory, and other aspects of the extended general variational inequalities, see [1–13] and the references therein. We would like to mention that one can show that the minimum of a sum of differentiable nonconvex (hg-convex) function and a nondifferentiable nonconvex (hg-convex) bifunction can be characterized by a class of variational inequality. Motivated by this result, we introduce a new class of mixed variational inequalities, which is called *extended general mixed quasi-variational inequality* involving four different operators. Due to the nature of the problem, one cannot use the projection and resolvent operator techniques to study the existence of such type of the variational inequalities. To overcome this drawback, one uses the auxiliary principle technique. This technique is mainly due to Glowinski et al. [14]. This technique is more flexible and has been used to develop several numerical methods for solving the variational inequalities and the equilibrium problems. Noor [1, 9] has used this technique to study the existence of the extended general mixed variational inequality and its variant forms. We again use the auxiliary principle technique to study the existence of a solution of the extended general mixed quasi-variational inequalities. Since the extended general variational inequalities include various classes of variational inequalities and complementarity problems as special cases, results proved in this paper continue to hold for these problems. Results proved in this paper may be viewed as important and significant improvement of the previously known results. It is interesting to explore the applications of these extended general variational inequalities in mathematical and engineering sciences with new and novel aspects. This may lead to new research in this field.

2. Preliminaries

Let H be a real Hilbert space whose inner product and norm are denoted by $\langle\cdot,\cdot\rangle$ and $\|\cdot\|$, respectively. Let K be a nonempty closed and convex set in H. Let $\varphi(\cdot,\cdot) : H\times \to R\cup\{\infty\}$ be a continuous bifunction.

For given nonlinear operators $T, g, h : H \to H$, consider the problem of finding $u \in H$ such that

$$\langle Tu, g(v) - h(u)\rangle + \varphi(g(v), h(u)) - \varphi(h(u), h(u)) \geq 0, \quad \forall v \in H. \tag{2.1}$$

Inequality of type (2.1) is called the extended general mixed quasi-variational inequality.

One can show that the minimum of a sum of differentiable nonconvex (hg-convex) function and a nondifferentiable nonconvex (hg-convex) function on the hg-convex set K in H can be characterized by the extended general mixed quasi-variational inequality (2.1). For this purpose, we recall the following well known concepts, see [1, 5, 15–18].

Definition 2.1. Let K be any set in H. The set K is said to be hg-convex, if there exist functions $g, h : H \to H$ such that

$$h(u) + t(g(v) - h(u)) \in K, \quad \forall u, v \in H : h(u),\ g(v) \in K,\ t \in [0,1]. \tag{2.2}$$

Note that every convex set is hg-convex, but the converse is not true, see [15]. If $g = h$, then the hg-convex set K is called the g-convex set, which was introduced by Youness [19]. See also Cristescu and Lupşa [15] for its various extensions and generalization.

Definition 2.2. The function $F : K \to H$ is said to be hg-convex on the hg-convex set K, if there exist two functions h, g such that

$$\begin{aligned} &F(h(u) + t(g(v) - h(u))) \\ &\leq (1-t)F(h(u)) + tF(g(v)), \quad \forall u, v \in H : h(u),\ g(v) \in K,\ t \in [0,1]. \end{aligned} \tag{2.3}$$

Clearly every convex function is hg-convex, but the converse is not true. For $g = h$, Definition 2.2 is due to Youness [19].

We now consider the function $I[v]$, defined by

$$I[v] = F(v) + \varphi(v, v), \quad \forall v \in H. \tag{2.4}$$

Using the technique of Noor [2, 4, 5], one can easily show that the minimum of a differentiable hg-convex function and a nondifferentiable hg-convex bifunction on a hg-convex set K in H can be characterized by the extended general mixed quasi-variational inequality (2.1).

Lemma 2.3. *Let $F : K \to H$ be a differentiable hg-convex function, and let $\varphi(\cdot,\cdot)$ be a nondifferentiable bifunction on the hg-convex set K. Then $u \in H : h(u) \in K$ is the minimum of the functional $I[v]$, defind by* (2.4) *on the hg-convex set K, if and only if, $u \in H : h(u) \in K$ satisfies the inequality*

$$\langle F'(h(u)), g(v) - h(u)\rangle + \varphi\big(g(v), h(u)\big) - \varphi(h(u), h(u)) \geq 0, \quad \forall v \in H : g(v) \in K, \tag{2.5}$$

where $F'(u)$ is the differential of F at $u \in K$.

Lemma 2.3 implies that hg-convex programming problem can be studied via the extended general mixed variational inequality (2.1) with $Tu = F'(h(u))$.

We now list some special cases of the extended general variational inequalities.

(i) If $g = h$, then Problem(2.1) is equivalent to finding $u \in H : g(u) \in H$ such that

$$\langle Tu, g(v) - g(u)\rangle + \varphi\big(g(v), g(u)\big) - \varphi\big(g(u), g(u)\big) \geq 0, \quad \forall v \in H : g(v) \in H, \tag{2.6}$$

which is known as general mixed quasi-variational inequality, introduced and studied by Noor [20, 21]. It turned out that odd order and nonsymmetric obstacle, free, moving, unilateral, and equilibrium problems arising in various branches of pure and applied sciences can be studied via general variational inequalities.

(ii) For $g \equiv I$, the identity operator, the extended general variational inequality (2.1) collapses to: find $u \in H : h(u) \in H$ such that

$$\langle Tu, v - h(u)\rangle + \varphi(v, h(u)) - \varphi(h(u), h(u)) \geq 0, \quad \forall v \in H, \tag{2.7}$$

which is also called the general mixed quasi-variational inequality and appears to be a new one.

(iii) For $h = I$, the identity operator, the extended general variational inequality (2.1) is equivalent to finding $u \in H$ such that

$$\langle Tu, g(v) - u\rangle + \varphi\big(g(u), u\big) - \varphi(u, u) \geq 0, \quad \forall v \in H : g(v) \in H, \tag{2.8}$$

which is also called the general mixed quasi-variational inequality involving two nonlinear operators, see M. A. Noor and K. I. Noor [13].

We would like to emphasize the fact that general variational inequalities (2.6), (2.7), and (2.8) are quite different from each other and have different applications.

(iv) For $g = h = I$, the identity operator, the extended general variational inequality (2.1) is equivalent to finding $u \in H$ such that

$$\langle Tu, v - u\rangle + \varphi(v, u) - \varphi(u, u) \geq 0, \quad \forall v \in H, \tag{2.9}$$

which is known as the classical mixed quasi-variational inequality. For the applications, formulation and numerical methods for solving the mixed quasi-variational inequalities (2.9), see [1–14, 16–18, 20–34].

(v) If the bifunction $\varphi(\cdot, \cdot)$ is the indicator function of a closed convex-valued set $K(u)$ in H, that is,

$$\varphi(u, u) = K_{(u)}(u) = \begin{cases} 0, & u \in K(u), \\ +\infty, & \text{otherwise,} \end{cases} \tag{2.10}$$

then problem (2.1) is equivalent to finding $u \in H : h(u) \in K(u)$ such that

$$\langle Tu, g(v) - h(u)\rangle \geq 0, \quad \forall v \in H : g(v) \in K(u). \tag{2.11}$$

Problems of type (2.11) are called extended general quasi-variational inequalities, studied by Noor [10].

(vi) If $K(u) = K$, the convex set in H, Then, problem (2.11) is equivalent to finding $u \in H : h(u) \in K$ such that

$$\langle Tu, g(v) - h(u)\rangle \geq 0, \quad \forall v \in H : g(v) \in K, \tag{2.12}$$

which is called the extended general variational inequality, introduced and studied by Noor [1–4]. For the formulation, applications, numerical method and other aspects of the extended general variational inequality (2.12), see [1–4, 6–12, 35, 36].

We would like to mention that one can obtain several known and new classes of variational inequalities as special cases of the problem (2.1). From the above discussion, it is clear that the extended general mixed quasi-variational inequalities (2.1) is most general and includes several previously known classes of variational inequalities and related optimization problems as special cases. These variational inequalities have important applications in mathematical programming and engineering sciences, see [1–40].

We also need the following concepts and results.

Definition 2.4. For all $u, v \in H$, an operator $T : H \to H$ is said to be:

(i) *strongly monotone*, if there exists a constant $\alpha > 0$ such that

$$\langle Tu - Tv, u - v\rangle \geq \alpha\|u - v\|^2, \tag{2.13}$$

(ii) *Lipschitz continuous*, if there exists a constant $\beta > 0$ such that

$$\|Tu - Tv\| \leq \beta\|u - v\|. \tag{2.14}$$

From (i) and (ii), it follows that $\alpha \leq \beta$.

Remark 2.5. It follows from the strong monotonicity of the operator T, that

$$\alpha\|u - v\|^2 \leq \langle Tu - Tv, u - v\rangle \leq \|Tu - Tv\|\|u - v\|, \quad \forall u, v \in H, \tag{2.15}$$

which implies that

$$\|Tu - Tv\| \geq \alpha\|u - v\|, \quad \forall u, v \in H. \tag{2.16}$$

This observation enables us to define the following concept.

Definition 2.6. The operator T is said to firmly expanding if

$$\|Tu - Tv\| \geq \|u - v\|, \quad \forall u, v \in H. \tag{2.17}$$

Definition 2.7. The bifunction $\varphi(\cdot,\cdot)$ is said to be skew-symmetric, if,

$$\varphi(u, u) - \varphi(u, v) - \varphi(v, u) + \varphi(v, v) \geq 0, \quad \forall u, v \in H. \tag{2.18}$$

Clearly, if the bifunction $\varphi(\cdot,\cdot)$ is linear in both arguments, then,

$$\varphi(u, u) - \varphi(u, v) - \varphi(v, u) + \varphi(v, v) = \varphi(u - v, u - v) \geq 0, \quad \forall u, v \in H, \tag{2.19}$$

which shows that the bifunction $\varphi(\cdot,\cdot)$ is nonnegative.

Remark 2.8. It is worth mentioning that the points (u, u), (u, v), (v, u), and (v, v) make up a set of the four vertices of the square. In fact, the skew-symmetric bifunction $\varphi(\cdot,\cdot)$ can be written in the form

$$\frac{1}{2}\varphi(u, u) + \frac{1}{2}\varphi(v, v) \geq \frac{1}{2}\varphi(u, v) + \frac{1}{2}\varphi(v, u), \quad \forall u, v \in H. \tag{2.20}$$

This shows that the arithmetic average value of the skew-symmetric bifunction calculated at the north-east and south-west vertices of the square is greater than or equal to the arithmetic average value of the skew-symmetric bifunction computed at the north-west and south-west vertices of the same square. The skew-symmetric bifunction have the properties which can be considered an analogs of monotonicity of gradient and nonnegativity of a second derivative for the convex functions.

3. Main Results

In this Section, we use the auxiliary principle technique of Glowinski et al. [14] as developed by Noor [1, 8, 20, 21] and M. A. Noor and K. I. Noor [13] to study the existence of a solution of the extended general mixed quasi-variational inequality (2.1).

Theorem 3.1. *Let T be a strongly monotone with constant $\alpha > 0$ and Lipschitz continuous with constant $\beta > 0$. Let g be a strongly monotone and Lipschitz continuous operator with constants $\sigma > 0$ and $\delta > 0$, respectively. Let the bifunction $\varphi(\cdot,\cdot)$ be skew-symmetric. If the operator h is firmly expanding and there exists a constant $\rho > 0$ such that*

$$\left|\rho - \frac{\alpha}{\beta^2}\right| < \frac{\sqrt{\alpha^2 - \beta^2 k(2-k)}}{\beta^2}, \quad \alpha > \beta\sqrt{k(2-k)}, \ k < 1, \tag{3.1}$$

where

$$\theta = k + \sqrt{1 - 2\rho\alpha + \rho^2\beta^2}, \tag{3.2}$$

$$k = \sqrt{1 - 2\sigma + \delta^2}, \tag{3.3}$$

then the extended general mixed quasi-variational inequality (2.1) *has a unique solution.*

Proof. We use the auxiliary principle technique to prove the existence of a solution of (2.1). For a given $u \in H : g(u) \in K$ satisfying the extended general mixed quasi-variational inequality (2.1), we consider the problem of finding a solution $w \in H : h(w) \in K$ such that

$$\begin{aligned} &\langle \rho Tu + h(w) - g(u), g(v) - h(w)\rangle + \rho\varphi(g(v), h(w)) - \rho\varphi(h(w), h(w)) \geq 0, \\ &\qquad \forall v \in H : g(v) \in K, \end{aligned} \tag{3.4}$$

where $\rho > 0$ is a constant.

The inequality of type (3.4) is called the auxiliary extended general mixed variational inequality associated with the problem (2.1). It is clear that the relation (3.4) defines a mapping $u \to w$. It is enough to show that the mapping $u \to w$ defined by the relation (3.4) has a unique fixed point belonging to H satisfying the general variational inequality (2.1). Let $w_1 \neq w_2$ be two solutions of (3.4) related to $u_1, u_2 \in H$, respectively. It is sufficient to show that for a well chosen $\rho > 0$,

$$\|w_1 - w_2\| \leq \theta\|u_1 - u_2\|, \tag{3.5}$$

with $0 < \theta < 1$, where θ is independent of u_1 and u_2. Taking $g(v) = h(w_2)$ (resp., $h(w_1)$) in (3.4) related to u_1 (resp., u_2), adding the resultant, and using the skew-symmetry of the bifunction $\varphi(\cdot,\cdot)$, we have

$$\langle h(w_1) - h(w_2), h(w_1) - h(w_2)\rangle \leq \langle g(u_1) - g(u_2) - \rho(Tu_1 - Tu_2), h(w_1) - h(w_2)\rangle, \tag{3.6}$$

from which we have

$$\begin{aligned}\|h(w_1)-h(w_2)\| &\le \left\|g(u_1)-g(u_2)-\rho(Tu_1-Tu_2)\right\| \\ &\le \left\|u_1-u_2-\left(g(u_1)-g(u_2)\right)\right\| + \left\|u_1-u_2-\rho(Tu_1-Tu_2)\right\|.\end{aligned} \tag{3.7}$$

Since T is both strongly monotone and Lipschitz continuous operator with constants $\alpha > 0$ and $\beta > 0$, respectively, it follows that

$$\begin{aligned}\left\|u_1-u_2-\rho(Tu_1-Tu_2)\right\|^2 &\le \|u_2-u_2\|^2 - 2\rho\langle u_1-u_2, Tu_1-Tu_2\rangle + \rho^2\|Tu_1-Tu_2\|^2 \\ &\le \left(1-2\rho\alpha+\rho^2\beta^2\right)\|u_1-u_2\|^2.\end{aligned} \tag{3.8}$$

In a similar way, using the strong monotonicity with constant $\sigma > 0$ and Lipschitz continuity with constant $\delta > 0$, we have

$$\left\|u_1-u_2-\left(g(u_1)-g(u_2)\right)\right\| \le \sqrt{1-2\sigma+\delta^2}\|u_1-u_2\|. \tag{3.9}$$

From (3.2), (3.3), (3.7), (3.8), and (3.9), and using the fact that the operator h is firmly expanding, we have

$$\begin{aligned}\|w_1-w_2\| &\le \left\{k+\sqrt{1-2\rho\alpha+\rho^2\beta^2}\right\}\|u_1-u_2\| \\ &= \theta\|u_1-u_2\|.\end{aligned} \tag{3.10}$$

From (3.1), it follows that $\theta < 1$ showing that the mapping defined by (3.4) has a fixed point belonging to K, which is solution of (2.1), the required result. □

We note that if $w = u$, then w is a solution of the extended general mixed quasi variational inequality (2.1). This observation enables us to suggest the following iterative method for solving the extended general mixed quasi-variational inequality (2.1).

Algorithm 3.2. For a given $u_0 \in H$, find the approximate solution u_{n+1} by the iterative scheme:

$$\left\langle \rho Tu_n + h(u_{n+1}) - g(u_n), g(v) - h(u_{n+1})\right\rangle + \rho\varphi\left(g(v), h(u_{n+1})\right) - \rho\varphi(h(u_{n+1}), h(u_{n+1})) \ge 0, \quad \forall v \in H, \tag{3.11}$$

where $\rho > 0$ is a constant. Algorithm 3.2 is called the explicit iterative method. For different and suitable choice of the operators and spaces, one can obtain various iterative methods for solving the quasi-variational inequalities and its variant forms. We leave this to the interested readers.

4. Conclusion

In this paper, we have introduced and studied a new class of variational inequalities, which is called the extended general mixed variational inequality. We have shown that this class is related to the optimality conditions of the nonconvex differentiable functions. One can easily obtain various classes of variational inequalities as special cases of this new class. We have used the auxiliary principle technique to study the existence of a solution of the extended general mixed quasi-variational inequalities under some suitable conditions. Our technique does not involve the projection or resolvent operator. We expect that the results proved in this paper may stimulate further research in this field. The interested readers are encouraged to find the novel and new applications of the extended general mixed quasi-variational inequalities in various branches of pure and applied sciences.

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