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Research Article

Permanence of a Single Species System with Distributed Time Delay and Feedback Control

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We study the permanence of a classof single species system with distributed time delay and feedback controls. General criteria on permanence are established in this paper. A very important fact is found in our results; that is, the feedback control is harmless to the permanence of species.

1. Introduction

Ecosystems in the real world are continuously disturbed by unpredictable forces which can result in changes in the biological parameters such as survival rates. Of practical interest in ecology is the question of whether or not an ecosystem can withstand those unpredictable disturbances which persist for a finite period of time. In the language of control variables, we call the disturbance functions as control variables. During the last decade, many scholars did works on the feedback control ecosystems. Motivated by those work especially [1], we consider the following single species system with distributed time delay and feedback control:

$$\begin{gathered} x'(t) = x(t)\left\{a(t) - b(t)x(t-\tau(t)) - c(t)\int_{-\infty}^{0} H(s)x(t+s)ds - h(t)u(t) - r(t)u(t-\lambda(t))\right\}, \\ u'(t) = -d(t)u(t) + e(t)\int_{-\infty}^{0} H(s)x^2(t+s)ds, \end{gathered} \tag{1.1}$$

where $x(t)$ is the density of the species at time t, $u(t)$ is the control variable; $a(t)$, $b(t)$, $c(t)$, $d(t)$, $e(t)$, $h(t)$, $r(t)$, $\tau(t)$ and $\lambda(t)$ are defined on $R_+ = [0, +\infty)$ and are bounded and continuous functions; $b(t)$, $c(t)$, $d(t)$, $e(t)$, $h(t)$, $r(t)$, $\tau(t)$, and $\lambda(t)$ are nonnegative for all $t \in R_+$. Further, we assume that the delay kernels $H(s)$ are nonnegative integral functions defined on $R_- = (-\infty, 0]$ such that $\int_{-\infty}^{0} H(s)ds = 1$ and $\sigma = \int_{-\infty}^{0} sH(s)ds < +\infty$.

We denote by BC_+ the space of all bounded continuous functions $\phi : R_- \to R_+$ with norm $|\phi| = \sup_{s\in R_-} |\phi(s)|$. In this paper, we always assume that all solutions of system (1.1) satisfy the following initial condition:

$$x(s) = \phi(s), \quad u(s) = \varphi(s) \quad \forall s \in R_-, \tag{1.2}$$

where $\phi(s) \in \mathrm{BC}_+$ and $\varphi(s) \in \mathrm{BC}_+$.

Let $(x(t), u(t))$ be the solution of system (1.1) satisfying initial condition (1.2). We easily prove $x(t) > 0$ and $u(t) > 0$ in maximal interval of the existence of the solution. For the sake of convenience, the solution of system (1.1) with initial condition (1.2) is said to be positive.

In addition, for a function $g(t)$ defined on set $I \subset R$, we denote

$$g^0 = \sup_{t\in I} g(t), \qquad \underline{g} = \inf_{t\in I} g(t). \tag{1.3}$$

In the theory of mathematical biology, systems such as (1.1) are very important in a single species system in time-fluctuating environments, the effect of time delays and feedback controls. We see that there has been a series of articles which deal with the dynamical behaviors of the autonomous, periodic, and general nonautonomous population growth systems with feedback controls, for example [1–12] and reference cited therein. In [1] the authors proposed the following single species model with feedback regulation and distributed time delay of the form:

$$\begin{aligned} x'(t) &= x(t)\left\{a(t) - b(t)\int_0^{\infty} H(s)x(t-s)ds - c(t)u(t)\right\}, \\ u'(t) &= -d(t)u(t) + e(t)\int_0^{\infty} H(s)x^2(t-s)ds. \end{aligned} \tag{1.4}$$

By using the continuation theorem of coincidence theory, a criterion which guarantees the existence of positive periodic solution of system (1.4) is obtained. In [2], the authors obtain sufficient condition which guarantees the global attractivity of the positive solution of system (1.4) by constructing a suitable Lyapunov functional. The aim of this paper is to establish new sufficient conditions on the permanence for all positive solutions of system (1.1) by improving the method given in [13–15].

2. Preliminaries

Throughout this paper, we will introduce the following assumptions:

(H_1) there exists a constant $\omega > 0$, such that

$$\liminf_{t\to\infty} \int_t^{t+\omega} a(s)ds > 0; \tag{2.1}$$

(H_2) there exists a constant $\beta > 0$, such that

$$\liminf_{t\to\infty} \int_t^{t+\beta} b(s)ds > 0; \tag{2.2}$$

(H_3) there exists a constant $\gamma > 0$ such that

$$\liminf_{t\to\infty} \int_t^{t+\gamma} c(s)ds > 0; \tag{2.3}$$

(H_4) there exists a constant $\lambda > 0$ such that

$$\liminf_{t\to\infty} \int_t^{t+\lambda} d(s)ds > 0; \tag{2.4}$$

(H_5) there exists a constant $\zeta > 0$ such that

$$\liminf_{t\to\infty} \int_t^{t+\zeta} e(s)ds > 0. \tag{2.5}$$

First, we consider the following nonautonomous logistic equation

$$x'(t) = x(t)(a(t) - b(t)x(t)), \tag{2.6}$$

where functions $a(t), b(t)$ are bounded and continuous on R_+. Furthermore, $b(t) \geq 0$ for all $t \geq 0$. We have the following result which is given in [15] by Teng and Li.

Lemma 2.1. *Suppose that assumptions (H_1)-(H_2) hold. Then,*

(a) *there exist positive constants m and M such that*

$$m \leq \liminf_{t\to\infty} x(t) \leq \limsup_{t\to\infty} x(t) \leq M \tag{2.7}$$

for any positive solution $x(t)$ of (2.6)*;*

(b) $\lim_{t\to\infty}(x^{(1)}(t) - x^{(2)}(t)) = 0$ *for any two positive solutions $x^{(1)}(t)$ and $x^{(2)}(t)$ of* (2.6).

Further, we consider the following nonautonomous linear equation

$$u'(t) = r(t) - d(t)u(t), \tag{2.8}$$

where functions $r(t)$ and $d(t)$ are bounded continuous defined on R_+, and $r(t) \geq 0$ for all $t \geq 0$. We have the following result.

Lemma 2.2. *Suppose that assumptions (H$_4$) hold. Then,*

(a) *there exists a positive constant* U *such that* $\limsup_{t\to\infty} u(t) \le U$ *for any positive solution* $u(t)$ *of* (2.8);

(b) $\lim_{t\to\infty}(u^{(1)}(t) - u^{(2)}(t)) = 0$ *for any two positive solutions* $u^{(1)}(t)$ *and* $u^{(2)}(t)$ *of* (2.8).

The proof of Lemma 2.2 is very simple, we hence omit it here.

Lemma 2.3. *Suppose that assumption (H$_4$) holds. Then for any constants* $\varepsilon > 0$ *and* $M > 0$ *there exist constants* $\delta = \delta(\varepsilon) > 0$ *and* $T = T(M) > 0$ *such that for any* $t_0 \in R_+$ *and* $u_0 \in R$ *with* $|u_0| \le M$, *when* $|r(t)| < \delta$ *for all* $t \ge t_0$, *one has*

$$|u(t, t_0, u_0)| < \varepsilon \quad \forall t \ge t_0 + T, \tag{2.9}$$

where $u(t, t_0, u_0)$ *is the solution of* (2.8) *with initial condition* $u(t_0) = u_0$.

The proof of Lemma 2.3 can be found as Lemma 2.4 in [16] by Wang et al.

Lemma 2.4. *Let* $x(t) : R \to R$ *be a nonnegative and bounded continuous function, and let* $H(s) : R_- \to R_+$ *be an integral function satisfying* $\int_{-\infty}^{0} H(s)ds = 1$. *Then one has*

$$\begin{aligned}\liminf_{t\to\infty} x(t) &\le \liminf_{t\to\infty} \int_{-\infty}^{0} H(s)x(t+s)ds \\ &\le \limsup_{t\to\infty} \int_{-\infty}^{0} H(s)x(t+s)ds \\ &\le \limsup_{t\to\infty}\, x(t).\end{aligned} \tag{2.10}$$

Lemma 2.4 is given in [17] by Montes de Oca and Vivas.

3. Main Results

Theorem 3.1. *Suppose that assumption (H$_1$) holds and (H$_2$) or (H$_3$) holds. Then there exists a positive constant* $M > 0$ *such that*

$$\limsup_{t\to\infty} x(t) < M, \qquad \limsup_{t\to\infty} u(t) < M, \tag{3.1}$$

for any positive solution $((x(t), u(t))$ *of system* (1.1).

Proof. Let $x(t)$ be a positive solution of the first equation system (1.1). Since

$$\frac{dx(t)}{dt} \le a(t)x(t) \tag{3.2}$$

for all $t > 0$ as long as the solution exists and the function $a(t)$ is bounded and continuous on R_+. We can obtain that the solution exists for all $t \in R_+$. For any $t \geq 0, s \leq 0$, and $t + s \geq 0$, by integrating (3.2) from $t + s$ to t, we have

$$x(t+s) \geq x(t) \exp \int_t^{t+s} a(u) du. \tag{3.3}$$

For any $t > \tau^0$, where $\tau^0 = \max\{\sup_{t\in R_+} \tau(t) \geq 0, \sup_{t\in R_+} \lambda(t) \geq 0\}$, by (3.3) we can directly from the first equation of (1.1)

$$\begin{aligned}\frac{dx(t)}{dt} &\leq x(t)\left(a(t) - b(t)x(t) \exp \int_t^{t-\tau(t)} a(u)du - c(t)\left(\int_{-t}^{0} H(s) \exp \int_t^{t+s} a(u)du\, ds\right)x(t)\right)\\ &= x(t)(a(t) - \theta(t)x(t)),\end{aligned} \tag{3.4}$$

where $\theta(t) = b(t) \exp \int_t^{t-\tau(t)} a(u)du + c(t) \int_{-t}^{0} H(s) \exp \int_t^{t+s} a(u)\, du\, ds$. Since for any $t \geq \tau^0$ and $s \in R_-$

$$\int_t^{t-\tau(t)} a(u)du \geq -\tau^0 a^0, \qquad \int_t^{t+s} a(u)du \geq s a^0. \tag{3.5}$$

We have

$$d(t) \geq b(t) \exp\left(-\tau^0 a^0\right) + c(t) \int_{-t}^{0} H(s) \exp\left(a^0 s\right) ds. \tag{3.6}$$

Let

$$\begin{aligned}f(t) &= b(t) \exp\left(-\tau^0 a^0\right) + c(t) \int_{-t}^{0} H(s) \exp\left(a^0 s\right) ds,\\ g(t) &= b(t) \exp\left(-\tau^0 a^0\right) + c(t) \int_{-\infty}^{0} H(s) \exp\left(a^0 s\right) ds.\end{aligned} \tag{3.7}$$

Since $0 < a^0$, we have

$$\begin{aligned}0 &< \int_{-\infty}^{0} H(s) \exp\left(a^0 s\right) ds < \int_{-\infty}^{0} H(s) ds < \infty,\\ \lim_{t\to\infty} (g(t) - f(t)) &= \lim_{t\to\infty} c(t) \int_{-\infty}^{-t} H(s) \exp\left(a^0 s\right) ds = 0.\end{aligned} \tag{3.8}$$

Hence, assumption (H_2) or (H_3) implies

$$\begin{aligned}\liminf_{t\to\infty}\int_t^{t+\sigma} f(s)ds &= \liminf_{t\to\infty}\left[\int_t^{t+\sigma} g(s)ds+\int_t^{t+\sigma}\left(f(s)-g(s)\right)ds\right]\\ &= \liminf_{t\to\infty}\int_t^{t+\sigma} g(s)ds>0,\end{aligned}\tag{3.9}$$

where the constant $\sigma=\beta$ or γ. Since $\theta(t)\geq f(t)$ for all $t\geq\tau^0$, we finally obtain

$$\liminf_{t\to\infty}\int_t^{t+\delta}\theta(s)ds\geq\liminf_{t\to\infty}\int_t^{t+\delta} f(s)ds>0.\tag{3.10}$$

We consider the following auxiliary equation:

$$\frac{dy(t)}{dt}\leq y(t)\left(a(t)-\theta(t)y(t)\right),\tag{3.11}$$

then by conclusion (a) of Lemma 2.1 and inequality (3.10) we obtain that there exist a constant M_1 such that $\limsup_{t\to\infty} y(t)\leq M_1$ for any solution of (3.11) with initial condition $y(0)>0$. Let $y^*(t)$ be the solution of (3.11) with initial condition $y^*(\tau^0)=x(\tau^0)$, then by the comparison theorem, we have from (3.4) that

$$x(t)<y^*(t),\quad \forall t>\tau^0.\tag{3.12}$$

Thus, we finally obtain that

$$\limsup_{t\to\infty} x(t)<M_1.\tag{3.13}$$

Moreover, by Lemma 2.4, for any positive constant $\varepsilon>0$, there exists a constant $T>0$ such that

$$\int_{-\infty}^0 H(s)x^2(t+s)ds<M_1^2+\varepsilon,\quad \forall t>T.\tag{3.14}$$

Considering the second equation of the system (1.1), we have

$$\frac{du(t)}{dt}\leq -d(t)u(t)+e(t)\left(M_1^2+\varepsilon\right).\tag{3.15}$$

We consider the following auxiliary equations:

$$\frac{dv(t)}{dt}=-d(t)v(t)+e(t)\left(M_1^2+\varepsilon\right),\tag{3.16}$$

then, by assumption (H_4) and conclusion (a) of the Lemma 2.2, we obtain that there is a constant M_2 such that $\limsup_{t\to\infty} v(t) \le M_2$ for any solution of (3.16) with initial condition $v(0) > 0$. Let $v^*(t)$ be the solution of (3.16) with initial condition $v^*(T') = u(T')$ and $T' > T$, then, by comparison theorem, we have from (3.15) that

$$u(t) \le v^*(t), \quad \forall t > T'. \tag{3.17}$$

Thus, we finally obtain that

$$\limsup_{t\to\infty} v(t) < M_2. \tag{3.18}$$

Choose the constant $M = \max\{M_1, M_2\}$, then we finally obtain

$$\limsup_{t\to\infty} x(t) < M, \qquad \limsup_{t\to\infty} u(t) < M. \tag{3.19}$$

This completes the proof. □

Theorem 3.2. *Suppose that assumptions (H_1)–(H_5) hold. Then there exists a constant $\eta_x > 0$ which is independent of the solution of system* (1.1) *such that*

$$\liminf_{t\to\infty} x(t) \ge \eta_x, \tag{3.20}$$

for any positive solutions $(x(t), u(t))$ of system (1.1).

Proof. Let $(x(t), u(t))$ be a solution of system (1.1); from Theorem 3.1 there exists a T_0 such that for all $t > T_0$ we have $x(t) \le M, u(t) \le M$. According to the assumption (H_1) we can choose positive constants $\varepsilon > 0$ and $T_0 > 0$ such that, for all $t \ge T_0$, we have

$$\int_t^{t+\delta} (a(s) - b(s)\varepsilon - 2c(s)\varepsilon - h(s)\varepsilon - r(s)\varepsilon)ds \ge \varepsilon. \tag{3.21}$$

Consider the following equation:

$$\frac{dv(t)}{dt} = -d(t)v(t) + e(t)\left(\alpha_0 + \alpha_0^2\right), \tag{3.22}$$

where $\alpha_0 > 0$ is a parameter. By Lemma 2.3, for given in above $\varepsilon > 0$ and positive constant M (Theorem 3.1), there exist constants $\delta_0 = \delta_0(\varepsilon)$ and $\widetilde{T}_0 = \widetilde{T}_0(M) > 0$ such that for any $t_0 \in R_+$ and $0 \le v_0 \le M$, when $e(t)(\alpha_0 + \alpha_0^2) < \delta_0$, for all $t \ge t_0$, we have

$$0 \le v(t, t_0, u_0) < \varepsilon \quad \forall t \ge t_0 + \widetilde{T}_0, \tag{3.23}$$

where $v(t,t_0,u_0)$ is the solution of (3.22) with initial condition $v(t_0) = v_0$. Hence, we can choose a positive constant $\alpha_0 \leq \min\{\varepsilon, \delta_0/2(e^0+1)\}$ such that, for all $t \geq T_0$,

$$\int_t^{t+\delta} (a(s) - b(s)\alpha_0 - 2c(s)\alpha_0 - h(s)\varepsilon - r(s)\varepsilon)ds \geq \alpha_0. \tag{3.24}$$

We first prove that

$$\limsup_{t\to\infty} x(t) \geq \alpha_0. \tag{3.25}$$

In fact, if (3.25) is not true, then there exists a constant $T_1 \geq T_0$ such that $x(t) < \alpha_0$ for all $t \geq T_1$. Choose constants $\tau_1 > \tau_0$, $\tau_2 > \tau_1$, such that

$$\int_{-\infty}^{-\tau_1} H(s)ds < \frac{\alpha_0}{M_0^2}, \qquad \int_{-\infty}^{-\tau_2} H(s)ds < \frac{\alpha_0}{M_0}, \tag{3.26}$$

where $M_0 = \sup\{x(t+s) : t \in R_+,\ S \in R_-\}$, then for $t \geq T_1 + \tau_1$ we have

$$\begin{aligned} u'(t) &= -d(t)u(t) + e(t)\left(\int_{-\infty}^{-\tau_1} + \int_{-\tau_1}^{0}\right) H(s)x^2(t+s)ds \\ &\leq -d(t)u(t) + e(t)\left(\frac{\alpha_0}{M_0^2}M_0^2 + \alpha_0^2\right) \\ &\leq -d(t)u(t) + e(t)\left(\alpha_0 + \alpha_0^2\right). \end{aligned} \tag{3.27}$$

Let $v(t)$ be the solution of (3.22) with initial condition $v(T_1+\tau_1) = u(T_1+\tau_1)$, then by the comparison theorem we have

$$u(t) \leq v(t), \quad \forall t \geq T_1 + \tau_1. \tag{3.28}$$

In (3.23), we choose $t_0 = T_1 + \tau_1$ and $v_0 = u(T_1+\tau_1)$, since $(\alpha_0 + \alpha_0^2)e(t) < \delta_0$ for all $t \geq T_1 + \tau_1$, we have

$$v(t) = v(t, T_1+\tau_1, v(T_1+\tau_1)) < \varepsilon \quad \forall t \geq T_1 + \tau_1 + \widetilde{T}_0. \tag{3.29}$$

Hence, we further have

$$u(t) < \varepsilon \quad \forall t \geq T_1 + \tau_1 + \widetilde{T}_0. \tag{3.30}$$

For any $t \geq T_1 + \tau_1 + \widetilde{T}_0 + \tau_2$, from the first equation of (1.1) we have

$$\begin{aligned} x'(t) &= x(t)\left\{ a(t) - b(t)x(t-\tau(t)) - c(t)\left(\int_{-\infty}^{-\tau_2} + \int_{-\tau_2}^{0}\right) H(s)x(t+s)ds - h(t)u(t) - r(t)u(t-\lambda(t)) \right\} \\ &\geq x(t)(a(s) - b(s)\alpha_0 - 2c(s)\alpha_0 - h(s)\varepsilon - r(s)\varepsilon). \end{aligned} \tag{3.31}$$

Integrating (3.31) from $T_1 + \tau_1 + \widetilde{T}_0 + \tau_2$ to $t \geq T_1 + \tau_1 + \widetilde{T}_0 + \tau_2$ we obtain

$$x(t) \geq x\left(T_1 + \tau_1 + \widetilde{T}_0 + \tau_2\right) \exp\left\{ \int_{T_1+\tau_1+\widetilde{T}_0+\tau_2}^{t} (a(s) - b(s)\alpha_0 - 2c(s)\alpha_0 - h(s)\varepsilon - r(s)\varepsilon)ds \right\}. \tag{3.32}$$

Obviously, inequality (3.24) implies that $x(t) \to \infty$ as $t \to \infty$, which leads to a contradiction.

Now, we prove the conclusion of Theorem 3.2 to hold. Assume that it is not true, then there exists a sequence $\{Z^{(m)}\} = \{(\varphi^{(m)}, \psi^{(m)})\}$ of initial functions of system (1.1) such that

$$\liminf_{t\to\infty} x\left(t, Z^{(m)}\right) < \frac{\alpha_0}{(m+1)^2} \quad \forall m = 1, 2, \ldots, \tag{3.33}$$

where $(x(t, Z^{(m)}), u(t, Z^{(m)}))$ is the solution of system (1.1) with initial condition

$$x(s) = \varphi^{(m)}(s), \quad u(s) = \psi^{(m)}(s) \quad \forall s \in R_-. \tag{3.34}$$

From (3.25) and (3.33), we obtain that for every m there are two time sequences $\{s_q^{(m)}\}$ and $\{t_q^{(m)}\}$, satisfying

$$0 < s_1^{(m)} < t_1^{(m)} < s_2^{(m)} < t_2^{(m)} < \cdots < s_q^{(m)} < t_q^{(m)} < \cdots, \tag{3.35}$$

$$s_q^{(m)} \longrightarrow \infty, \qquad t_q^{(m)} \longrightarrow \infty \quad \text{as } q \longrightarrow \infty, \tag{3.36}$$

$$x\left(s_q^{(m)}, \phi^{(m)}, \psi^{(m)}\right) = \frac{\alpha_0}{m+1}, \qquad x\left(t_q^{(m)}, \phi^{(m)}, \psi^{(m)}\right) = \frac{\alpha_0}{(m+1)^2}, \tag{3.37}$$

$$\frac{\alpha_0}{(m+1)^2} \leq x\left(t, \phi^{(m)}, \psi^{(m)}\right) \leq \frac{\alpha_0}{m+1} \quad \forall t \in \left(s_q^{(m)}, t_q^{(m)}\right). \tag{3.38}$$

Let $M_m = \sup\{x(t+s, \phi^{(m)}, \psi^{(m)}) : t \in R_+, S \in R_-\}$. For each $m = 1, 2, \ldots$ we can choose a constant $\tau_1^{(m)} > \tau^0$ such that

$$\int_{-\infty}^{-\tau_1^{(m)}} H(s)x\left(t+s, \phi^{(m)}, \psi^{(m)}\right)ds \leq M_m \int_{-\infty}^{-\tau_1^{(m)}} H(s)ds < \alpha_0. \tag{3.39}$$

By Theorem 3.1, there exists a constant M such that for each $m = 1, 2, \ldots$ there exists a $T_1^{(m)} > T_0$ such that

$$x\left(t, \phi^{(m)}, \psi^{(m)}\right) \leq M, \quad u\left(t, \phi^{(m)}, \psi^{(m)}\right) \leq M \quad \forall t > T_1^{(m)}. \tag{3.40}$$

Further, from (3.36) there is an integer $K_1^{(m)} > 0$ such that $s_q^{(m)} > T_1^{(m)} + \tau_1^{(m)}$ for all $q \geq K_1^{(m)}$ for all $m = 1, 2, \ldots$. Hence, for any $t \in (s_q^{(m)}, t_q^{(m)})$ and $q \geq K_1^{(m)}$ by (3.39) and (3.40) we have

$$\begin{aligned}
&\frac{dx(t, \phi^{(m)}, \psi^{(m)})}{dt} \\
&\quad \geq x\left(t, \phi^{(m)}, \psi^{(m)}\right)\left\{a(t) - b(t)M - c(t)\left(\int_{-\infty}^{-\tau_1^{(m)}} + \int_{-\tau_1^{(m)}}^{0}\right)H(s)x(t+s)ds - h(t)M - r(t)M\right\} \\
&\quad \geq x\left(t, \phi^{(m)}, \psi^{(m)}\right)(a(t) - b(t)M - c(t)(\alpha_0 + M) - h(t)M - r(s)M) \\
&\quad \geq -\gamma_1 x\left(t, \phi^{(m)}, \psi^{(m)}\right),
\end{aligned} \tag{3.41}$$

where $\gamma_1 = \sup_{t \in R_+}\{|a(t)| + b(t)M + c(t)(\alpha_0 + M_0) + r(t)M + h(t)M\} > 0$. Therefore, for any $q \geq K_1^{(m)}$ and $m = 1, 2, \ldots$ integrating the above inequality from $s_q^{(m)}$ to $t_q^{(m)}$, we further have

$$\begin{aligned}
\frac{\alpha_0}{(m+1)^2} = x\left(t_q^{(m)}, \phi^{(m)}, \psi^{(m)}\right) &\geq x\left(s_q^{(m)}, \phi^{(m)}, \psi^{(m)}\right)\exp\left\{-\gamma_1\left(t_q^{(m)} - s_q^{(m)}\right)\right\} \\
&= \frac{\alpha_0}{m+1}\exp\left\{-\gamma_1\left(t_q^{(m)} - s_q^{(m)}\right)\right\}.
\end{aligned} \tag{3.42}$$

Consequently,

$$t_q^{(m)} - s_q^{(m)} \geq \frac{\ln(m+1)}{\gamma_1} \quad \forall q \geq K_1^{(m)}, \; m = 1, 2, \ldots. \tag{3.43}$$

For any $q \geq K_1^{(m)}$ and $t > s_q^{(m)}$ we can obtain

$$\begin{aligned}
&\int_{-\infty}^{T_1^{(m)}} H(\theta - t)x\left(\theta, \phi^{(m)}, \psi^{(m)}\right)d\theta \leq M_m \int_{-\infty}^{T_1^{(m)} - t} H(s)ds, \\
&\int_{-\infty}^{T_1^{(m)}} H(\theta - t)x^2\left(\theta, \phi^{(m)}, \psi^{(m)}\right)d\theta \leq M_m^2 \int_{-\infty}^{T_1^{(m)} - t} H(s)ds,
\end{aligned}$$

$$\int_{T_1^{(m)}}^{s_q^{(m)}} H(\theta - t)x\left(\theta, \phi^{(m)}, \varphi^{(m)}\right)d\theta \le M \int_{-\infty}^{s_q^{(m)}-t} H(s)ds,$$
$$\int_{T_1^{(m)}}^{s_q^{(m)}} H(\theta - t)x^2\left(\theta, \phi^{(m)}, \varphi^{(m)}\right)d\theta \le M^2 \int_{-\infty}^{s_q^{(m)}-t} H(s)ds. \tag{3.44}$$

For each $m = 1, 2, \ldots$ by (3.36) there exists a $K_2^{(m)} \ge K_1^{(m)}$ and constant $L > \tau^0$ such that

$$M_m \int_{-\infty}^{T_1^{(m)}-s_q^{(m)}} H(s)ds < M_m^2 \int_{-\infty}^{T_1^{(m)}-s_q^{(m)}} H(s)ds \le \frac{\alpha_0}{2}, \quad \forall q \ge K_2^{(m)},$$
$$M \int_{-\infty}^{-L} H(s)ds < M^2 \int_{-\infty}^{-L} H(s)ds \le \frac{\alpha_0}{2}. \tag{3.45}$$

By (3.43) there exists a large enough $\widehat{m}_1$ such that

$$t_q^{(m)} - s_q^{(m)} \ge L + \delta + \widetilde{T}_0, \quad \forall m \ge \widehat{m}_1, \ q \ge K_2^{(m)}. \tag{3.46}$$

Hence, for any $m \ge \widehat{m}_1$, $q \ge K_2^{(m)}$ and $t \in [s_q^{(m)} + L, t_q^{(m)}]$, by (3.38), (??), and (3.45) we have

$$\begin{aligned}
\frac{du(t, Z^{(m)})}{dt} &= -d(t)u\left(t, Z^{(m)}\right) + e(t)\left(\int_{-\infty}^{T_1^{(m)}} + \int_{T_1^{(m)}}^{s_q^{(m)}} + \int_{s_q^{(m)}}^{t}\right)H(\theta - t)x^2(\theta)d\theta \\
&\le -d(t)u\left(t, Z^{(m)}\right) + e(t)M_m^2 \int_{-\infty}^{T_1^{(m)}-t} H(\theta)d\theta + e(t)M \int_{-\infty}^{s_q^{(m)}-t} H(\theta)d\theta \\
&\quad + e(t)\frac{\alpha_0^2}{(m+1)^2}\int_{s_q^{(m)}}^{t} H(\theta - t)d\theta \\
&\le -d(t)u\left(t, Z^{(m)}\right) + e(t)\left(\frac{\alpha_0}{2} + \frac{\alpha_0}{2} + \frac{\alpha_0^2}{(m+1)^2}\right) \\
&\le -d(t)u\left(t, Z^{(m)}\right) + e(t)\left(\alpha_0 + \alpha_0^2\right).
\end{aligned} \tag{3.47}$$

Assume that $v(t)$ is the solution of (3.22) satisfying initial condition $v(s_q^{(m)} + L) = u(s_q^{(m)} + L)$, then we have

$$u\left(t, Z^{(m)}\right) \le v(t), \quad \forall t \in \left[s_q^{(m)} + L, t_q^{(m)}\right], \ m \ge \widehat{m}_1, \ q \ge K_2^{(m)}. \tag{3.48}$$

In (3.23), we choose $t_0 = s_q^{(m)} + L$ and $v_0 = u(s_q^{(m)} + L)$, since $(\alpha_0 + \alpha_0^2)e(t) < \delta_0$ for all $t \geq s_q^{(m)} + L$, we have

$$v(t) = v\left(t, s_q^{(m)} + L, v\left(s_q^{(m)} + L\right)\right) < \varepsilon \quad \forall t \in \left[s_q^{(m)} + L + \widetilde{T}_0, t_q^{(m)}\right]. \tag{3.49}$$

Hence, we further have

$$u\left(t, Z^{(m)}\right) < \varepsilon \tag{3.50}$$

for all $t \in [s_q^{(m)} + L + \widetilde{T}_0, t_q^{(m)}]$, $q \geq K_2^{(m)}$ and $m \geq \widehat{m}_1$.

For any $q \geq K_2^{(m)}$, $m \geq \widehat{m}_1$, and $t \in [s_q^{(m)} + L + \widetilde{T}_0, t_q^{(m)}]$, from (3.38)–(3.40), we have

$$\begin{aligned}
&\frac{dx(t, \phi^{(m)}, \varphi^{(m)})}{dt} \\
&\quad \geq x\left(t, \phi^{(m)}, \varphi^{(m)}\right) \Bigg\{ a(t) - b(t)x(t - T(t)) \\
&\qquad\qquad - c(t)\left(\int_{-\infty}^{T_1^{(m)}} + \int_{T_1^{(m)}}^{s_q^{(m)}} + \int_{s_q^{(m)}}^{t}\right) H(s)x(t+s)ds - h(t)\alpha_0 - r(t)\alpha_0 \Bigg\} \\
&\quad > x(t)\Bigg(a(t) - b(t)\frac{\alpha_0}{m+1} - c(t)M_m \int_{-\infty}^{T_1^{(m)} - t} H(\theta)ds - c(t)M \int_{T_1^{(m)}}^{s_q^{(m)} - t} H(\theta)ds \\
&\qquad\qquad - c(t)\frac{\alpha_0}{m+1}\int_{s_q^{(m)}}^{t} H(\theta - t)ds - h(t)\varepsilon - r(t)\varepsilon \Bigg) \\
&\quad \geq x(t)(a(t) - b(t)\alpha_0 - 2c(t)\alpha_0 - h(t)\varepsilon - r(t)\varepsilon).
\end{aligned} \tag{3.51}$$

Integrating the above inequality from $t_q^{(m)} - \delta$ to $t_q^{(m)}$, then by (3.24), (3.37) and (3.38) we obtain

$$\begin{aligned}
\frac{\alpha_0}{(m+1)^2} &= x\left(t_q^{(m)}, Z^{(m)}\right) \\
&\geq x\left(t_q^{(m)} - \delta, Z^{(m)}\right) \exp \int_{t_q^{(m)} - \delta}^{t_q^{(m)}} (a(s) - b(s)\alpha_0 - 2c(s)\alpha_0 - h(s)\varepsilon - r(s)\varepsilon)ds \\
&\geq \frac{\alpha_0}{(m+1)^2}\exp\{\alpha_0\} > \frac{\alpha_0}{(m+1)^2}
\end{aligned} \tag{3.52}$$

which leads to a contradiction. Therefore, this contradiction shows that there exists constant $\eta_x > 0$ such that

$$\liminf_{t \to \infty} x(t) \geq \eta_x, \tag{3.53}$$

for any positive solutions $(x(t), u(t))$ of system (1.1). This completes the proof. □

Applying Theorem 3.2 to system (1.4), we have the following corollary.

Corollary 3.3. *Suppose that assumptions* (H_1)–(H_5) *hold, then system* (1.4) *is permanent.*

Obviously, we will first consider system (1.1) which is more general than system (1.4). Moreover, Corollary 3.3 is a very good improvement of Theorem 2.1 in [2]. From Corollary 3.3, we find that it is established that a very weak sufficient condition for the permanent of system (1.4).

Remark 3.4. From Theorem 3.2 we directly see that for system (1.1) the feedback control is harmless to the permanence of system (1.1).

Remark 3.5. In this paper, the biological model about a single species is considered. However, complex networks have attracted increasing attention from various fields of science and engineering in recent years. Meanwhile, in some sense, the coupled species systems can be treated as a typical complex networks. The real world biological systems have more complex structures and relationships. Motivated by above work [18–21], our future work is that how to apply the current complex networks theory to improve the current work.

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