Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2012, Article ID 735675, 9 pages
doi:10.1155/2012/735675

Research Article

Monostable-Type Travelling Wave Solutions of the Diffusive FitzHugh-Nagumo-Type System in $\mathbb{R}^N$

Chih-Chiang Huang

Department of Mathematics, National Taiwan University, Taipei 10617, Taiwan

Correspondence should be addressed to Chih-Chiang Huang, loveworldsteven@hotmail.com

Received 30 March 2012; Accepted 25 May 2012

Academic Editor: Norimichi Hirano

This paper is concerned with monostable-type travelling wave solutions of the diffusive FitzHugh-Nagumo-type system (FHN) in $\mathbf{R}^N$ for the two components u and v. By solving v in terms of u, this system can be reduced to a nonlocal single equation for u. When the diffusion coefficients in the system are equal, we construct travelling wave solutions for the non-local equation by the method of super- and subsolutions developed by Morita and Ninomiya (2008) Moreover, we propose a condition for γ, which is similar to the condition Reinecke and Sweers (1999) used to transform (FHN) into a quasimonotone system.

1. Introduction

In the present paper, we are concerned with the diffusive FitzHugh-Nagumo-type system (FHN) in $\mathbf{R}^N$ that is,

$$\begin{aligned} u_t &= u_{\xi\xi} + \Delta_y u + f(y,u) - v, \\ v_t &= d v_{\xi\xi} + \Delta_y v + \delta(u - \gamma v), \end{aligned} \tag{1.1}$$

where $(\xi, y) \in \mathbf{R}^N = \mathbf{R}^1 \times \mathbf{R}^{N-1}$, $N \geq 2$, $\delta, \gamma > 0$ and $d \geq 0$. A typical example of $f(y,u)$ is $f(y,u) = u(1-u)(u-\beta)$ for $0 < \beta < 1/2$. Throughout the paper we assume that f is a C^2 function in u and f, f_u, and f_{uu} are bounded in $\{(y,u) \mid y \in \Omega_y,\ |u| \leq K\}$ for some large constant $K > 0$. In addition, f satisfies (H1)–(H5).

FHN derived from the Hodgkin-Huxley model is a typical model for excitable media. In many fields, such as physics, chemistry, and biology, FHN has become one of the frequently used-reaction diffusion systems to describe interesting phenomena. The solutions of interest here are traveling wave solutions. Let $x = \xi - ct$, then travelling wave solutions of (1.1) satisfy

$$u_{xx} + cu_x + \Delta_y u + f(y,u) - v = 0, \tag{1.2}$$

$$dv_{xx} + cv_x + \Delta_y v + \delta(u - \gamma v) = 0. \tag{1.3}$$

Over the past decades, this system has been extensively studied. For instance, as $N = 1$, under different assumptions, Systems (1.2) and (1.3) admit standing pulses in [1–3], infinitely many periodic solutions in [3], fronts, back waves in [4, 5] and travelling pulses in [5]. For the higher dimension case $N \geq 2$, symmetric standing waves were established by Reinecke and Sweers [6] and Wei and Winter [7].

As $\gamma \to \infty$, if the solutions are assumed to be bounded, (1.2) and (1.3) tend to the single equation

$$u_{xx} + cu_x + \Delta_y u + f(y,u) = 0. \tag{1.4}$$

Let $f(y,u)$ be a C^2 function $g(u)$ which has the property that for some $\in (0,1)$ $g(0) = g(\theta) = g(1) = 0$, $g_u(0) < 0$, $g_u(\theta) > 0$, $g_u(1) < 0$, $g < 0$ on $(0,\theta)$ and $g > 0$ on $(\theta,1)$. In addition to the planar waves, (1.4) admits other types of solutions, including travelling curved fronts ($N = 2$), conical shapes and pyramidal shapes ($N \geq 3$) in [8–11]. Moreover, Hamel and Roquejoffre [12] established travelling wave solutions of (1.4) in $\mathbf{R}^2$ which connect one unstable periodic solution at $x \to \infty$ $(-\infty)$ and one stable constant solution at $x \to -\infty$ (∞). On the other hand, travelling wave solutions of (1.4) in $\mathbf{R}^N$ connecting a unstable one-peak solution at $x \to \infty$ $(-\infty)$ and a stable constant solution $x \to -\infty$ (∞) were obtained by Morita and Ninomiya [13].

In this paper, we use the method of super- and subsolutions developed in [13]. Due to technical restriction, we assume $d = 1$. Since (1.3) is linear, v can be solved formally in terms of u. With v expressed in terms of u, Systems (1.2) and (1.3) are reduced to the non-local equation

$$\mathcal{F}[u] := u_{xx} + cu_x + \Delta_y u + f(y,u) - B_c[u] = 0, \tag{1.5}$$

where we denote v by $B_c[u] := \delta(-\partial^2/\partial x^2 - c(\partial/\partial x) - \Delta_y + \delta\gamma)^{-1}u$. It is readily seen that if u is independent of x, then by the uniqueness theorem $B_c[u] = \delta(-\Delta_y + \delta\gamma)^{-1}u$. As $x \to \pm\infty$, the asymptotic behaviors of travelling wave solutions of (1.5) formally satisfy

$$\Delta_y u + f(y,u) - B_c[u] = 0, \tag{1.6}$$

where $B_c[u] = \delta(-\Delta_y + \delta\gamma)^{-1}u$. Our main purpose is to look for monostable-type travelling wave solutions $u(x,y)$ which connect a stable solution of (1.6) as $x \to -\infty$ (∞) and a unstable one as $x \to \infty$ $(-\infty)$. Without loss of generality, we may assume that $u(+\infty,y)$ is an unstable solution. Throughout this paper, the following hypotheses are assumed.

(H1) There are two solutions $u_{\pm}(y)$ of (1.6) satisfying $u_-(y) \geq u_+(y)$. Moreover, there exist an eigenvalue $\mu > 0$ and its corresponding eigenfunction $\phi(y) > 0$ with $\max_{\{y\in\mathbf{R}^{N-1}\}}\phi(y) = 1$ and $\lim_{|y|\to\infty}\phi(y) = 0$ such that

$$\Delta_y\phi + f_u(y,u_+)\phi - B_c[\phi] = \mu\phi. \tag{1.7}$$

(H2) There exists no other solution $u(y)$ of (1.6) with the property $u_-(y) \geq u(y) \geq u_+(y)$.

(H3) $u_-(y) \geq u_+(y) + \epsilon\phi(y)$ for some $\epsilon > 0$.

(H4) For all small $\eta > 0$, there exists solutions $u_+^{\eta}(y)$ satisfying $\lim_{\eta\to 0}u_+^{\eta}(y) = u_+(y)$,

$$\begin{gathered}\Delta_y u_+^{\eta} + f\left(y,u_+^{\eta}\right) - B_c\left[u_+^{\eta}\right] + \eta = 0,\\ u_+^{\eta}(y) \geq u_+(y) + \frac{\eta}{M},\end{gathered} \tag{1.8}$$

for some constant $M > 0$.

(H5)

$$\Delta_y\psi_i - \left(K_1 + \sqrt{\delta}\right)\psi_i \leq 0, \quad i = 1,2,3, \tag{1.9}$$

where $K_1 = -\min_{\{u_-(y)\geq u\geq u_+(y),y\in\mathbf{R}^{N-1}\}}f_u(y,u) > 0$, $\psi_1 = \phi$, $\psi_2 = u_- - u_+$ and $\psi_3 = u_+^{\eta} - u_+$.

To simplify the proof of the main theorem in this paper, we modify the nonlinear term $f(y,u)$ such that the minimum and maximum of $f_u(y,u)$ in $\{u(y) \in \mathbf{R},\ y \in \mathbf{R}^{N-1}\}$ are the same as those in $\{u_-(y) \geq u \geq u_+(y),\ y \in \mathbf{R}^{N-1}\}$. For convenience, we still denote $f(y,u)$ for the new modification of f. Let $K^* := \max_{\{u_-(y)\geq u\geq u_+(y),y\in\mathbf{R}^{N-1}\}}f_u(y,u) > 0$ and let $K_2 > 0$ satisfy $K_2 + \delta/(\delta\gamma + K_2) = K^*$. We state the main theorem as follows.

Theorem 1.1. *Assume $\gamma \geq 2/\sqrt{\delta} + (K_1 + \mu)/\delta$ and* (H1)–(H5) *hold. Then there exists $c^* = \max\{2\sqrt{\mu}, 2\sqrt{K_2}\} > 0$ such that for all $c \geq c^*$, Systems* (1.2) *and* (1.3) *admit a pair of smooth solutions (u^*,v^*) which satisfie $u_x^* \leq 0$, $v_x^* \leq 0$ and the boundary conditions $(u^*,v^*)(\pm\infty,y) = (u_{\pm}(y),v_{\pm}(y))$, where $v_{\pm}(y) = B_c[u_{\pm}(y)]$.*

Remark 1.2. In (H1), when the inequality $u_-(y) \geq u_+(y)$ is reversed, that is, $u_-(y) \leq u_+(y)$, a result similar to Theorem 1.1 can be proved except that the inequalities $u_x^* \leq 0$ and $v_x^* \leq 0$ in Theorem 1.1 need to be replaced by $u_x^* \geq 0$ and $v_x^* \geq 0$, respectively.

Remark 1.3. In fact, (H5) can be weakened to the following assumption:

$$\Delta_y\psi_i - M_i\psi_i \leq 0, \quad \text{for some constants } M_i > 0. \tag{1.10}$$

This condition holds if $\Delta_y\psi_i$ does not decay faster than ψ_i as $|y| \to \infty$. In this case, if we choose $\gamma \geq 1/\sqrt{\delta} + (K_3 + \mu)/\delta$, where $K_3 = \max\{M_1, M_2, M_3, K_1 + \sqrt{\delta}\}$, then a similar result can be proved.

It is not easy to find an example which satisfies assumptions (H1)–(H5) even for the case $f(y,u) = u(1-u)(u-\beta)$ since the stability of the radially symmetric solutions obtained in [6, 7] has not yet been studied. However, we believe that for $\gamma \gg 1$ the structure of System (1.2) and (1.3) are similar to that of (1.5). Accordingly, we extend the result of Theorem 2.1 in [13] to the one in Theorem 1.1.

2. Proof of the Main Theorem

To prove the Theorem 1.1, we use the super- and subsolutions constructed in [13]. By considering the following equation, we construct subsolutions of $\mathcal{F}[u]$. Let $w(x)$ satisfy

$$\begin{gathered} w_{xx} + cw_x + \mu w - w^2 = 0, \\ w(-\infty) = \mu, \qquad w(\infty) = 0. \end{gathered} \tag{2.1}$$

For all $c \geq 2\sqrt{\mu}$, the above boundary value problem admits a unique solution $w(x)$ (up to a translation) which is strictly increasing in x. Subsolutions of $\mathcal{F}[u]$ are established as follows.

Lemma 2.1. *Let $\underline{U}(x,y) = u_+(y) + \sigma\phi(y)w(x)$. Then there exists $\sigma_1 > 0$ such that $\mathcal{F}[\underline{U}] \geq 0$ for all $0 < \sigma \leq \sigma_1$ and $c \geq 2\sqrt{\mu}$.*

Proof. Let $V := wB_c[\phi] - B_c[\phi w] \geq 0$, then $V \geq 0$. Indeed, it is easy to see that $B_c[\phi] \geq 0$ by the maximum principle and $\phi > 0$. A straightforward calculation gives

$$V_{xx} + cV_x + \Delta_y V - \delta\gamma V = -w(\mu - w)B_c[\phi] \leq 0. \tag{2.2}$$

Using the maximum principle, we obtain $V \geq 0$. Therefore by (H1)

$$\begin{aligned} \mathcal{F}[\underline{U}] &= \sigma\phi(w_{xx} + cw_x) + (\Delta_y u_+ - B_c[u_+]) + \sigma w \Delta_y \phi + f(y, u_+ + \sigma\phi w) - \sigma B_c[\phi w] \\ &= \sigma\phi(w_{xx} + cw_x + \mu w) + f(y, u_+ + \sigma\phi w) - f(y, u_+) - f_u(y, u_+)\sigma\phi w + \sigma V \\ &\geq \sigma\phi w^2 + G, \end{aligned} \tag{2.3}$$

where $G = f(y, u_+ + \sigma\phi w) - f(y, u_+) - f_u(y, u_+)\sigma\phi w$.

Let $M_1 = \min_{\{u_-(y) \geq u \geq u_+(y), y \in \mathbf{R}^{N-1}\}} f_{uu}(y,u)$. By choosing $\sigma \leq \epsilon/\mu$ and using (H3), we obtain $u_+ \leq u_+ + \sigma\phi w \leq u_+ + \epsilon\phi \leq u_-$. According to the mean value theorem, we have $G \geq 0$ if $M_1 \geq 0$ and $G \geq M_1 \sigma^2 \phi^2 w^2$ if $M_1 < 0$. Therefore $\mathcal{F}[\underline{U}] \geq 0$ if $\sigma \leq \sigma_1$, where $\sigma_1 = \epsilon/\mu$ as $M_1 \geq 0$ and $\sigma_1 = \min\{\epsilon/\mu, -1/M_1\}$ as $M_1 < 0$. The proof is completed. □

In what follows we construct supersolutions of $\mathcal{F}[u]$.

Lemma 2.2. *Let $Q(x) = e^{-((c-\sqrt{(c^2-4K_2)}/2))x}$ and $U^+(x,y) = u_+^{\eta}(y) + Q(x)$, where $K_2 > 0$ satisfies $K_2 + \delta/(\delta\gamma + K_2) = K^*$ and $c \geq 2\sqrt{K_2}$. Then $\mathcal{F}[U^+] < 0$.*

Proof. Note that $Q_{xx}+cQ_x+K_2Q=0$ and $0<B_c[Q]<\infty$. Indeed, by the uniqueness theorem we have $B_c[Q(x)]=\delta(-\partial^2/\partial x^2-c(\partial/\partial x)+\delta\gamma)^{-1}Q$ and

$$B_c[Q]=\frac{\delta}{\sqrt{c^2+4\gamma\delta}}\int_{-\infty}^{+\infty}e^{-(\sqrt{c^2+4\gamma\delta}/2)|x-\xi|+(c/2)(\xi-x)}Q(\xi)d\xi=\frac{\delta}{\delta\gamma+K_2}Q(x). \tag{2.4}$$

It follows from (H4) that

$$\begin{aligned}\mathcal{F}[U^+]&=(Q_{xx}+cQ_x)+\left(\Delta_y u_+^\eta-B_c\left[u_+^\eta\right]\right)+f\left(y,u_+^\eta+Q\right)-B_c[Q]\\&=-K_2Q+f\left(y,u_+^\eta+Q\right)-f\left(y,u_+^\eta\right)-\eta-B_c[Q]\\&=\left\{-K_2+f_u\left(y,u_+^\eta+\theta Q\right)-\frac{\delta}{\delta\gamma+K_2}\right\}Q-\eta\le-\eta<0,\end{aligned} \tag{2.5}$$

where $0\le\theta\le1$. The last second inequality is due to

$$K_2+\frac{\delta}{\delta\gamma+K_2}=\max_{\{u_-(y)\ge u\ge u_+(y),y\in\mathbf{R}^{N-1}\}}f_u(y,u). \tag{2.6}$$

We complete the proof of the lemma. □

Let

$$\mathcal{L}[u]=u_{xx}+cu_x+\Delta_y u-\left(K_1+\mu+\sqrt{\delta}\right)u, \tag{2.7}$$

where $K_1=-\min_{\{u_-(y)\ge u\ge u_+(y),y\in\mathbf{R}^{N-1}\}}f_u(y,u)>0$.

To show the existences of travelling wave solutions of (1.6), we use the following iteration process:

$$\begin{gathered}u_n(x,y)=\mathcal{L}^{-1}\left(-f(u_{n-1})+B_c[u_{n-1}]-\left(K_1+\mu+\sqrt{\delta}\right)u_{n-1}\right),\quad n=1,2,\ldots,\\u_0(x,y)=\underline{U}.\end{gathered} \tag{2.8}$$

In the following lemma, we assert that the supersolutions of $\mathcal{F}$ are greater than or equal to the subsolutions of $\mathcal{F}$. Moreover, we show that both $U^+-\underline{U}$ and $u_--\underline{U}$ are supersolutions of $\mathcal{L}$, which is useful in the proof of iteration process.

Lemma 2.3. *Assume $\gamma\ge 2/\sqrt{\delta}+(K_1+\mu)/\delta$ and let $\overline{U}:=\min\{U^+(x,y),u_-(y)\}$. Then for all $\eta>0$ there exists $\sigma_2>0$ depending on η such that for all $0<\sigma\le\sigma_2$ one has*

$$\overline{U}\ge\underline{U},\qquad\mathcal{L}[U^+-\underline{U}]\le0,\qquad\mathcal{L}[u_--\underline{U}]\le0. \tag{2.9}$$

Proof. For the case $\overline{U} = u_-(y)$ we take $\sigma \le \epsilon/\mu$, then

$$\overline{U} - \underline{U} = u_-(y) - u_+(y) - \sigma\phi(y)w(x) \ge u_-(y) - u_+(y) - \epsilon\phi(y) \ge 0. \tag{2.10}$$

The last inequality holds by (H3). On the other hand,

$$\mathcal{L}[u_- - \underline{U}] = \Delta_y(u_- - u_+) - \left(K_1 + \mu + \sqrt{\delta}\right)(u_- - u_+) + A, \tag{2.11}$$

where $A = -\sigma\phi(w_{xx} + cw_x) + (K_1 + \mu + \sqrt{\delta})\sigma\phi w - \sigma w \Delta_y \phi$. According to (H5), $|A| \le \sigma C\phi$ for some positive constant $C = C(\mu, \delta, K_1)$. By choosing $\sigma \le \epsilon\mu/C$, we obtain

$$\begin{aligned} \mathcal{L}[u_- - \underline{U}] &\le \Delta_y(u_- - u_+) - \left(K_1 + \sqrt{\delta}\right)(u_- - u_+) - \mu(u_- - u_+) + \sigma C\phi \\ &\le -\epsilon\mu\phi + \sigma C\phi \le 0, \end{aligned} \tag{2.12}$$

which holds due to assumptions (H3) and (H5).

For the case $\overline{U} = u_+^\eta(y) + Q(x)$, given $\eta > 0$ we choose $\sigma \le \eta/\mu M$ and use assumption (H4), then

$$\overline{U} - \underline{U} = u_+^\eta(y) + Q(x) - u_+(y) - \sigma\phi(y)w(x) \ge \frac{\eta}{M} - \sigma\mu \ge 0. \tag{2.13}$$

Moreover,

$$\begin{aligned} \mathcal{L}[U^+ - \underline{U}] = \Delta_y\left(u_+^\eta - u_+\right) - \left(K_1 + \mu + \sqrt{\delta}\right)\left(u_+^\eta - u_+\right) + A + Q_{xx} + Q_x \\ - \left(K_1 + \mu + \sqrt{\delta}\right)Q. \end{aligned} \tag{2.14}$$

It is readily seen that $Q_{xx} + Q_x - (K_1 + \mu + \sqrt{\delta})Q \le 0$. By (H4) and (H5),

$$\mathcal{L}[U^+ - \underline{U}] \le -\frac{\eta\mu}{M} + \sigma C \le \quad \text{if } \sigma \le \frac{\eta\mu}{MC}. \tag{2.15}$$

Setting $\sigma_2 = \min\{\epsilon/\mu, \epsilon\mu/C, \eta/\mu M, \eta\mu/MC\}$, the lemma holds. □

To generalize the result of Theorem 2.1 in [13], the nonlocal term of (1.5) needs to be better estimated. More precisely, we point wisely control $B_c[u]$ by the local term u such that the iterative sequence u_n is comparable with u_{n-1}.

Lemma 2.4. *Let $u \in C^2(\mathbf{R}^N)$ be nonnegative and solve $u_{xx} + cu_x + \Delta_y u - au \le 0$ for some constant a. Assume $\gamma \ge a/\delta + 1/b$ for some b. Then $bu - B_c[u] \ge 0$.*

Proof. Let $v = B_c[u]$ and $U = bu - v$. Then $v \geq 0$ because of $u \geq 0$ and the maximum principle. Our main purpose is to claim $U \geq 0$. By the assumption of u and the definition of v, we have

$$U_{xx} + cU_x + \Delta_y U - \frac{ab+\delta}{b} U \leq -\left(\delta\gamma - a - \frac{\delta}{b}\right) v \leq 0. \tag{2.16}$$

The last inequality follows from the hypothesis of γ and the nonnegativity of v. By the maximum principle, $U \geq 0$. □

As γ becomes large, we claim that the iterative sequence u_n is increasing.

Lemma 2.5. *Assume $\gamma \geq 2/\sqrt{\delta} + (K_1 + \mu)/\delta$ and $c \geq c^* = \max\{2\sqrt{\mu}, 2\sqrt{K_2}\}$, then for all $\eta > 0$ and $0 < \sigma \leq \min\{\sigma_1, \sigma_2\}$ one has $u_{n,x} \leq 0$ and*

$$u_0 \leq u_1 \leq \cdots \leq u_n \leq \cdots \leq \overline{U}. \tag{2.17}$$

Proof. We first claim that $u_n \leq \overline{U}$ for all n. Indeed, by Lemmas 2.3 and 2.4 (take $a = K_1 + \mu + \sqrt{\delta}$ and $b = \sqrt{\delta}$) we obtain

$$\sqrt{\delta}(U^+ - u_0) - B_c[U^+ - u_0] \geq 0. \tag{2.18}$$

Therefore Lemmas 2.2 and 2.3 yield

$$\begin{aligned}\mathcal{L}[U^+ - u_1] &\leq -f(U^+) + B_c[U^+] + f(u_0) - B_c[u_0] - \left(K_1 + \mu + \sqrt{\delta}\right)(U^+ - u_0) \\ &\leq \left\{-f_u(\theta U^+(1-\theta)u_0) - K_1\right\}(U^+ - u_0) \leq 0,\end{aligned} \tag{2.19}$$

where $0 \leq \theta \leq 1$. According to the maximum principle, $U^+ - u_1 \geq 0$. It follows form the proof of $U^+ - u_1 \geq 0$ that $u_- - u_1 \geq 0$. Therefore $u_1 \leq \overline{U}$. Continuing this process, we have $u_n \leq \overline{U}$ for all n by induction.

Next obvert that $\mathcal{L}[u_1 - u_0] = -\mathcal{F}[\underline{U}] \leq 0$ due to Lemma 2.1. By the maximum principle, $u_1 - u_0 \geq 0$. Applying Lemma 2.4 to $u_1 - u_0$, we have

$$\sqrt{\delta}(u_1 - u_0) - B_c[u_1 - u_0] \geq 0. \tag{2.20}$$

Therefore

$$\begin{aligned}\mathcal{L}[u_2 - u_1] &= -\left(f(u_1) - f(u_0)\right) + B_c[u_1 - u_0] - \left(K_1 + \mu + \sqrt{\delta}\right)(u_1 - u_0) \\ &\leq \left\{-f_u(\theta u_1 + (1-\theta)u_0 - K_1\right\}(u_1 - u_0) - \sqrt{\delta}(u_1 - u_0) + B_c[u_{1-}u_0] \\ &\leq 0,\end{aligned} \tag{2.21}$$

where $0 \leq \theta \leq 1$. Thus $u_2 \geq u_1$. By induction, the sequence of functions $\{u_n\}$ is nondecreasing. On the other hand, obvert that $u_{0,x} = \sigma\phi w_x < 0$. Therefore by (H5), we obtain

$$\begin{aligned}\mathcal{L}[-u_{0,x}] &= \sigma\phi(\mu w_x - 2ww_x) - \sigma w_x \Delta_y \phi + \left(K_1 + \mu + \sqrt{\delta}\right)\sigma\phi w_x \\ &= -\sigma w_x \left\{\Delta_y \phi - \left(K_1 + \sqrt{\delta}\right)\phi + (-2\mu + 2w)\phi\right\} \leq 0.\end{aligned} \tag{2.22}$$

Using Lemma 2.4 again, we have

$$\begin{aligned}\sqrt{\delta}(-u_{0,x}) - B_c[-u_{0,x}] \geq 0, \\ \mathcal{L}[u_{1,x}] = -f_u(u_0)u_{0,x} + B_c[u_{0,x}] - \left(K_1 + \mu + \sqrt{\delta}\right)u_{0,x} \geq 0.\end{aligned} \tag{2.23}$$

Then $u_{1,x} \leq 0$ by the maximum principle. Inducting in n, we obtain $u_{n,x} \leq 0$. □

Proof of Theorem 1.1. By Lemma 2.5, we define $u^*(x,y) = \lim_{n\to\infty} u_n(x,y)$. Following the proof of Theorem 2.1 in [13], (H2) and (H3), for all $c \geq c^*$ we obtain that $u^*(x,y)$ is a smooth solution of (1.5), $u_x^* \leq 0$ and $u^*(\pm\infty, y) = u_\pm(y)$. Let $v^* = B_c[u^*]$, then $v_x^* = B_c[u_x^*] \leq 0$ by the maximum principle. We complete the proof of the theorem. □

Acknowledgments

C.-C. Huang wishes to express his sincere gratitude to Dr. Li-Chang Hung for his careful reading of the paper and valuable suggestions for improvement, and to his supervisor, Professor Chiun-Chuan Chen for his warm encouragement and invaluable advice throughout his Ph.D. program.

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