Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2012, Article ID 860921, 9 pages
doi:10.1155/2012/860921

Research Article

Some Relations of the Twisted q-Genocchi Numbers and Polynomials with Weight α and Weak Weight β

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Received 12 February 2012; Accepted 9 March 2012

Academic Editor: Natig Atakishiyev

Recently many mathematicians are working on Genocchi polynomials and Genocchi numbers. We define a new type of twisted q-Genocchi numbers and polynomials with weight α and weak weight β and give some interesting relations of the twisted q-Genocchi numbers and polynomials with weight α and weak weight β. Finally, we find relations between twisted q-Genocchi zeta function and twisted Hurwitz q-Genocchi zeta function.

1. Introduction

The Genocchi numbers and polynomials possess many interesting properties and arising in many areas of mathematics and physics. Recently, many mathematicians have studied in the area of the q-Genocchi numbers and polynomials (see [1–16]).

Throughout this paper we use the following notations. By $\mathbb{Z}_p$ we denote the ring of p-adic rational integers, $\mathbb{Q}_p$ denotes the field of p-adic rational numbers, $\mathbb{C}_p$ denotes the completion of algebraic closure of $\mathbb{Q}_p$, $\mathbb{N}$ denotes the set of natural numbers, $\mathbb{Z}$ denotes the ring of rational integers, $\mathbb{Q}$ denotes the field of rational numbers, $\mathbb{C}$ denotes the set of complex numbers, and $\mathbb{Z}^+ = \mathbb{N} \cup \{0\}$. Let ν_p be the normalized exponential valuation of $\mathbb{C}_p$ with $|p|_p = p^{-\nu_p(p)} = p^{-1}$. When one talks of q-extension, q is considered in many ways such as an indeterminate, a complex number $q \in \mathbb{C}$, or p-adic number $q \in \mathbb{C}_p$. If $q \in \mathbb{C}$, one normally assume that $|q| < 1$. If $q \in \mathbb{C}_p$, we normally assume that $|q-1|_p < p^{-1/(p-1)}$ so that $q^x = \exp(x \log q)$ for $|x|_p \leq 1$. Throughout this paper we use the following notation:

$$[x]_q = \frac{1-q^x}{1-q}, \qquad [x]_{-q} = \frac{1-(-q)^x}{1+q} \tag{1.1}$$

(cf. [1–13]).

Hence, $\lim_{q\to 1}[x] = x$ for any x with $|x|_p \leq 1$ in the present p-adic case.

For

$$f \in UD(\mathbb{Z}_p) = \{f \mid f : \mathbb{Z}_p \longrightarrow \mathbb{C}_p \text{ is uniformly differentiable function}\}, \tag{1.2}$$

the fermionic p-adic q-integral on $\mathbb{Z}_p$ is defined by Kim as follows:

$$I_{-q}(f) = \int_{\mathbb{Z}_p} f(x)d\mu_{-q}(x) = \lim_{N\to\infty} \frac{1}{[p^N]_{-q}} \sum_{x=0}^{p^N-1} f(x)(-q)^x \tag{1.3}$$

(cf. [11–14]).

If we take $f_1(x) = f(x+1)$ in (1.1), then we easily see that

$$qI_{-q}(f_1) + I_{-q}(f) = [2]_q f(0). \tag{1.4}$$

From (1.4), we obtain

$$q^n I_{-q}(f_n) + (-1)^{n-1} I_{-q}(f) = [2]_q \sum_{l=0}^{n-1} (-1)^{n-1-l} q^l f(l), \tag{1.5}$$

where $f_n(x) = f(x+n)$ (cf. [5–9]).

Let $C_{p^n} = \{w \mid w^{p^n} = 1\}$ be the cyclic group of order p^n and let

$$T_p = \lim_{n\to\infty} C_{p^n} = C_{p^\infty} = \cup_{n\geq 0} C_{p^n} \tag{1.6}$$

be the locally constant space. For $w \in T_p$, we denote by $\phi_w : \mathbb{Z}_p \to \mathbb{C}_p$ the locally constant function $x \mapsto w^x$.

As well-known definition, the Genocchi polynomials are defined by

$$\begin{gathered} F(t) = \frac{2t}{e^t+1} = e^{Gt} = \sum_{n=0}^{\infty} G_n \frac{t^n}{n!}, \\ F(t,x) = \frac{2t}{e^t+1} e^{xt} = e^{G(x)t} = \sum_{n=0}^{\infty} G_n(x) \frac{t^n}{n!}, \end{gathered} \tag{1.7}$$

with the usual convention of replacing $G^n(x)$ by $G_n(x)$. $G_n(0) = G_n$ are called the nth Genocchi numbers (cf. [2–5, 14]).

These numbers and polynomials are interpolated by the Genocchi zeta function and Hurwitz-type Genocchi zeta function, respectively:

$$\zeta_G(s) = 2\sum_{n=1}^{\infty}\frac{(-1)^n}{n^s},$$
$$\zeta_G(s,x) = 2\sum_{n=0}^{\infty}\frac{(-1)^n}{(n+x)^s}. \tag{1.8}$$

Our aim in this paper is to define twisted q-Genocchi numbers $G_{n,q,w}^{(\alpha,\beta)}$ and polynomials $G_{n,q,w}^{(\alpha,\beta)}(x)$ with weight α and weak weight β. We investigate some properties which are related to $G_{n,q,w}^{(\alpha,\beta)}$ and $G_{n,q,w}^{(\alpha,\beta)}(x)$. We also derive the existence of a specific interpolation function which interpolate $G_{n,q,w}^{(\alpha,\beta)}$ and $G_{n,q,w}^{(\alpha,\beta)}(x)$ at negative integers.

2. Generating Functions of Twisted q-Genocchi Numbers and Polynomials with Weight α and Weak Weight β

Our primary goal of this section is to define twisted q-Genocchi numbers $G_{n,q,w}^{(\alpha,\beta)}$ and polynomials $G_{n,q,w}^{(\alpha,\beta)}(x)$ with weight α and weak weight β. We also find generating functions of $G_{n,q,w}^{(\alpha,\beta)}$ and $G_{n,q,w}^{(\alpha,\beta)}(x)$.

Definition 2.1. For $\alpha,\beta\in\mathbb{Q}$ and $q\in\mathbb{C}_p$ with $|1-q|_p\le 1$,

$$G_{n,q,w}^{(\alpha,\beta)} = n\int_{\mathbb{Z}_p}\phi_w(x)[x]_{q^\alpha}^{n-1}d\mu_{-q^\beta}(x). \tag{2.1}$$

We call $G_{n,q,w}^{(\alpha,\beta)}$ twisted q-Genocchi numbers with weight α and weak weight β.

By using p-adic q-integral on $\mathbb{Z}_p$, we obtain

$$\begin{aligned} n\int_{\mathbb{Z}_p}\phi_w(x)[x]_{q^\alpha}^{n-1}d\mu_{-q^\beta}(x) &= n\lim_{N\to\infty}\frac{1}{[p^N]_{-q^\beta}}\sum_{x=0}^{p^N-1}w^x[x]_{q^\alpha}^{n-1}\left(-q^\beta\right)^x \\ &= n[2]_{q^\beta}\sum_{m=0}^{\infty}(-1)^m q^{\beta m}w^m[m]_{q^\alpha}^{n-1}. \end{aligned} \tag{2.2}$$

From (2.1) and (2.2), we have

$$G_{n,q,w}^{(\alpha,\beta)} = n[2]_{q^\beta}\left(\frac{1}{1-q^\alpha}\right)^{n-1}\sum_{l=0}^{n-1}\binom{n-1}{l}(-1)^l\frac{1}{1+wq^{\beta+\alpha l}}. \tag{2.3}$$

We set

$$F_{q,w}^{(\alpha,\beta)}(t) = \sum_{n=0}^{\infty}G_{n,q,w}^{(\alpha,\beta)}\frac{t^n}{n!}. \tag{2.4}$$

By using the previous equation and (2.3), we have

$$\begin{aligned} F_{q,w}^{(\alpha,\beta)}(t) &= \sum_{n=0}^{\infty}\left(n[2]_{q^{\beta}}\left(\frac{1}{1-q^{\alpha}}\right)^{n-1}\sum_{l=0}^{n-1}\binom{n-1}{l}(-1)^{l}\frac{1}{1+wq^{\beta+\alpha l}}\right)\frac{t^{n}}{n!} \\ &= [2]_{q^{\beta}}t\sum_{m=0}^{\infty}(-1)^{m}q^{\beta m}w^{m}e^{[m]_{q^{\alpha}}t}. \end{aligned} \tag{2.5}$$

Thus twisted q-Genocchi numbers $G_{n,q,w}^{(\alpha,\beta)}$ with weight α and weak weight β are defined by means of the generating function:

$$F_{q,w}^{(\alpha,\beta)}(t) = [2]_{q^{\beta}}t\sum_{n=0}^{\infty}(-1)^{n}q^{\beta n}w^{n}e^{[n]_{q^{\alpha}}t}. \tag{2.6}$$

By using (2.2), we have

$$\sum_{n=0}^{\infty}G_{n,q,w}^{(\alpha,\beta)}\frac{t^{n}}{n!} = t\int_{\mathbb{Z}_p}\phi_{w}(x)e^{[x]_{q^{\alpha}}t}d\mu_{-q^{\beta}}(x). \tag{2.7}$$

From (2.5) and (2.7), we have

$$\sum_{n=0}^{\infty}G_{n,q,w}^{(\alpha,\beta)}\frac{t^{n}}{n!} = [2]_{q^{\beta}}t\sum_{m=0}^{\infty}(-1)^{m}q^{\beta m}w^{m}e^{[m]_{q^{\alpha}}t}. \tag{2.8}$$

Next, we introduce twisted q-Genocchi polynomials $G_{n,q,w}^{(\alpha,\beta)}(x)$ with weight α and weak weight β.

Definition 2.2. For $\alpha,\beta\in\mathbb{Q}$ and $q\in\mathbb{C}_p$ with $|1-q|_p\le 1$,

$$G_{n,q,w}^{(\alpha,\beta)}(x) = n\int_{\mathbb{Z}_p}\phi_{w}(y)[x+y]_{q^{\alpha}}^{n-1}d\mu_{-q^{\beta}}(y). \tag{2.9}$$

We call $G_{n,q,w}^{(\alpha,\beta)}(x)$ twisted q-Genocchi polynomials with weight α and weak weight β.

By using p-adic q-integral, we have

$$\begin{aligned} n\int_{\mathbb{Z}_p}\phi_{w}(y)[x+y]_{q^{\alpha}}^{n-1}d\mu_{-q^{\beta}}(y) &= n\lim_{N\to\infty}\frac{1}{[p^{N}]_{-q^{\beta}}}\sum_{y=0}^{p^{N}-1}w^{y}[x+y]_{q^{\alpha}}^{n-1}\left(-q^{\beta}\right)^{y} \\ &= n[2]_{q^{\beta}}\sum_{m=0}^{\infty}(-1)^{m}q^{\beta m}w^{m}[x+m]_{q^{\alpha}}^{n-1}. \end{aligned} \tag{2.10}$$

By using (2.9) and (2.10), we obtain

$$G_{n,q,w}^{(\alpha,\beta)}(x) = n[2]_{q^\beta}\left(\frac{1}{1-q^\alpha}\right)^{n-1}\sum_{l=0}^{n-1}\binom{n-1}{l}(-1)^l q^{\alpha x l}\frac{1}{1+wq^{\beta+\alpha l}}. \tag{2.11}$$

We set

$$F_{q,w}^{(\alpha,\beta)}(t,x) = \sum_{n=0}^{\infty} G_{n,q,w}^{(\alpha,\beta)}(x)\frac{t^n}{n!}. \tag{2.12}$$

By using the previous equation and (2.11), we have

$$\begin{aligned} F_{q,w}^{(\alpha,\beta)}(t,x) &= \sum_{n=0}^{\infty}\left(n[2]_{q^\beta}\left(\frac{1}{1-q^\alpha}\right)^{n-1}\sum_{l=0}^{n-1}\binom{n-1}{l}(-1)^l q^{\alpha x l}\frac{1}{1+wq^{\beta+\alpha l}}\right)\frac{t^n}{n!} \\ &= [2]_{q^\beta} t\sum_{m=0}^{\infty}(-1)^m q^{\beta m} w^m e^{[x+m]_{q^\alpha} t}. \end{aligned} \tag{2.13}$$

Thus twisted q-Genocchi polynomials $G_{n,q,w}^{(\alpha,\beta)}(x)$ with weight α and weak weight β are defined by means of the generating function:

$$F_{q,w}^{(\alpha,\beta)}(t,x) = [2]_{q^\beta} t\sum_{m=0}^{\infty}(-1)^m q^{\beta m} w^m e^{[x+m]_{q^\alpha} t}. \tag{2.14}$$

By using (2.9), we have

$$\sum_{n=0}^{\infty} G_{n,q,w}^{(\alpha,\beta)}(x)\frac{t^n}{n!} = t\int_{\mathbb{Z}_p}\phi_w(y)e^{[x+y]_{q^\alpha} t}\,d\mu_{-q^\beta}(y). \tag{2.15}$$

By (2.13) and (2.15) we have

$$\sum_{n=0}^{\infty} G_{n,q,w}^{(\alpha,\beta)}(x)\frac{t^n}{n!} = [2]_{q^\beta} t\sum_{m=0}^{\infty}(-1)^m q^{\beta m} w^m e^{[x+m]_{q^\alpha} t}. \tag{2.16}$$

Remark 2.3. In (2.14), we simply identify that

$$\begin{aligned} \lim_{q\to 1} F_{q,w}^{(\alpha,\beta)}(t,x) &= 2t\sum_{n=0}^{\infty}(-1)^n w^n e^{(x+n)t} \\ &= F_w(t,x). \end{aligned} \tag{2.17}$$

Observe that if $q \to 1$, then $F_{q,w}^{(\alpha,\beta)}(t) \to F_w(t)$ and $F_{q,w}^{(\alpha,\beta)}(t,x) \to F_w(t,x)$. Note that if $q \to 1$ and $w = 1$, then $G_{n,q,w}^{(\alpha,\beta)} \to G_n$ and $G_{n,q,w}^{(\alpha,\beta)}(x) \to G_n(x)$.

3. Some Relations between Twisted q-Genocchi Numbers and Polynomials with Weight α and Weak Weight β

By (2.11), we have the following complement relation.

Theorem 3.1. *One has the property of complement*

$$G_{n,q^{-1},w^{-1}}^{(\alpha,\beta)}(1-x) = (-1)^n w q^{\alpha(n-1)} G_{n,q,w}^{(\alpha,\beta)}(x). \tag{3.1}$$

Also, by (2.11), *we have the following distribution relation.*

Theorem 3.2. *For any positive integer* m(=odd)*, one has*

$$G_{n,q,w}^{(\alpha,\beta)}(x) = \frac{[2]_{q^\beta}}{[2]_{q^{\beta m}}}[m]_{q^\alpha}^{n-1}\sum_{i=0}^{m-1}(-1)^i w^i q^{\beta i} G_{n,q^m,w^m}^{(\alpha,\beta)}\left(\frac{i+x}{m}\right), \qquad n \in \mathbb{Z}^+. \tag{3.2}$$

Let $f(x) = tw^x e^{[x]_{q^\alpha} t}$*. Then by* (1.5)*, left-hand side is in the following form:*

$$q^{\beta n} I_{-q^\beta}(f_n) + (-1)^{n-1} I_{-q^\beta}(f) = \sum_{m=0}^{\infty}\left(q^{\beta n} w^n G_{m,q,w}^{(\alpha,\beta)}(n) + (-1)^{n-1} G_{m,q,w}^{(\alpha,\beta)}\right)\frac{t^m}{m!}. \tag{3.3}$$

And right-hand side in (1.5) *is in the following form:*

$$[2]_{q^\beta}\sum_{l=0}^{n-1}(-1)^{n-1-l} q^{\beta l} f(l) = \sum_{m=0}^{\infty}[2]_{q^\beta} m \sum_{l=0}^{n-1}(-1)^{n-1-l} q^{\beta l} w^l [l]_{q^\alpha}^{m-1}\frac{t^m}{m!}. \tag{3.4}$$

By (3.3) *and* (3.4)*, one easily sees that*

$$q^{\beta n} w^n G_{m,q,w}^{(\alpha,\beta)}(n) + (-1)^{n-1} G_{m,q,w}^{(\alpha,\beta)} = [2]_{q^\beta} m \sum_{l=0}^{n-1}(-1)^{n-1-l} q^{\beta l} w^l [l]_{q^\alpha}^{m-1}. \tag{3.5}$$

Hence, we have the following theorem.

Theorem 3.3 (Let $m \in \mathbb{Z}^+$). *If* $n \equiv 0 \pmod 2$*, then*

$$q^{\beta n} w^n G_{m,q,w}^{(\alpha,\beta)}(n) - G_{m,q,w}^{(\alpha,\beta)} = [2]_{q^\beta} m \sum_{l=0}^{n-1}(-1)^{l+1} q^{\beta l} w^l [l]_{q^\alpha}^{m-1}. \tag{3.6}$$

If $n \equiv 1 \pmod 2$*, then*

$$q^{\beta n} w^n G_{m,q,w}^{(\alpha,\beta)}(n) + G_{m,q,w}^{(\alpha,\beta)} = [2]_{q^\beta} m \sum_{l=0}^{n-1}(-1)^{l} q^{\beta l} w^l [l]_{q^\alpha}^{m-1}. \tag{3.7}$$

Since $[x+y]_{q^\alpha} = [x]_{q^\alpha} + q^{\alpha x}[y]_{q^\alpha}$, *one easily obtains that*

$$\begin{aligned} G_{n+1,q,w}^{(\alpha,\beta)}(x) &= (n+1)\int_{\mathbb{Z}_p} \phi_w(y)\,[x+y]_{q^\alpha}^n d\mu_{-q^\beta}(y) \\ &= (n+1)[2]_{q^\beta}\sum_{m=0}^{\infty}(-1)^m q^{\beta m} w^m [x+m]_{q^\alpha}^n. \end{aligned} \tag{3.8}$$

From (1.4), *one notes that*

$$\begin{aligned} [2]_{q^\beta} t &= q^\beta \int_{\mathbb{Z}_p} t w^{(x+1)} e^{[x+1]_{q^\alpha} t} d\mu_{-q^\beta}(x) + \int_{\mathbb{Z}_p} t w^x e^{[x]_{q^\alpha} t} d\mu_{-q^\beta}(x) \\ &= \sum_{n=0}^{\infty}\left(q^\beta w G_{n,q,w}^{(\alpha,\beta)}(1) + G_{n,q,w}^{(\alpha,\beta)}\right)\frac{t^n}{n!}. \end{aligned} \tag{3.9}$$

By using comparing coefficients of $t^n/n!$ in the previous equation, we easily obtain the following theorem.

Theorem 3.4. *For* $n \in \mathbb{Z}^+$, *one has*

$$q^\beta w G_{n,q,w}^{(\alpha,\beta)}(1) + G_{n,q,w}^{(\alpha,\beta)} = \begin{cases} [2]_{q^\beta}, & \text{if } n = 1, \\ 0, & \text{if } n \neq 1. \end{cases} \tag{3.10}$$

By (3.8) *and* (3.10), *we have the following corollary.*

Corollary 3.5. *For* $n \in \mathbb{Z}^+$, *one has*

$$q^{\beta-\alpha} w \left(q^\alpha G_{q,w}^{(\alpha,\beta)} + 1\right)^n + G_{n,q,w}^{(\alpha,\beta)} = \begin{cases} [2]_{q^\beta}, & \text{if } n = 1, \\ 0, & \text{if } n \neq 1, \end{cases} \tag{3.11}$$

with the usual convention of replacing $(G_{q,w}^{(\alpha,\beta)})^n$ *by* $G_{n,q,w}^{(\alpha,\beta)}$.

4. The Analogue of the Genocchi Zeta Function

By using q-Genocchi numbers and polynomials with weight α and weak weight β, q-Genocchi zeta function and Hurwitz q-Genocchi zeta functions are defined. These functions interpolate the q-Genocchi numbers and q-Genocchi polynomials with weight α and weak weight β, respectively. In this section we assume that $q \in \mathbb{C}$ with $|q| < 1$.

From (2.5), we note that

$$\left.\frac{d^{k+1}}{dt^{k+1}} F_{q,w}^{(\alpha,\beta)}(t)\right|_{t=0} = (k+1)[2]_{q^\beta}\sum_{m=0}^{\infty}(-1)^m q^{\beta m} w^m [m]_{q^\alpha}^k = G_{k+1,q,w}^{(\alpha,\beta)} \quad (k \in \mathbb{N}). \tag{4.1}$$

By using the previous equation, we are now ready to define q-Genocchi zeta functions.

Definition 4.1. Let $s \in \mathbb{C}$. One has

$$\zeta_{q,w}^{(\alpha,\beta)}(s) = [2]_{q^\beta} \sum_{n=1}^{\infty} \frac{(-1)^n q^{\beta n} w^n}{[n]_{q^\alpha}^s}. \tag{4.2}$$

Note that $\zeta_{q,w}^{(\alpha,\beta)}(s)$ is a meromorphic function on $\mathbb{C}$. Observe that if $q \to 1$, then $\lim_{q\to 1} \zeta_{q,w}^{(\alpha,\beta)}(s) = \zeta_w(s)$.

Theorem 4.2. *Relation between $\zeta_{q,w}^{(\alpha,\beta)}(s)$ and $G_{k,q,w}^{(\alpha,\beta)}$ is given by*

$$\zeta_{q,w}^{(\alpha,\beta)}(-k) = \frac{G_{k+1,q,w}^{(\alpha,\beta)}}{k+1}. \tag{4.3}$$

Observe that $\zeta_{q,w}^{(\alpha,\beta)}(s)$ interpolates $G_{k,q,w}^{(\alpha,\beta)}$ at nonnegative integers.

By using (2.14), one notes that

$$\frac{d^{k+1}}{dt^{k+1}} F_{q,w}^{(\alpha,\beta)}(t,x)\bigg|_{t=0} = (k+1)[2]_{q^\beta} \sum_{m=0}^{\infty} (-1)^m q^{\beta m} w^m [x+m]_{q^\alpha}^k = G_{k+1,q,w}^{(\alpha,\beta)}(x), \tag{4.4}$$

$$\left(\frac{d}{dt}\right)^{k+1} \left(\sum_{n=0}^{\infty} G_{n,q,w}^{(\alpha,\beta)}(x) \frac{t^n}{n!} \right)\Bigg|_{t=0} = G_{k+1,q,w}^{(\alpha,\beta)}(x), \quad \text{for } k \in \mathbb{N}. \tag{4.5}$$

By (4.5), we are now ready to define the twisted Hurwitz q-Genocchi zeta functions.

Definition 4.3. Let $s \in \mathbb{C}$. One has

$$\zeta_{q,w}^{(\alpha,\beta)}(s,x) = [2]_{q^\beta} \sum_{n=0}^{\infty} \frac{(-1)^n q^{\beta n} w^n}{[x+n]_{q^\alpha}^s}. \tag{4.6}$$

Note that $\zeta_{q,w}^{(\alpha,\beta)}(s,x)$ is a meromorphic function on $\mathbb{C}$. Observe that if $q \to 1$, then $\lim_{q\to 1} \zeta_{q,w}^{(\alpha,\beta)}(s,x) = \zeta_w(s,x)$.

Theorem 4.4. *Relation between $\zeta_{q,w}^{(\alpha)}(s,x)$ and $G_{k,q,w}^{(\alpha)}(x)$ is given by*

$$\zeta_{q,w}^{(\alpha,\beta)}(-k,x) = \frac{G_{k+1,q,w}^{(\alpha,\beta)}(x)}{k+1}. \tag{4.7}$$

Observe that $\zeta_{q,w}^{(\alpha,\beta)}(-k,x)$ interpolates $G_{k,q,w}^{(\alpha,\beta)}(x)$ at nonnegative integers.

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