Hindawi Publishing Corporation
Abstract and Applied Analysis
Volume 2012, Article ID 631932, 19 pages
doi:10.1155/2012/631932

Research Article

Global Synchronization of Neutral-Type Stochastic Delayed Complex Networks

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Received 5 September 2012; Revised 24 October 2012; Accepted 24 October 2012

Academic Editor: Sabri Arik

This paper is concerned with the delay-dependent synchronization criterion for neutral-type stochastic delayed complex networks. Firstly, expectations of stochastic crossterms containing the Itô integral are investigated. In fact, for stochastic delay systems, if we want to obtain the delay-dependent condition with less conservatism, how to deal with expectations of stochastic cross terms properly is of vital importance, and many existing results did not deal with expectations of these stochastic cross terms correctly. Then, based on this, this paper establishes a novel delay-dependent synchronization criterion for neutral-type stochastic delayed complex networks. In the derivation process, the mathematical development avoids bounding stochastic cross terms. Thus, this method shows less conservatism. Finally, a numerical example is provided to demonstrate the effectiveness of the proposed approach.

1. Introduction

In the real world, many systems can be described as complex networks such as Internet networks, biological networks, epidemic spreading networks, collaborative networks, social networks, neural networks, and so forth [1–4]. Thus, during the past years, the study of complex networks has become a very active area, see, for example, [5, 6] and the references therein. In particular, for complex networks, the major collective behavior is the synchronization phenomena, because many problems in practice have close relationships with synchronization [7]. Recently, growing research results, that focused on synchronization problems for complex networks, have been reported in [8–12] and the references therein.

Up to now, it has been well realized that in spreading information through complex networks, there always exist time delays caused by the finite speed of information transmission and the limit of bandwidth, which often decrease the quality of the system and even lead to oscillation, divergence, and instability. Accordingly, synchronization problems for many delayed complex networks have been studied in [13–17]. It is worth mentioning that in the above results for delayed complex networks, each dynamical node is modeled as a retarded functional differential equation coupling with other nodes. However, in some cases, in order to reflect dynamical behaviors for some realistic networks models, the information about derivatives of the past state variables of the networks should be utilized. Therefore, the dynamic of the complex networks should be described by a group of neutral-type functional differential equations. This kind of delayed complex network is termed as the neutral-type delayed complex network. As a matter of fact, neutral-type delays exist in many fields such as the population ecology, distributed networks containing lossless transmission lines, and a typical neutral-type delayed complex network example which is the stock transaction system [18]. Consequently, synchronization problems of neutral-type delayed complex networks were studied in [18–20]. For instance, a delay-dependent synchronization criterion for complex networks with neutral-type coupling delay was presented in [18], and the robust synchronization criterion for a class of uncertain neutral-type delayed complex networks was given in [19]. And [20] discussed the synchronization problem for the neutral-type complex networks with coupling time-varying delays.

On the other hand, in the real world, complex networks are often subject to stochastic disturbances. For example, the signal transfer in a real complex network could be perturbed randomly from the release of probabilistic causes such as neurotransmitters and packet dropouts [21]. Hence, such a stochastic disturbance phenomenon that typically occurs in complex networks has attracted considerable attention during the past years, and synchronization problems for delayed complex networks with stochastic disturbances have been investigated in [21–24]. For instance, the synchronization problems of discrete-time delayed complex networks with stochastic disturbances were investigated in [21, 22]. Reference [24] designed an adaptive feedback controller to solve the synchronization problem for an array of linearly stochastically coupled networks with time delays. Although the above results have discussed delayed complex networks under the influence of stochastic noises, it should be pointed out that as to the neutral-type delayed complex networks, there is still *no* paper to investigate the influence of stochastic disturbances on this kind of complex networks.

Moreover, for delay systems including delayed complex networks, a very active research topic is to obtain the delay-dependent conditions. The reason is that the delay-dependent condition makes use of the information on the size of time delays, and the delay-dependent condition is generally less conservative than the delay-independent one [25–27]. However, when we used the existing effective methods, such as the model transformation method [25, 26] and the free-weighting matrix method [27], to give the delay-dependent condition for stochastic delay systems including stochastic delayed complex (or neural) networks, the following stochastic cross terms containing the Itô integral will appear:

$$x(t)^T\mathbb{J}\int_{t-h}^{t}\mu(s,x_s)dw(s),\qquad x(t-h)^T\mathbb{K}\int_{t-h}^{t}\mu(s,x_s)dw(s),$$
$$\left(\int_{t-h}^{t}\kappa(s,x_s)ds\right)^T\mathbb{L}\int_{t-h}^{t}\mu(s,x_s)dw(s). \tag{1.1}$$

It is still very difficult to calculate expectations of these stochastic cross terms up to now. The results in [28–31] resorted to bounding techniques, which obviously can bring the conservatism. Some papers such as [32–34] considered that expectations of these stochastic cross terms are all equal to zero. However, these results are not given by strict mathematical proofs, and we can find examples to illustrate that expectations of some stochastic cross terms are not equal to zero in Remark 3.3. Therefore, in order to obtain the delay-dependent synchronization criterion with less conservatism for neutral-type stochastic delayed complex networks, there is a strong need to investigate the expectations of stochastic cross terms containing the Itô integral firstly.

Motivated by the discussion mentioned above, this paper investigates the delay-dependent synchronization problem for neutral-type stochastic delayed complex networks. The main contributions of this paper are summarized as follows. (1) Expectations of stochastic cross terms containing the Itô integral are investigated by stochastic analysis techniques in Lemma 3.1 and Corollary 3.2. We prove that the expectation of $x(t-h)^T\mathbb{K}\int_{t-h}^{t}\mu(s,x_s)dw(s)$ is equal to zero and expectations of other stochastic cross terms are not. (2) Based on this conclusion, this paper establishes a delay-dependent synchronization criterion that guarantees the globally asymptotic synchronization of neural-type stochastic delayed complex networks. In the derivation process, the mathematical development avoids bounding stochastic cross terms. Thus, this method leads to a criterion with less conservatism. Finally, a numerical example is provided to demonstrate the effectiveness of the proposed approach.

Notation. Throughout the paper, unless otherwise specified, we will employ the following notation. Let $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq 0},\mathbb{P})$ be a complete probability space with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$, and let $\mathcal{E}(\cdot)$ be the expectation operator with respect to the probability measure. If A is a vector or matrix, its transpose is denoted by A^T. If P is a square matrix, then $P>0$ $(P<0)$ means that it is a symmetric positive (negative) definite matrix of appropriate dimensions while $P\geq 0$ $(P\leq 0)$ is a symmetric positive (negative) semidefinite matrix. I stands for the identity matrix of appropriate dimensions. Denote by $\lambda_{\min}(\cdot)$ the minimum eigenvalue of a given matrix. Let $|\cdot|$ denote the Euclidean norm of a vector and its induced norm of a matrix. Unless explicitly specified, matrices are assumed to have real entries and compatible dimensions. $L^2(\Omega)$ denotes the space of all random variables X with $\mathcal{E}|X|^2<\infty$, it is a Banach space with norm $\|X\|_2=(\mathcal{E}|X|^2)^{1/2}$. Let $h>0$ and $C([-h,0];\mathcal{R}^n)$ denote the family of all continuous $\mathcal{R}^n$-valued functions φ on $[-h,0]$ with the norm $\|\varphi\|=\sup\{|\varphi(\theta)|:-h\leq\theta\leq 0\}$. Let $L^2_{\mathcal{F}_0}([-h,0];\mathcal{R}^n)$ be the family of all $\mathcal{F}_0$-measurable $C([-h,0];\mathcal{R}^n)$-valued random variables ϕ such that $\mathcal{E}(\|\phi\|^2)<\infty$, and let $\mathcal{L}^2([a,b];\mathcal{R}^n)$ be the family of all $\mathcal{R}^n$-valued $\mathcal{F}_t$-adapted processes $\{f(t)\}_{a\leq t\leq b}$ such that $\int_a^b|f(t)|^2dt<\infty$ a.s. Let $\mathcal{M}^2([a,b];\mathcal{R}^n)$ be the family of processes $\{f(t)\}_{a\leq t\leq b}$ in $\mathcal{L}^2([a,b];\mathcal{R}^n)$ such that $\mathcal{E}(\int_a^b|f(t)|^2dt)<\infty$, and $\mathcal{M}^2([a,b])$ is the 1-dimensional case of $\mathcal{M}^2([a,b];\mathcal{R}^n)$.

2. Problem Formulation and Preliminaries

In this paper, we consider the following neutral-type stochastic delayed complex networks consisting of N identical nodes:

$$
\begin{aligned}
& d[x_i(t) - Dx_i(t-h)] \\
& \quad = \left[Ax_i(t) + Bf(x_i(t)) + Cf(x_i(t-h)) + \sum_{j=1}^{N} g_{ij}\Gamma x_j(t) + \sum_{j=1}^{N} h_{ij}\Upsilon x_j(t-h) \right] dt \\
& \qquad + \sigma_i(t, x_i(t), x_i(t-h)) dw(t), \quad i = 1, 2, \ldots, N,
\end{aligned} \tag{2.1}
$$

where $x_i(t) = [x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t)]^T \in \mathcal{R}^n$ represents the state vector of the ith node; the scalar $h > 0$ is the time delay; A is a known connection matrix; B and C denote, respectively, the connection weight matrix and the delayed connection weight matrix; $\Gamma, \Upsilon \in \mathcal{R}^{n\times n}$ are matrices describing the inner coupling between the subsystems at time t and $t-h$, respectively; $G = (g_{ij})_{N\times N}$ and $H = (h_{ij})_{N\times N}$ are called the outer-coupling configuration matrices representing the coupling strength and the topological structure of the complex networks; D is a known real matrix, and the spectrum radius of the matrix D, $\rho(D)$, satisfies $\rho(D) < 1$. $\sigma_i(\cdot,\cdot,\cdot) : \mathcal{R} \times \mathcal{R}^n \times \mathcal{R}^n \to \mathcal{R}^n$ which is the noise intensity function vector; $w(t)$ is a scalar standard Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$ with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$. $f(x_i(t)) = (f_1(x_{i1}(t)), \ldots, f_n(x_{in}(t)))^T$, is an unknown but sector-bounded nonlinear function.

The initial conditions associated with system (2.1) are given by

$$
x_i(s) = \varphi_i(s), \quad -h \leq s \leq 0, \ i = 1, 2, \ldots, N, \tag{2.2}
$$

where $\varphi_i(\cdot) \in L^2_{\mathcal{F}_0}([-h, 0]; \mathcal{R}^n)$.

Let

$$
\begin{aligned}
x(t) &= \left(x_1(t)^T, \ldots, x_N(t)^T \right)^T, \\
F(x(t)) &= \left(f(x_1(t))^T, \ldots, f(x_N(t))^T \right)^T, \\
F(x(t-h)) &= \left(f(x_1(t-h))^T, \ldots, f(x_N(t-h))^T \right)^T, \\
\sigma(t) &= \left(\sigma_1(t, x_1(t), x_1(t-h))^T, \ldots, \sigma_N(t, x_N(t), x_N(t-h))^T \right)^T, \\
\overline{D} &= \operatorname{diag}\left(\overbrace{D, D, \ldots, D}^{N} \right).
\end{aligned} \tag{2.3}
$$

With the Kronecker product "$\otimes$" for matrices, system (2.1) can be rearranged as

$$
\begin{aligned}
d\left[x(t) - \overline{D}x(t-h) \right] = {} & [(I_N \otimes A + G \otimes \Gamma)x(t) + (H \otimes \Upsilon)x(t-h) \\
& + (I_N \otimes B)F(x(t)) + (I_N \otimes C)F(x(t-h))]dt + \sigma(t)dw(t).
\end{aligned} \tag{2.4}
$$

Before stating our main results, we need the following definitions, assumptions, and propositions.

Definition 2.1. The neutral-type stochastic delayed complex network (2.1) is globally asymptotically synchronized in the mean square if, for all $\varphi_i(\cdot), \varphi_j(\cdot) \in L^2_{\mathcal{F}_0}([-h, 0]; \mathcal{R}^n)$, the following holds:

$$\lim_{t\to\infty} \mathcal{E}\left\{ \left| x_i(t,\varphi_i) - x_j(t,\varphi_j) \right|^2 \right\} = 0, \quad 1 \le i < j \le N. \tag{2.5}$$

Definition 2.2 (see [35]). If a stochastic process $\{\nu(t)\}_{a\le t\le b}$ belongs to $\mathcal{M}^2([a,b])$, then its Itô integral (from a to b) is defined by

$$\int_a^b \nu(t)dw(t) = \lim_{n\to\infty} \int_a^b \nu_n(t)dw(t) \quad \left(\lim \text{in } L^2(\Omega)\right), \tag{2.6}$$

where $\{\nu_n(t)\}_{a\le t\le b}$ $(n = 1, 2, \ldots)$ are the step stochastic processes and belong to $\mathcal{M}^2([a,b])$ such that

$$\lim_{n\to\infty} \mathcal{E}\left(\int_a^b |\nu(t) - \nu_n(t)|^2 dt \right) = 0. \tag{2.7}$$

Definition 2.3 (see [36]). Let $\{\mathcal{F}_t\}_{t\in T}$ be an increasing family of σ-algebras of subset of Ω. A stochastic process $\{X_t\}_{t\in T}$ is said to be adapted to $\{\mathcal{F}_t\}_{t\in T}$ if for each t, the random variable X_t is $\mathcal{F}_t$-measurable.

Assumption 2.4. The outer-coupling configuration matrices of the complex networks (2.1) satisfy

$$\begin{gathered} g_{ij} = g_{ji} \ge 0, \quad h_{ij} = h_{ji} \ge 0, \quad (i \ne j), \\ g_{ii} = -\sum_{j=1, j\ne i}^{N} g_{ij}, \quad h_{ii} = -\sum_{j=1, j\ne i}^{N} h_{ij}, \quad i, j = 1, 2, \ldots, N. \end{gathered} \tag{2.8}$$

Assumption 2.5. The noise intensity function vector $\sigma_i : \mathcal{R} \times \mathcal{R}^n \times \mathcal{R}^n \to \mathcal{R}^n$ satisfies the Lipschitz condition, that is, there exist constant matrices W_1 and W_2 of appropriate dimensions such that

$$\left| \sigma_i(t, x_1, y_1) - \sigma_j(t, x_2, y_2) \right|^2 \le |W_1(x_1 - x_2)|^2 + \left| W_2(y_1 - y_2) \right|^2, \tag{2.9}$$

for all $i, j = 1, 2, \ldots, N$ and $x_1, y_1, x_2, y_2 \in \mathcal{R}^n$.

Assumption 2.6. For all $x, y \in \mathcal{R}^n$, the nonlinear function $f(\cdot)$ is assumed to satisfy the following condition:

$$\left(f(x) - f(y) - U(x-y)\right)^T \left(f(x) - f(y) - V(x-y)\right) \le 0, \tag{2.10}$$

where U and V are real constant matrices with U-V being symmetric and positive definite.

Proposition 2.7 (see [14]). *The Kronecker product has the following properties:*

$$\begin{aligned}
(\alpha A) \otimes B &= A \otimes (\alpha B), \\
(A + B) \otimes C &= A \otimes C + B \otimes C, \\
(A \otimes B)(C \otimes D) &= (AC) \otimes (BD), \\
(A \otimes B)^T &= A^T \otimes B^T.
\end{aligned} \tag{2.11}$$

Proposition 2.8 (see [19]). *Let $\mathcal{U} = (\alpha_{ij})_{n\times n}, P \in \mathcal{R}^{m\times m}, x = (x_1^T, x_2^T, \ldots, x_n^T)^T, y = (y_1^T, y_2^T, \ldots, y_n^T)^T$, where $x_i = (x_{i1}, x_{i2}, \ldots, x_{im})^T \in \mathcal{R}^m$, $y_i = (y_{i1}, y_{i2}, \ldots, y_{im})^T \in \mathcal{R}^m$ $(i = 1, 2, \ldots, n)$. If $\mathcal{U} = \mathcal{U}^T$ and each row sum of $\mathcal{U}$ is equal to zero, then*

$$x^T (\mathcal{U} \otimes P) y = -\sum_{1\le i<j\le n} \alpha_{ij} (x_i - x_j)^T P (y_i - y_j). \tag{2.12}$$

Proposition 2.9 (see [35]). *Let $\{\vartheta(t)\}_{a\le t\le b}$ be a stochastic process and belong to $\mathcal{M}^2([a,b])$, then*

$$\mathcal{E}\left(\int_a^b \vartheta(t) dw(t)\right) = 0. \tag{2.13}$$

3. Main Results

Then, we give the following lemma and corollary which will play a key role in the proof of our main results.

Lemma 3.1. *If a stochastic process $\{\nu(t)\}_{a\le t\le b} \in \mathcal{M}^2([a,b])$ and ϖ is a bounded and $\mathcal{F}_a$-measurable random variable, then*

$$\mathcal{E}\left(\varpi \int_a^b \nu(t) dw(t)\right) = 0. \tag{3.1}$$

Proof. Firstly, in order to prove the above results, we will prove that if $\{\nu(t)\}_{a\le t\le b} \in \mathcal{M}^2([a,b])$ and ϖ is a bounded and $\mathcal{F}_a$-measurable random variable, then

$$\varpi \int_a^b \nu(t) dw(t) = \int_a^b \varpi \nu(t) dw(t). \tag{3.2}$$

Most important of all, since ϖ is a bounded and $\mathcal{F}_a$-measurable random variable, it is easy to verify $\{\varpi\nu(t)\}_{a\le t\le b} \in \mathcal{M}^2([a,b])$. Then, we will prove (3.2) by the following two steps.

Step 1. If $\{\nu(t)\}_{a\le t\le b}$ is a step stochastic process, then we let, without loss of generality,

$$\nu(t) = \sum_{i=1}^n \varsigma_{i-1} 1_{[t_{i-1}, t_i)}(t), \tag{3.3}$$

where $t_0 = a, t_n = b, \varsigma_{i-1}$ is $\mathcal{F}_{t_{i-1}}$-measurable and $\mathcal{E}(\varsigma_{i-1}^2) < \infty$. In this case,

$$\int_a^b \varpi v(t)dw(t) = \sum_{i=1}^{n} \varpi \varsigma_{i-1}(w(t_i) - w(t_{i-1})) = \varpi \sum_{i=1}^{n} \varsigma_{i-1}(w(t_i) - w(t_{i-1})) = \varpi \int_a^b v(t)dw(t). \tag{3.4}$$

Step 2. If $\{v(t)\}_{a\le t\le b} \in \mathcal{M}^2([a,b])$ is not a step stochastic process, then by Definition 2.2, we can find a sequence of step stochastic processes in $\mathcal{M}^2([a,b])$: $\{v_1(t)\}_{a\le t\le b}, \{v_2(t)\}_{a\le t\le b}, \ldots, \{v_n(t)\}_{a\le t\le b}, \ldots$ such that

$$\int_a^b v(t)dw(t) = \lim_{n\to\infty} \int_a^b v_n(t)dw(t) \quad \left(\lim \text{in } L^2(\Omega)\right), \tag{3.5}$$

where $\{v(t)\}_{a\le t\le b}$ and $\{v_n(t)\}_{a\le t\le b}$ satisfy

$$\lim_{n\to\infty} \mathcal{E}\left(\int_a^b |v(t) - v_n(t)|^2 dt\right) = 0. \tag{3.6}$$

Because ϖ is bounded, by Definition 2.2 and (3.5)-(3.6), it is easy to prove that

$$\begin{aligned} \int_a^b \varpi v(t)dw(t) &= \lim_{n\to\infty} \int_a^b \varpi v_n(t)dw(t) \quad \left(\lim \text{in } L^2(\Omega)\right), \\ \varpi \int_a^b v(t)dw(t) &= \lim_{n\to\infty} \varpi \int_a^b v_n(t)dw(t) \quad \left(\lim \text{in } L^2(\Omega)\right). \end{aligned} \tag{3.7}$$

From Step 1, it follows that for each step stochastic process $\{v_n(t)\}_{a\le t\le b}$, we have

$$\int_a^b \varpi v_n(t)dw(t) = \varpi \int_a^b v_n(t)dw(t). \tag{3.8}$$

Therefore, it is easy to obtain

$$\lim_{n\to\infty} \int_a^b \varpi v_n(t)dw(t) = \lim_{n\to\infty} \varpi \int_a^b v_n(t)dw(t) \quad \left(\lim \text{in } L^2(\Omega)\right). \tag{3.9}$$

Then, we can get by (3.7) and (3.9) that

$$\int_a^b \varpi v(t)dw(t) = \varpi \int_a^b v(t)dw(t). \tag{3.10}$$

Due to $\{\varpi v(t)\}_{a\leq t\leq b} \in \mathcal{M}^2([a,b])$, then by Proposition 2.9, we can know that

$$\mathcal{E}\left(\varpi \int_a^b v(t)dw(t)\right) = \mathcal{E}\left(\int_a^b \varpi v(t)dw(t)\right) = 0. \tag{3.11}$$

This completes the proof. □

Corollary 3.2. *Let one consider the following neutral stochastic functional differential equation:*

$$d[x(t) - \mathfrak{D}x(t-h)] = \kappa(t,x_t)dt + \mu(t,x_t)dw(t), \tag{3.12}$$

on $t \geq 0$ with the initial data $x_0 = \xi \in L^2_{\mathcal{F}_0}([-h,0];\mathcal{R}^n)$. $\kappa(\cdot,\cdot)$ and $\mu(\cdot,\cdot)$ satisfy the local Lipschitz condition and the linear growth condition. If $x(t)$ is the solution of (3.12) *and $\mathbb{K}$ is any compatible dimensional matrix, then*

$$\mathcal{E}\left(x(t-h)^T \mathbb{K}\left[\int_{t-h}^t \mu(s,x_s)dw(s)\right]\right) = 0, \quad t \geq h. \tag{3.13}$$

Especially when $\mathfrak{D} = 0$ in (3.12)*, that is,*

$$dx(t) = \kappa(t,x_t)dt + \mu(t,x_t)dw(t). \tag{3.14}$$

Equation (3.14) *is a common stochastic functional equation. For this case,* (3.13) *is also tenable.*

Proof. Since $\kappa(\cdot,\cdot)$ and $\mu(\cdot,\cdot)$ satisfy the local Lipschitz condition and the linear growth condition, we can know that, for all $T > 0$, (3.12) has a unique continuous solution on $[-h,T]$ denoted by $\{x(t)\}_{-h\leq t\leq T}$ that is adapted to $\{\mathcal{F}_t\}_{-h\leq t\leq T}$ and $\{x(t)\}_{-h\leq t\leq T} \in \mathcal{M}^2([-h,T])$ [37]. Therefore, it can be derived that for $t \geq h$, $x(t-h)$ is a bounded random variable and $x(t-h)$ is $\mathcal{F}_{t-h}$-measurable. Then, by Lemma 3.1, it is easy to obtain (3.13). If $\mathfrak{D} = 0$ in (3.12) that is a common stochastic functional equation, then we can easily prove that (3.13) is also tenable for this case. □

Remark 3.3. Lemma 3.1 has proved

$$\mathcal{E}\left(x(t-h)^T \mathbb{K}\left[\int_{t-h}^t \mu(s,x_s)dw(s)\right]\right) = 0, \quad t \geq h. \tag{3.15}$$

However, for any compatible dimensional matrix $\mathbb{J}$ or $\mathbb{L}$, the following results are *not* correct:

$$\begin{aligned} &\mathcal{E}\left(x(t)^T\mathbb{J}\left[\int_{t-h}^{t}\mu(s,x_s)dw(s)\right]\right)=0, \\ &\mathcal{E}\left(\left(\int_{t-h}^{t}\kappa(s,x_s)ds\right)^T\mathbb{L}\left[\int_{t-h}^{t}\mu(s,x_s)dw(s)\right]\right)=0, \end{aligned} \qquad t\geq h. \tag{3.16}$$

We will give two examples to illustrate it.

Example 3.4. Consider the following one-dimensional Langevin equation in [36] that can be regarded as a special class of neutral stochastic delay systems as follows:

$$d[x(t)-0x(t-h)]=\kappa(t,x_t)dt+\mu(t,x_t)dw(t), \qquad x(0)=\xi, \tag{3.17}$$

where $\kappa(t,x_t)=-\beta x(t),\mu(t,x_t)=\alpha$ and $\alpha>0,\beta>0$. This equation has a solution

$$x(t)=e^{-\beta(t-u)}x(u)+\alpha\int_{u}^{t}e^{-\beta(t-s)}dw(s), \quad u\leq t. \tag{3.18}$$

Then by (3.18), we can know that

$$\begin{aligned} \mathcal{E}\left(x(t)\mathbb{J}\int_{t-h}^{t}\mu(s,x_s)dw(s)\right) &= \mathcal{E}\left(\left(e^{-\beta h}x(t-h)+\alpha\int_{t-h}^{t}e^{-\beta(t-s)}dw(s)\right)\right. \\ &\quad \left.\times\mathbb{J}\left[\int_{t-h}^{t}\alpha\, dw(s)\right]\right) \\ &= e^{-\beta h}\mathcal{E}\left(x(t-h)\mathbb{J}\left[\int_{t-h}^{t}\alpha\, dw(s)\right]\right) \\ &\quad +\mathcal{E}\left(\alpha\int_{t-h}^{t}e^{-\beta(t-s)}dw(s)\mathbb{J}\int_{t-h}^{t}\alpha\, dw(s)\right) \\ &= 0+\alpha^2\mathbb{J}e^{-\beta t}\int_{t-h}^{t}e^{\beta s}ds \\ &= \frac{\alpha^2\mathbb{J}}{\beta}\left(1-e^{-\beta h}\right)\neq 0, \quad \forall\mathbb{J}\neq 0, \\ \mathcal{E}\left(\int_{t-h}^{t}\kappa(s,x_s)ds\mathbb{L}\left[\int_{t-h}^{t}\mu(s,x_s)dw(s)\right]\right) &= \mathcal{E}\left(\left(x(t)-x(t-h)-\int_{t-h}^{t}\mu(s,x_s)dw(s)\right)\right. \\ &\quad \left.\times\mathbb{L}\left[\int_{t-h}^{t}\mu(s,x_s)dw(s)\right]\right) \\ &= \mathcal{E}\left(x(t)\mathbb{L}\int_{t-h}^{t}\mu(s,x_s)dw(s)\right) \end{aligned}$$

$$
\begin{aligned}
&- \mathcal{E}\left(x(t-h)\mathbb{L}\int_{t-h}^{t} \mu(s,x_s)dw(s) \right) \\
&- \mathcal{E}\left(\int_{t-h}^{t} \mu(s,x_s)dw(s)\mathbb{L}\int_{t-h}^{t} \mu(s,x_s)dw(s) \right) \\
&= \frac{\alpha^2\mathbb{L}}{\beta}\left(1-e^{-\beta h}\right) - 0 - \mathbb{L}\int_{t-h}^{t} \alpha^2 ds \\
&= \frac{\alpha^2\mathbb{L}}{\beta}\left(1-e^{-\beta h}-\beta h\right) \neq 0, \quad \forall \mathbb{L} \neq 0.
\end{aligned}
\tag{3.19}
$$

Example 3.5. Consider the following one-dimensional stochastic equation:

$$
d[x(t) - 0x(t-h)] = dw(t), \tag{3.20}
$$

which has a one solution $x(t) = w(t)$. However, we can easily verify that

$$
\mathcal{E}\left(x(t)^T \mathbb{J}\int_{t-h}^{t} \mu(s,x_s)dw(s) \right) = \mathcal{E}\left(w(t)\mathbb{J}\int_{t-h}^{t} dw(s) \right) = \mathbb{J}h \neq 0, \quad \forall \mathbb{J} \neq 0. \tag{3.21}
$$

We should point out that in recent years, some papers such as [32–34] considered that the expectations of these stochastic terms are all equal to zero. However, this is not the case. From the above examples and Corollary 3.2, we can see that $x(t-h)^T\mathbb{K}\int_{t-h}^{t}\mu(s,x_s)dw(s)$ is the only one whose expectation is equal to zero.

Then, we are in the position to present our main result for the synchronization criterion of the neutral-type delayed complex networks with stochastic disturbances.

Theorem 3.6. *Under the Assumptions 2.4–2.6, the dynamical system* (2.1) *is globally asymptotically synchronized in the mean square if there exist matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $R > 0$, $Z > 0$, S and scalars $\epsilon > 0$, $\lambda > 0$ such that the following LMIs hold for all $1 \le i < j \le N$:*

$$
P < \lambda I, \tag{3.22}
$$

$$
\Xi = \begin{pmatrix}
\Xi_{11} & \Xi_{12} & 0 & \Xi_{14} & PC & \Xi_{16} & 0 \\
* & \Xi_{22} & \Xi_{23} & -D^TPB & -D^TPC & \Xi_{26} & \Xi_{27} \\
* & * & -Q_2 & 0 & 0 & 0 & 0 \\
* & * & * & R-2\epsilon I & 0 & B^TS^T & 0 \\
* & * & * & * & -R & C^TS^T & 0 \\
* & * & * & * & * & hZ-S^T-S & 0 \\
* & * & * & * & * & * & -hZ
\end{pmatrix} < 0, \tag{3.23}
$$

where

$$\Xi_{11} = PA + A^TP - Ng_{ij}P\Gamma - Ng_{ij}\Gamma^TP + \lambda W_1^TW_1 + Q_1 + Q_2 - \epsilon U^TV - \epsilon V^TU,$$
$$\Xi_{12} = -A^TPD + Ng_{ij}\Gamma^TPD, \qquad \Xi_{14} = PB + \epsilon U^T + \epsilon V^T, \qquad \Xi_{16} = A^TS^T - Ng_{ij}\Gamma^TS^T,$$
$$\Xi_{22} = \lambda W_2^TW_2 - Q_1 - Nh_{ij}P\Upsilon - Nh_{ij}\Upsilon^TP,$$
$$\Xi_{23} = Nh_{ij}\Upsilon^TPD, \qquad \Xi_{26} = -Nh_{ij}\Upsilon^TS^T, \qquad \Xi_{27} = -hNh_{ij}\Upsilon^TP. \tag{3.24}$$

Proof. Firstly, set

$$y(t) = (I_N \otimes A + G \otimes \Gamma)x(t) + (H \otimes \Upsilon)x(t-h) + (I_N \otimes B)F(x(t)) + (I_N \otimes C)F(x(t-h)), \tag{3.25}$$

then, (2.1) can be rewritten as

$$d\left[x(t) - \overline{D}x(t-h)\right] = y(t)dt + \sigma(t)dw(t). \tag{3.26}$$

From (3.26), we can have

$$\left[x(t) - \overline{D}x(t-h)\right] - \left[x(t-h) - \overline{D}x(t-2h)\right] = \int_{t-h}^{t} y(s)ds + \int_{t-h}^{t} \sigma(s)dw(s). \tag{3.27}$$

Consider the following Lyapunov functional for the system (3.26):

$$\begin{aligned} V(x_t,t) = {} & \left[x(t) - \overline{D}x(t-h)\right]^T (U \otimes P)\left[x(t) - \overline{D}x(t-h)\right] + \int_{t-h}^{t} x(s)^T(U \otimes Q_1)x(s)ds \\ & + \int_{t-2h}^{t} x(s)^T(U \otimes Q_2)x(s)ds + \int_{-h}^{0}\int_{t+\theta}^{t} y(s)^T(U \otimes Z)y(s)ds\, d\theta \\ & + \int_{t-h}^{t} F(x(s))^T(U \otimes R)F(x(s))ds, \quad t \geq h, \end{aligned} \tag{3.28}$$

where

$$U = \begin{pmatrix} N-1 & -1 & \cdots & -1 \\ -1 & N-1 & \cdots & -1 \\ \cdots & \cdots & \cdots & \cdots \\ -1 & -1 & \cdots & N-1 \end{pmatrix}. \tag{3.29}$$

Then, by the Itô's formula, the stochastic differential $dV(x_t,t)$ can be obtained

$$dV(x_t,t) = \mathcal{L}V(x_t,t)dt + 2\left[x(t) - \overline{D}x(t-h)\right]^T (U \otimes P)\sigma(t)dw(t), \tag{3.30}$$

where

$$\begin{aligned}\mathcal{L}V(x_t,t) = {} & 2\Big[x(t)-\overline{D}x(t-h)\Big]^T(U\otimes P)y(t)+\sigma(t)^T(U\otimes P)\sigma(t)+x(t)^T(U\otimes Q_1)x(t)\\ & -x(t-h)^T(U\otimes Q_1)x(t-h)+x(t)^T(U\otimes Q_2)x(t)-x(t-2h)^T(U\otimes Q_2)x(t-2h)\\ & +F(x(t))^T(U\otimes R)F(x(t))-F(x(t-h))^T(U\otimes R)F(x(t-h))+hy(t)^T(U\otimes Z)y(t)\\ & -\int_{t-h}^{t}\Big[y(s)^T(U\otimes Z)y(s)\Big]ds.\end{aligned}\tag{3.31}$$

By (3.27), we have

$$\begin{aligned}& 2\Big[x(t)-\overline{D}x(t-h)\Big]^T(U\otimes P)y(t)\\ & \quad = 2\Big[x(t)-\overline{D}x(t-h)\Big]^T(U\otimes P)\\ & \qquad \times[(I_N\otimes A+G\otimes\Gamma)x(t)+(I_N\otimes B)F(x(t))+(I_N\otimes C)F(x(t-h))]\\ & \qquad +2\Big[x(t)-\overline{D}x(t-h)\Big]^T(U\otimes P)(H\otimes\Upsilon)x(t-h)\\ & \quad = 2\Big[x(t)-\overline{D}x(t-h)\Big]^T(U\otimes P)\\ & \qquad \times[(I_N\otimes A+G\otimes\Gamma)x(t)+(I_N\otimes B)F(x(t))+(I_N\otimes C)F(x(t-h))]\\ & \qquad +2\left[x(t-h)-\overline{D}x(t-2h)+\int_{t-h}^{t}y(s)ds+\int_{t-h}^{t}\sigma(s)dw(s)\right]^T(U\otimes P)(H\otimes\Upsilon)x(t-h).\end{aligned}\tag{3.32}$$

From Corollary 3.2, it follows that

$$\begin{aligned}& \mathcal{E}\left(2\Big[x(t)-\overline{D}x(t-h)\Big]^T(U\otimes P)y(t)\right)\\ & \quad = \mathcal{E}\Bigg(2\Big[x(t)-\overline{D}x(t-h)\Big]^T(U\otimes P)\\ & \qquad\quad \times[(I_N\otimes A+G\otimes\Gamma)x(t)+(I_N\otimes B)F(x(t))+(I_N\otimes C)F(x(t-h))]\\ & \qquad\quad +2\left[x(t-h)-\overline{D}x(t-2h)+\int_{t-h}^{t}y(s)ds\right]^T(U\otimes P)(H\otimes\Upsilon)x(t-h)\Bigg).\end{aligned}\tag{3.33}$$

By (3.25), it is easy to know that for any matrix S, we have

$$2y(t)^T(U\otimes S)\left[(I_N\otimes A+G\otimes\Gamma)x(t)+(H\otimes\Upsilon)x(t-h)+(I_N\otimes B)F(x(t))\right.$$
$$\left.+(I_N\otimes C)F(x(t-h))-y(t)\right]=0. \tag{3.34}$$

From (3.31)–(3.34) and by the Propositions 2.7 and 2.8, it is easy to get

$$\begin{aligned}\mathcal{E}(\mathcal{L}V(x_t,t)) = \mathcal{E}\Bigg(\frac{1}{h}\int_{t-h}^{t}\Big[&2\left(x(t)-\overline{D}x(t-h)\right)^T(U\otimes P)\\
&\times[(I_N\otimes A+G\otimes\Gamma)x(t)+(I_N\otimes B)F(x(t))\\
&\quad+(I_N\otimes C)F(x(t-h))]+2\left(x(t-h)-\overline{D}x(t-2h)+hy(s)\right)^T\\
&\times(U\otimes P)(H\otimes\Upsilon)x(t-h)\\
&+\sigma(t)^T(U\otimes P)\sigma(t)+x(t)^T(U\otimes Q_1)x(t)-x(t-h)^T\\
&\times(U\otimes Q_1)x(t-h)+x(t)^T(U\otimes Q_2)x(t)\\
&-x(t-2h)^T(U\otimes Q_2)x(t-2h)+F(x(t))^T(U\otimes R)F(x(t))\\
&-F(x(t-h))^T(U\otimes R)F(x(t-h))\\
&+hy(t)^T(U\otimes Z)y(t)-hy(s)^T(U\otimes Z)y(s)+2y(t)^T(U\otimes S)\\
&\times((I_N\otimes A+G\otimes\Gamma)x(t)+(H\otimes\Upsilon)x(t-h)+(I_N\otimes B)F(x(t))\\
&\quad+(I_N\otimes C)F(x(t-h))-y(t))\Big]ds\Bigg)\\
=\mathcal{E}\Bigg(\frac{1}{h}\int_{t-h}^{t}\Bigg[\sum_{1\le i<j\le N}&\Big(2\left(x_i(t)-x_j(t)-D\left(x_i(t-h)-x_j(t-h)\right)\right)^T\\
&\times\left(PA-Ng_{ij}P\Gamma\right)\left(x_i(t)-x_j(t)\right)\\
&+2\left(x_i(t)-x_j(t)-D\left(x_i(t-h)-x_j(t-h)\right)\right)^T\\
&\times PB\left(f(x_i(t))-f(x_j(t))\right)\\
&+2\left(x_i(t)-x_j(t)-D\left(x_i(t-h)-x_j(t-h)\right)\right)^T\\
&\times PC\left(f(x_i(t-h))-f(x_j(t-h))\right)\\
&-2\left(x_i(t-h)-x_j(t-h)-D\left(x_i(t-2h)-x_j(t-2h)\right)\right)^T\\
&\times\left(Nh_{ij}P\Upsilon\right)\left(x_i(t-h)-x_j(t-h)\right)
\end{aligned}$$

$$
\begin{aligned}
&- 2h\big(y_i(s) - y_j(s)\big)^T N h_{ij} P \Upsilon \big(x_i(t-h) - x_j(t-h)\big)\Big) \\
&+ \big(\sigma_i(t, x_i(t), x_i(t-h)) - \sigma_j\big(t, x_j(t), x_j(t-h)\big)\big)^T \\
&\times P\big(\sigma_i(t, x_i(t), x_i(t-h)) - \sigma_j\big(t, x_j(t), x_j(t-h)\big)\big) \\
&+ \big(x_i(t) - x_j(t)\big)^T Q_1 \big(x_i(t) - x_j(t)\big) - \big(x_i(t-h) - x_j(t-h)\big)^T \\
&\times Q_1 \big(x_i(t-h) - x_j(t-h)\big) \\
&+ \big(x_i(t) - x_j(t)\big)^T Q_2 \big(x_i(t) - x_j(t)\big) \\
&- \big(x_i(t-2h) - x_j(t-2h)\big)^T Q_2 \big(x_i(t-2h) - x_j(t-2h)\big) \\
&+ \big(f(x_i(t)) - f\big(x_j(t)\big)\big)^T R\big(f(x_i(t)) - f\big(x_j(t)\big)\big) \\
&- \big(f(x_i(t-h)) - f\big(x_j(t-h)\big)\big)^T \\
&\times R\big(f(x_i(t-h)) - f\big(x_j(t-h)\big)\big) \\
&+ h\big(y_i(t) - y_j(t)\big)^T Z\big(y_i(t) - y_j(t)\big) \\
&- \ h\big(y_i(s) - y_j(s)\big)^T Z\big(y_i(s) - y_j(s)\big) \\
&+ 2\big(y_i(t) - y_j(t)\big)^T \big(SA - N g_{ij} S\Gamma\big)\big(x_i(t) - x_j(t)\big) \\
&- 2\big(y_i(t) - y_j(t)\big)^T \big(N h_{ij} S \Upsilon\big)\big(x_i(t-h) - x_j(t-h)\big) \\
&+ 2\big(y_i(t) - y_j(t)\big)^T SB\big(f(x_i(t)) - f\big(x_j(t)\big)\big) \\
&+ 2\big(y_i(t) - y_j(t)\big)^T SC\big(f(x_i(t-h)) - f\big(x_j(t-h)\big)\big) \\
&-2\big(y_i(t) - y_j(t)\big)^T S\big(y_i(t) - y_j(t)\big)\Bigg] ds \Bigg).
\end{aligned}
\tag{3.35}
$$

According to Assumptions 2.5 and (3.22), it is clear that

$$
\begin{aligned}
&\big(\sigma_i(t, x_i(t), x_i(t-h)) - \sigma_j\big(t, x_j(t), x_j(t-h)\big)\big)^T P\big(\sigma_i(t, x_i(t), x_i(t-h)) - \sigma_j\big(t, x_j(t), x_j(t-h)\big)\big) \\
&\quad \le \lambda\big(x_i(t) - x_j(t)\big)^T W_1^T W_1 \big(x_i(t) - x_j(t)\big) \\
&\qquad + \ \lambda\big(x_i(t-h) - x_j(t-h)\big)^T W_2^T W_2 \big(x_i(t-h) - x_j(t-h)\big).
\end{aligned}
\tag{3.36}
$$

By Assumption 2.6, we can obtain

$$
\begin{aligned}
0 \le{}& 2\epsilon\big(x_i(t) - x_j(t)\big)^T U^T \big(f(x_i(t)) - f\big(x_j(t)\big)\big) + 2\epsilon\big(f(x_i(t)) - f\big(x_j(t)\big)\big)^T V\big(x_i(t) - x_j(t)\big) \\
&- 2\epsilon\big(x_i(t) - x_j(t)\big)^T U^T V\big(x_i(t) - x_j(t)\big) - 2\epsilon\big(f(x_i(t)) - f\big(x_j(t)\big)\big)^T \big(f(x_i(t)) - f\big(x_j(t)\big)\big).
\end{aligned}
\tag{3.37}
$$

Combining (3.35)–(3.37), we have

$$\mathcal{E}(\mathcal{L}V(x_t,t)) \leq \mathcal{E}\left[\frac{1}{h}\int_{t-h}^{t}\sum_{1\leq i<j\leq N}\xi_{ij}^{T}\Xi\xi_{ij}ds\right], \tag{3.38}$$

where

$$\xi_{ij} = \begin{pmatrix} x_i(t)-x_j(t) \\ x_i(t-h)-x_j(t-h) \\ x_i(t-2h)-x_j(t-2h) \\ f(x_i(t))-f(x_j(t)) \\ f(x_i(t-h))-f(x_j(t-h)) \\ y_i(t)-y_j(t) \\ y_i(s)-y_j(s) \end{pmatrix}. \tag{3.39}$$

Since $\Xi < 0$, it is guaranteed that all the subsystems in (2.1) are globally asymptotically synchronized in the mean square. The proof is completed. □

Remark 3.7. We note here that if $D = 0$ in (2.1), then system (2.1) describes a kind of stochastic delayed complex networks considered in [32]. Our result can be applied to this case, and we have pointed out that [32] made a mistake when dealing with expectations of stochastic cross terms in Remark 3.3. If we let A be a diagonal and negative matrix and let $D = 0$ in (2.1), the system (2.1) will be an array of coupled neural networks consisting of N nodes, in which each node is an n-dimensional stochastic delayed Hopfield neural network. As to stochastic Hopfield neural networks with time delays, [30, 38] have investigated the stability problems, respectively. Furthermore, if we don't consider stochastic disturbances and time delays in stochastic delayed Hopfield neural networks, then this kind of neural networks is the famous Hopfield neural network.

Remark 3.8. If we do not consider the stochastic disturbances in (2.1), then the system will be a kind of determinate neutral-type delayed complex networks, that have been considered in the [18–20]. If we let A be a diagonal and negative matrix in this kind of determinate neutral-type delayed complex networks, each node will be an n-dimensional neutral-type delayed neural network. For neutral-type neural networks with time delays, [39, 40] have discussed the stability problems and presented the new and effective stability conditions, respectively.

Remark 3.9. For neutral stochastic delay systems, a very active topic is to obtain the delay-dependent condition. For example, [28, 29] considered delay-dependent stability problems for neutral stochastic delay systems. However, these two papers used bounding techniques including the Jensen inequality to deal with stochastic cross terms contain the Itô integral. Obviously, bounding techniques will increase the conservatism. In the derivation process of Theorem 3.6, we don't use any bounding technique to deal with stochastic cross terms. Therefore, this method can show less conservatism and can also be extended to solve delay-dependent stability problems for neutral stochastic delay systems.

Remark 3.10. In Theorem 3.6, we give a delay-dependent synchronization criterion by the linear matrix inequalities (LMIs), because LMIs can be easily solved by using the Matlab

LMI toolbox and no tuning of parameters is required. Moreover, we can easily get the maximum possible upper bound on the delay by the LMI toolbox. The maximum possible upper bound on the delay is the main criterion for judging the conservatism of a delay-dependent condition.

4. Numerical Example

In this section, we present a simulation example to illustrate the effectiveness of our approach.

Example 4.1. Consider the following complex network consisting of three identical nodes:

$$\begin{aligned} &d[x_i(t) - Dx_i(t-h)] \\ &\quad = \left[Ax_i(t) + Bf(x_i(t)) + Cf(x_i(t-h)) + \sum_{j=1}^{3} g_{ij}\Gamma x_j(t) + \sum_{j=1}^{3} h_{ij}\Upsilon x_j(t-h)\right]dt \\ &\qquad + \sigma_i(t, x_i(t), x_i(t-h))dw(t), \end{aligned} \tag{4.1}$$

for all $i = 1, 2, 3$, where $x_i(t) = [x_{i1}(t), x_{i2}(t)]^T \in \mathcal{R}^2$ is the state vector of the ith subsystem, and

$$\begin{gathered} A = \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}, \quad B = \begin{pmatrix} 0.6 & -0.1 \\ -0.3 & 0.5 \end{pmatrix}, \quad C = \begin{pmatrix} -0.5 & -0.1 \\ 0.2 & -1.5 \end{pmatrix}, \quad D = \begin{pmatrix} -0.6 & 0 \\ 0 & -0.6 \end{pmatrix}, \\ G = \begin{pmatrix} -3 & 1 & 2 \\ 1 & -2 & 1 \\ 2 & 1 & -3 \end{pmatrix}, \quad H = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix}, \quad \Gamma = \begin{pmatrix} 0.5 & 0 \\ 0.1 & 0.5 \end{pmatrix}, \quad \Upsilon = \begin{pmatrix} 0.5 & 0.1 \\ 0 & 0.4 \end{pmatrix}, \\ \sigma(t, x(t), x(t-h)) = \begin{pmatrix} \sqrt{0.15} & 0 & \sqrt{0.2} & 0 \\ 0 & \sqrt{0.15} & 0 & \sqrt{0.2} \end{pmatrix} \begin{pmatrix} x(t) \\ x(t-h) \end{pmatrix}, \\ f(x_i(t)) = \left(f_1(x_{i1}(t)), f_2(x_{i2}(t))\right)^T = (\tanh(x_{i1}(t)), \tanh(x_{i2}(t)))^T. \end{gathered} \tag{4.2}$$

Thus, the matrices U, V, W_1, W_2, in the Assumptions 2.5 and 2.6 are

$$U = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad V = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad W_1 = \begin{pmatrix} \sqrt{0.3} & 0 \\ 0 & \sqrt{0.3} \end{pmatrix}, \quad W_2 = \begin{pmatrix} \sqrt{0.4} & 0 \\ 0 & \sqrt{0.4} \end{pmatrix}. \tag{4.3}$$

According to Theorem 3.6, the allowable maximum delay h, that can guarantee the globally asymptotic mean-square synchronization of the neutral-type stochastic delayed complex networks, is 0.33. When we randomly choose the the initial states in $[0, 1] \times [0, 1]$, the synchronization errors are plotted in Figures 1 and 2, which can confirm that the neutral-type stochastic delayed complex system is globally synchronized in the mean square.

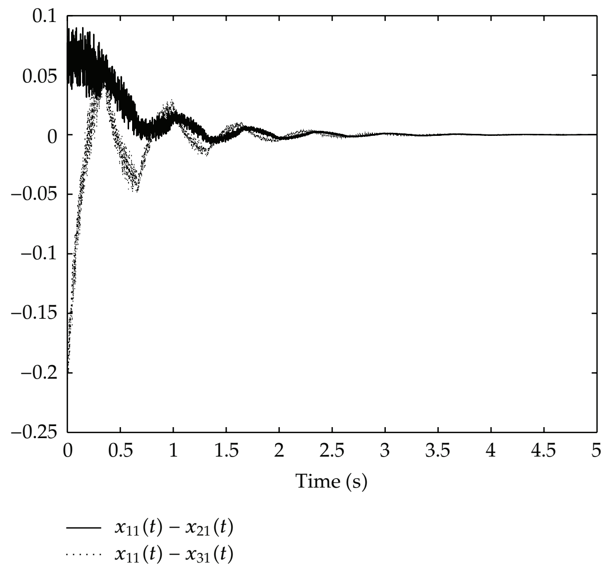

Figure 1: State error of $x_{11}(t) - x_{i1}(t), i = 2, 3$.

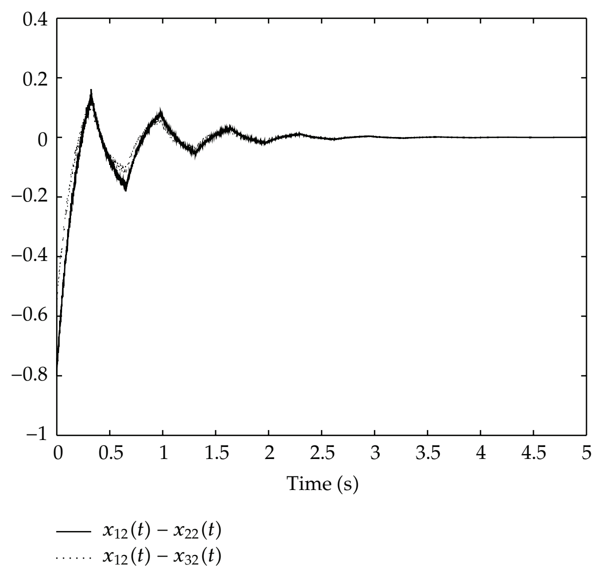

Figure 2: State error of $x_{12}(t) - x_{i2}(t), i = 2, 3$.

5. Conclusions

This paper has investigated the problem of delay-dependent synchronization criterion for neutral-type stochastic delayed complex networks. Most important of all, this paper is concerned with expectations of stochastic cross terms containing the Itô integral. By stochastic analysis techniques, we prove that among these stochastic cross terms, $x(t-h)^T \mathbb{K} \int_{t-h}^{t} \mu(s, x_s) dw(s)$ is the only one whose expectation is equal to zero. Then, this paper has utilized this conclusion to give a delay-dependent synchronization criterion for neutral-type stochastic delayed complex networks. In the derivation process, the mathematical development avoids bounding stochastic cross terms. Thus, the method in our paper can lead to a criterion with less conservatism, and a numerical example is provided to demonstrate the effectiveness of the proposed approach.

Acknowledgments

The work of Y. Zhang and B. Song was supported by the National Natural Science Foundation of China under Grants no. 61104221 and the Natural Science Foundation of the Jiangsu Higher Education Institutions of China under Grant no. 10KJB120004. The work of J. H. Park was supported by 2012 Yeungnam University Research Grant. The work of Z.-G. Wu was supported by the National Natural Science Foundation of China under Grant no. 61174029.

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