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Research Article

Delay-Independent Stability of Switched Linear Systems with Unbounded Time-Varying Delays

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This paper is focused on delay-independent stability analysis for a class of switched linear systems with time-varying delays that can be unbounded. When the switched system is not necessarily positive, we first establish a delay-independent stability criterion under arbitrary switching signal by using a new method that is different from the methods to positive systems in the literature. We also apply this method to a class of time-varying switched linear systems with mixed delays.

1. Introduction

The theory of switched systems has historically assumed a position of great importance in systems theory and has been studied extensively in recent years [1–6]. A switched system is a type of hybrid dynamic system that consists of a family of continuous-time (discrete-time) subsystems and a switching signal, which determines the switching between subsystems. The stability of switched linear systems under arbitrary switching signal is a very important problem, which is usually studied by a common Lyapunov functional approach, especially a common quadratic Lyapunov functional approach [7–10].

Very recently, the stability of positive switched linear system has attracted a lot of attention [11–16]. As usual, a system is said to be positive if its state and outputs are nonnegative whenever the initial condition and inputs are nonnegative. For stability of positive switched linear system under arbitrary switching signal, a common linear copositive Lyapunov function is usually applied [17–20]. A switched linear copositive Lyapunov function has been used in discrete-time positive switched systems in [21]. When the positive switched linear system involves multiple time-varying delays that can be unbounded, it has been proved in [22] that the stability of such systems under any switching signal does not depend on delays if the switched system shares a common linear copositive Lyapunov function, which generalizes the early results in [23, 24].

For the general switched linear systems with unbounded time-varying delays, it is necessary to consider whether the similar delay-independent stability criterion under arbitrary switching signal can also be derived. Note that the system is not necessarily positive; the methods to positive systems in [22] usually do not hold. Consequently, to answer this problem, we need a new approach that is different from those methods to positive systems in the literature.

The main purpose of this paper is to establish a delay-independent stability criterion under arbitrary switching signal for the general switched linear systems with time-varying delays that can be unbounded. Since the switched systems are not necessarily positive, a new method based on some smart techniques of real analysis is proposed. By using this method, we not only present a delay-independent stability criterion for the system, but also extend the main result to a class of time-varying switched system with mixed delays, where one kind of delays is time-varying state delay that can be unbounded and the others are bounded time-varying distributed delay. Another advantage of the new method used in this paper lies in that it imposes less constraint on unbounded state delays than that given in [22] (see the corresponding discussion in Section 2).

Notations. Say $A \succcurlyeq 0 (\prec 0)$ if all elements of matrix A are nonnegative (negative). We write $A \succcurlyeq B$ if and only if $A - B \succcurlyeq 0$. Denote by $\mathbb{M}$ the the set of Metzler matrices whose off-diagonal entries are nonnegative. $\mathbb{R}^n$ is an n-dimensional real vector space, $\mathbb{R}^n_+$ is the set of positive vectors, and $\mathbb{R}^{n\times n}$ is the set of real $n \times n$-dimensional matrices. For $x \in \mathbb{R}^n$, denote $\|x\| = \max_{1\leq i\leq n}\{|x_i|\}$. For positive integers p, q, n, and m, denote $\underline{p} = \{1, 2, \ldots, p\}$, $\underline{q} = \{1, 2, \ldots, q\}$, $\underline{n} = \{1, 2 \ldots, n\}$, and $\underline{m} = \{1, 2, \ldots, m\}$.

2. Problem Statements and Preliminaries

Consider the following switched linear system with time-varying delays:

$$\begin{aligned} \dot{x}(t) &= A_{0\varrho}x(t) + \sum_{l=1}^{p} A_{l\varrho}x\left(t - \tau_{l\varrho}(t)\right), \quad t \geq 0, \\ x(t) &= \phi(t), \quad t \in [-\alpha, 0], \end{aligned} \tag{2.1}$$

where $x \in \mathbb{R}^n$ is the state; the piecewise continuous function $\varrho : [0, \infty) \to \underline{m}$ is the switching signal; $A_{ls} \in \mathbb{R}^{n\times n}$ are constant matrices for $l \in \underline{p} \cup \{0\}$ and $s \in \underline{m}$; time-varying delays $\tau_{ls}(t) \geq 0$, $l \in \underline{p}$, $s \in \underline{m}$, are continuous on $[0, \infty)$; $\phi(t)$ is the continuous vector-valued initial function on $[-\alpha, 0]$ with $\alpha = \max_{l\in\underline{p}, s\in\underline{m}} \sup_{t\geq 0}\{\tau_{ls}(t) - t\}$.

Unlike the assumptions on the system matrices [22], we here do not require $A_{0s} \in \mathbb{M}$ and $A_{ls} \succcurlyeq 0$ for $l \in \underline{p}$ and $s \in \underline{m}$. What is more, we make a less restrictive assumption on time-varying delays $\tau_{ls}(t)$ as follows:

(H1) $\lim_{t\to+\infty} t - \tau_{ls}(t) = +\infty$, $l \in \underline{p}$, $s \in \underline{m}$.

We recall to introduce another assumption on $\tau_{ls}(t)$ in [22] as follows:

(H1′) there exist $T > 0$ and a scaler $0 \leq \theta < 1$ such that $\theta = \sup_{t>T, l\in\underline{p}, s\in\underline{m}} \tau_{ls}(t)/t$.

We show that (H1) is less constrained than (H1′). In fact, it is not difficult to see that (H1′) implies (H1). However, (H1) does not yield (H1′). For example, let $\tau_{ls}(t) = t - \sqrt{t} \geq 0$ for $t \geq 0$.

Since

$$\lim_{t\to+\infty} t - \tau_{ls}(t) = \lim_{t\to+\infty} \sqrt{t} = +\infty,$$
$$\lim_{t\to+\infty} \frac{\tau_{ls}(t)}{t} = \lim_{t\to+\infty} \frac{t-\sqrt{t}}{t} = 1, \tag{2.2}$$

we see that (H1) holds while (H1′) does not hold.

In the sequel, we say system (2.1) is asymptotically stable under arbitrary switching signal, if for any $\epsilon > 0$, there exists $\delta > 0$ such that any solution $x(t,\phi,\varrho)$ of system (2.1) under arbitrary switching signal satisfies $\|x(t,\phi,\varrho)\| < \epsilon$ when $\|\phi\| < \delta$, and $\lim_{t\to+\infty}\|x(t,\phi,\varrho)\| = 0$.

Generally speaking, due to the less constraint on delays $\tau_{ls}(t)$ and system matrices $A_{ls} \not\succcurlyeq 0$ for $l \in \underline{p}$ and $s \in \underline{m}$, the methods for positive systems in the literature usually become invalid. Consequently, a new method should be introduced to analyze the delay-independent stability for system (2.1) under arbitrary switching signal.

3. Main Result

In the sequel, we denote $\widetilde{A}_{ls} = [\widetilde{a}_{ij}^{(ls)}]$ and $A_{ls} = [a_{ij}^{(ls)}]$ for $l \in \underline{p} \cup \{0\}$ and $s \in \underline{m}$, where

$$\widetilde{a}_{ij}^{(0s)} = a_{ij}^{(0s)}, \quad i = j,$$
$$\widetilde{a}_{ij}^{(0s)} = \left|a_{ij}^{(0s)}\right|, \quad i \neq j, \tag{3.1}$$
$$\widetilde{a}_{ij}^{(ls)} = \left|a_{ij}^{(ls)}\right|, \quad l \in \underline{p}.$$

It is easy to see that $\widetilde{A}_{0s} \in \mathbb{M}$ and $\widetilde{A}_{ls} \succcurlyeq 0$ for $s \in \underline{m}$.

We now present the main result of this paper.

Theorem 3.1. *Assume that (H1) holds. If there exists a vector $\xi \in \mathbb{R}_+^n$ such that*

$$\mathcal{A}_s \xi \prec 0, \quad s \in \underline{m}, \tag{3.2}$$

where $\mathcal{A}_s = \sum_{l=0}^{p} \widetilde{A}_{ls}$, then system (2.1) *is asymptotically stable under arbitrary switching signal.*

Proof. Denote $\xi = [\xi_1, \xi_2, \ldots, \xi_n]^T$ and

$$\mathcal{A}_s \xi = -\left[\eta_{1s}, \eta_{2s}, \ldots, \eta_{ns}\right]^T, \quad s \in \underline{m}, \tag{3.3}$$

where $\eta_{is} > 0$ for $i \in \underline{n}$ by (3.2). The remaining proof is divided into two parts.

(i) For any constant $\epsilon > 0$, there exists a constant $\delta > 0$ such that $\|x(t,\phi,\varrho)\| < \epsilon$ when $\|\phi\| < \delta$. In the sequel, we denote the ith element of the solution $x(t,\phi,\varrho)$ of system (2.1) by $x_i(t)$ for $i \in \underline{n}$.

In fact, for any given $\epsilon > 0$, let $\delta = (d_1/d_2)\epsilon$, where

$$d_1 = \min_{i\in\underline{n}}\{\xi_1,\xi_2,\ldots,\xi_n\},$$
$$d_2 = \max_{i\in\underline{n}}\{\xi_1,\xi_2,\ldots,\xi_n\}. \tag{3.4}$$

When $\|\phi\| < \delta$, we prove that

$$|x_i(t)| < \frac{\xi_i}{d_1}\delta, \quad t \geq 0,\ i \in \underline{n}. \tag{3.5}$$

Note that $\xi_i/d_1 \geq 1$ for $i \in \underline{n}$; then

$$|x_i(0)| \leq \|\phi\| < \delta \leq \frac{\xi_i}{d_1}\delta, \quad i \in \underline{n}. \tag{3.6}$$

By the continuity of the solution of system (2.1), we have that there exists $t' > 0$ such that

$$|x_i(t)| < \frac{\xi_i}{d_1}\delta, \quad t \in [0,t'],\ i \in \underline{n}. \tag{3.7}$$

We further show that (3.5) holds if $\|\phi\| < \delta$. Otherwise, there exists $t^* > 0$ and at least one index $k \in \underline{n}$ such that

$$|x_k(t^*)| = \frac{\xi_k}{d_1}\delta, \quad |x_i(t)| < \frac{\xi_i}{d_1}\delta, \quad 0 \leq t < t^*,\ i \in \underline{n}, \tag{3.8}$$

which implies $D_-|x_k(t^*)| \geq 0$, where D_- means the left derivative. Set the left limitation $\varrho(t^*-) = s_1 \in \underline{m}$. By (2.1), (3.1), (3.3), and (3.8), we get

$$\begin{aligned} D_-|x_k(t^*)| &= D_-x_k(t^*)\,\mathrm{sign}\ x_k(t^*) \\ &\leq a_{kk}^{(0s_1)}\left|x_j(t^*)\right| + \sum_{j=1,j\neq k}^{n}\left|a_{kj}^{(0s_1)}\right|\left|x_j(t^*)\right| + \sum_{l=1}^{p}\sum_{j=1}^{n}\left|a_{kj}^{(ls_1)}\right|\left|x_j(t^*-\tau_{ls_1}(t^*))\right| \\ &\leq \frac{\delta}{d_1}\sum_{l=0}^{p}\sum_{j=1}^{n}\tilde{a}_{kj}^{(ls_1)}\xi_j = -\frac{\delta}{d_1}\eta_{ks_1} < 0. \end{aligned} \tag{3.9}$$

From (3.9), we get a contradiction with the fact $D_{-}|x_k(t^*)| \geq 0$. Therefore, for any $\epsilon > 0$, by choosing $\delta = (d_1/d_2)\epsilon$ and using (3.5), we have that

$$\left\|x(t,\phi,\varrho)\right\| < \max_{i\in\underline{n}}\left\{\frac{\xi_i}{d_1}\delta\right\} \leq \frac{d_2}{d_1}\delta = \epsilon, \quad t \geq 0, \tag{3.10}$$

if $\|\phi\| < \delta$. This completes the proof of part (i).

(ii) For any solution $x(t,\phi,\varrho)$ of system (2.1), $\lim_{t\to+\infty}\|x(t,\phi,\varrho)\| = 0$.

Let $x_i(t) = \xi_i y_i(t)$ for $i \in \underline{n}$. Denote the upper limitation of $|y_i(t)|$ by $\overline{\lim}_{t\to+\infty}|y_i(t)| = \overline{\beta}_i$ and the lower limitation of $|y_i(t)|$ by $\underline{\lim}_{t\to+\infty}|y_i(t)| = \beta_i$ for $i \in \underline{n}$. Set $\overline{\beta}_k = \max_{i\in\underline{n}}\{\overline{\beta}_i\}$ for some $k \in \underline{n}$ and

$$c_{ks} = \left|\tilde{a}_{kk}^{(0s)}\right|\xi_k + \sum_{j=1,j\neq k}^{n}\tilde{a}_{kj}^{(0s)}\xi_j + \sum_{l=1}^{p}\sum_{j=1}^{n}\tilde{a}_{kj}^{(ls)}\xi_j. \tag{3.11}$$

We first show that $\overline{\beta}_k = \beta_k$. Assume to the contrary that $\overline{\beta}_k > \beta_k$. Choose a sufficiently small ϵ satisfying

$$0 < \epsilon < \min\left\{\overline{\beta}_k, \frac{\min_{s\in\underline{m}}\{\eta_{ks}\}\overline{\beta}_k}{\max_{s\in\underline{m}}\{c_{ks}\}}\right\}. \tag{3.12}$$

By the definition of $\overline{\beta}_k$, we have that $|y_i(t)| \leq \overline{\beta}_k + \epsilon$, $i \in \underline{n}$, hold for sufficiently large t. Since $\lim_{t\to+\infty} t - \tau_{ls}(t) = +\infty$ for $l \in \underline{p}$ and $s \in \underline{m}$, we have that there exists sufficiently large $t_1 > 0$ such that

$$\begin{aligned} \left|y_i(t)\right| &\leq \overline{\beta}_k + \epsilon, \quad t \geq t_1, \\ \left|y_i(t-\tau_{ls}(t))\right| &\leq \overline{\beta}_k + \epsilon, \quad t \geq t_1, \end{aligned} \tag{3.13}$$

where $i \in \underline{n}$, $l \in \underline{p}$ and $s \in \underline{m}$.

On the other hand, by the assumption that $\overline{\beta}_k > \beta_k$ and the choice of ϵ, there exists a sufficiently large $t_2 > t_1 > 0$ such that

$$\begin{aligned} \left|y_k(t_2)\right| &\geq \overline{\beta}_k - \epsilon > 0, \\ D_{+}\left|y_k(t_2)\right| &\geq 0, \end{aligned} \tag{3.14}$$

where D_{+} means the right derivative. Otherwise, we have $|y_k(t)| < \overline{\beta}_k - \epsilon$ or $D_{+}|y_k(t)| < 0$ eventually, which contradicts with the assumption $\overline{\lim}_{t\to+\infty} y_k(t) = \overline{\beta}_k > \beta_k = \underline{\lim}_{t\to+\infty} y_k(t)$.

By (3.3), it is easy to see that $\widetilde{a}_{kk}^{0s} < 0$. Denote the right limitation $\varrho(t_2+) = s_2 \in \underline{m}$. By (3.3), (3.13), and (3.14), we get

$$\begin{aligned} D_+\left|y_k(t_2)\right| &= \xi_k^{-1} D_+ x_k(t_2) \operatorname{sign} x_k(t_2) \\ &\leq \xi_k^{-1}\left[\sum_{j=1}^{n} \widetilde{a}_{kj}^{(0s_2)} \xi_j \left|y_j(t_2)\right| + \sum_{l=1}^{p}\sum_{j=1}^{n} \widetilde{a}_{kj}^{(ls_2)} \xi_j \left|y_j(t_2 - \tau_{ls_2}(t_2))\right|\right] \\ &\leq \xi_k^{-1}\left[\overline{\beta}_k \sum_{l=0}^{p}\sum_{j=1}^{n} \widetilde{a}_{kj}^{(ls_2)} \xi_j + \epsilon c_{ks_2}\right] \\ &= \xi_k^{-1}\left[-\overline{\beta}_k \eta_{ks_2} + \epsilon c_{ks_2}\right] \\ &\leq -\frac{\omega_k}{\xi_k} < 0, \end{aligned} \tag{3.15}$$

where $\omega_k = \overline{\beta}_k \min_{s\in\underline{m}}\{\eta_{ks}\} - \epsilon \max_{s\in\underline{m}}\{c_{ks}\} > 0$ by (3.12). This is a contradiction with the fact that $D_+|y_k(t_2)| \geq 0$. Therefore, $\overline{\beta}_k = \beta_k$.

Next, we show that $\overline{\beta}_k = \beta_k = 0$. Otherwise, $\overline{\beta}_k = \beta_k > 0$. Then, for sufficiently small ϵ satisfying (3.12), there exists $t_3 > t_1$ such that (3.13) holds, and

$$\left|y_k(t)\right| \geq \overline{\beta}_k - \epsilon > 0, \quad t \geq t_3. \tag{3.16}$$

Here, (3.16) is concluded from the property of the lower limitation $\underline{\lim}_{t\to+\infty}|y_k(t)| = \beta_k = \overline{\beta}_k$. Similar to the above analysis, we have

$$D_+\left|y_k(t)\right| \leq -\frac{\omega_k}{\xi_k}, \quad t \geq t_3. \tag{3.17}$$

Integrating (3.17) from t_3 to t on both sides, we get the following contradiction:

$$\left|y_k(t)\right| \leq \left|y_k(t_3)\right| - \frac{\omega_k}{\xi_k}(t - t_3) \longrightarrow -\infty \quad (t \longrightarrow +\infty). \tag{3.18}$$

Thus, $\lim_{t\to+\infty}|y_k(t)| = 0$. By the choice of k and the definition of $y_i(t)$, we have that $\lim_{t\to+\infty}|x_i(t)| = 0$ for $i \in \underline{n}$, which implies that $\lim_{t\to+\infty}\|x(t,\phi,\varrho)\| = 0$.

By (i) and (ii), system (2.1) is asymptotically stable under arbitrary switching signal. This completes the proof of Theorem 3.1. □

Remark 3.2. For the particular case when $m = 1$, condition (3.2) holds if and only if $\mathcal{A}_1$ is a Hurwitz matrix [25]. When $m > 1$, it requires that all $\mathcal{A}_s$, $s \in \underline{r}$, share a common $\xi \in \mathbb{R}_+^n$ such that $-\mathcal{A}_s\xi \in \mathbb{R}_+^n$. This problem has been studied in [20], where necessary and sufficient conditions for the existence of such a vector ξ were established.

4. Extension to Time-Varying Switched Systems with Mixed Delays

We now extend Theorem 3.1 to a class of time-varying switched system with mixed delays:

$$\begin{gathered}\dot{x}(t) = A_{0\varrho}(t)x(t) + \sum_{l=1}^{p} A_{l\varrho}(t)x\left(t-\tau_{l\varrho}(t)\right) + \sum_{r=1}^{q} B_{r\varrho}(t)\int_{t-\sigma_{r\varrho}(t)}^{t} x(u)du, \quad t \geq 0, \\ x(t) = \phi(t), \quad t \in [-\gamma, 0],\end{gathered} \tag{4.1}$$

where $A_{ls}(t) = [a_{ij}^{(ls)}(t)]$, $B_{rs}(t) = [b_{ij}^{(rs)}(t)]$ are continuous matrix function on $[0,\infty)$, delays $\sigma_{rs}(t)$ are continuous on $[0,\infty)$, and $\gamma = \max_{l\in\underline{p},r\in\underline{q},s\in\underline{m}}\{\sup_{t\geq 0}\{\tau_{ls}(t)-t\},\sup_{t\geq 0}\{\sigma_{rs}(t)-t\}\}$.

Assume that

(H2) there exist constants $\theta_{rs} > 0$ such that $0 \leq \sigma_{rs}(t) \leq \theta_{rs}$ for $r \in \underline{q}$ and $s \in \underline{m}$;

(H3) there exist constant matrices $\widehat{A}_{0s} = [\widehat{a}_{ij}^{(0s)}] \in \mathbb{M}$ and $\widehat{A}_{ls} = [\widehat{a}_{ij}^{(ls)}] \succcurlyeq 0$, $\widehat{B}_{rs} = [\widehat{b}_{ij}^{(rs)}] \succcurlyeq 0$ such that, for $t \geq 0$ and $i, j \in \underline{n}$,

$$\begin{aligned} a_{ij}^{(0s)}(t) &\leq \widehat{a}_{ij}^{(0s)}, \quad i = j, \\ \left|a_{ij}^{(0s)}(t)\right| &\leq \widehat{a}_{ij}^{(0s)}, \quad i \neq j, \\ \left|a_{ij}^{(ls)}(t)\right| &\leq \widehat{a}_{ij}^{(ls)}, \quad l \in \underline{p}, \\ \left|b_{ij}^{(rs)}(t)\right| &\leq \widehat{b}_{ij}^{(rs)}, \quad r \in \underline{q}. \end{aligned} \tag{4.2}$$

When $x_i(t) \neq 0$, a straightforward computation based on (4.2) yields that

$$\begin{aligned} D_{\pm}|x_i(t)| = \operatorname{sign} x_i(t) &\left[\sum_{j=1}^{n} a_{ij}^{(0\varrho_\pm)}(t)x_j(t) + \sum_{l=1}^{p}\sum_{j=1}^{n} a_{ij}^{(l\varrho_\pm)}(t)x_j\left(t-\tau_{l\varrho_\pm}(t)\right)\right. \\ &\left. + \sum_{r=1}^{q}\sum_{j=1}^{n} b_{ij}^{(r\varrho_\pm)}(t)\int_{t-\sigma_{r\varrho_\pm}(t)}^{t} x_j(u)du\right] \\ \leq \sum_{j=1}^{n} \widehat{a}_{ij}^{(0\varrho_\pm)}\left|x_j(t)\right| &+ \sum_{l=1}^{p}\sum_{j=1}^{n} \widehat{a}_{ij}^{(l\varrho_\pm)}\left|x_j\left(t-\tau_{l\varrho_\pm}(t)\right)\right| \\ + \sum_{r=1}^{q}\theta_{r\varrho_\pm} &\sum_{j=1}^{n} \widehat{b}_{ij}^{(r\varrho_\pm)} \max_{t-\theta_{r\varrho_\pm}\leq u\leq t}\left\{\left|x_j(u)\right|\right\}, \end{aligned} \tag{4.3}$$

where $\varrho_\pm = \varrho(t\pm)$. Then, similar to the analysis in Theorem 3.1, it is not difficult to get the following stability criterion for system (4.1).

Theorem 4.1. *Assume that (H1)–(H3) hold. If there exists a vector $\xi \in \mathbb{R}_+^n$ such that*

$$\mathcal{B}_s \xi \prec 0, \quad s \in \underline{m}, \tag{4.4}$$

where $\mathcal{B}_s = \sum_{l=0}^{p} \widehat{A}_{ls} + \sum_{r=1}^{q} \theta_{rs} \widehat{B}_{rs}$, then system (4.1) *is asymptotically stable under arbitrary switching signal.*

Consider the following uncertain switched system:

$$\begin{aligned} \dot{x}(t) &= \left[A_{0\varrho} + \Delta A_{0\varrho}(t)\right] x(t) + \sum_{l=1}^{p} \left[A_{l\varrho} + \Delta A_{l\varrho}(t)\right] x\left(t - \tau_{l\varrho}(t)\right) \\ &\quad + \sum_{r=1}^{q} \left[B_{r\varrho} + \Delta B_{r\varrho}(t)\right] \int_{t-\sigma_{r\varrho}(t)}^{t} x(u) du, \quad t \geq 0 \\ x(t) &= \phi(t), \quad t \in [-\gamma, 0], \end{aligned} \tag{4.5}$$

where $\Delta A_{ls}(t) = [\Delta a_{ij}^{(ls)}(t)]$ and $\Delta B_{rs}(t) = [\Delta b_{ij}^{(ls)}(t)]$ are uncertain matrices satisfying

$$\Delta \left| a_{ij}^{(ls)}(t) \right| \leq \Delta a_{ij}^{(ls)}, \quad \left| \Delta b_{ij}^{(ls)}(t) \right| \leq \Delta b_{ij}^{(ls)}, \quad t \geq 0. \tag{4.6}$$

Set

$$\begin{aligned} \widehat{a}_{ij}^{(0s)} &= a_{ij}^{(0s)} + \Delta a_{ij}^{(0s)}, \quad i = j, \\ \widehat{a}_{ij}^{(0s)} &= \left| a_{ij}^{(0s)} \right| + \Delta a_{ij}^{(0s)}, \quad i \neq j, \\ \widehat{a}_{ij}^{(ls)} &= \left| a_{ij}^{(ls)} \right| + \Delta a_{ij}^{(ls)}, \quad l \in \underline{p}, \\ \widehat{b}_{ij}^{(rs)} &= \left| b_{ij}^{(rs)} \right| + \Delta b_{ij}^{(rs)}, \quad r \in \underline{q}. \end{aligned} \tag{4.7}$$

Then, based on the same analysis as above, we have the following result for the uncertain system (4.5).

Theorem 4.2. *Assume that (H1) and (H2) hold. If there exists a vector $\xi \in \mathbb{R}_+^n$ such that* (4.4) *holds, then system* (4.5) *is asymptotically stable under arbitrary switching signal.*

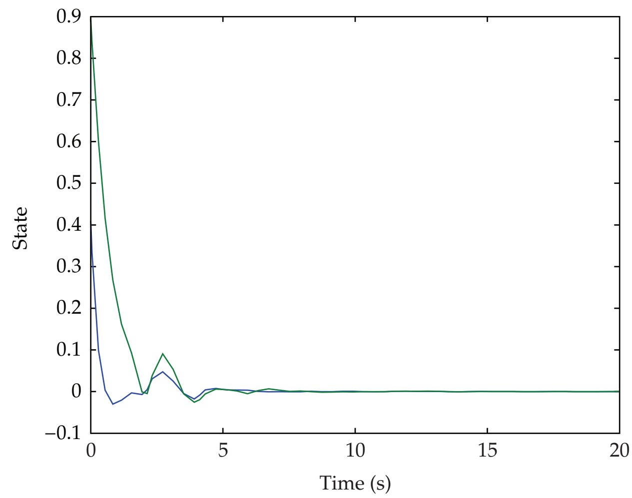

Figure 1: The state of the system under stochastic initial condition and switching.

5. A Numerical Example

To illustrate Theorem 3.1, we present a simple numerical example of system (2.1) with

$$A_{01} = \begin{bmatrix} -3 & -0.5 \\ 0.5 & -1.5 \end{bmatrix}, \qquad A_{11} = \begin{bmatrix} -1 & 0.5 \\ -0.5 & 0.5 \end{bmatrix},$$
$$A_{02} = \begin{bmatrix} -1.5 & 0.25 \\ -0.25 & -1.5 \end{bmatrix}, \qquad A_{12} = \begin{bmatrix} 0.5 & -0.25 \\ 0.25 & -0.5 \end{bmatrix}, \tag{5.1}$$

and $\tau_{11}(t) = \tau_{12}(t) = 0.4t + 1$ for $t \geq 0$. By (3.1), we have that

$$\mathcal{A}_1 = \begin{bmatrix} -2 & 1 \\ 1 & -1 \end{bmatrix}, \qquad \mathcal{A}_2 = \begin{bmatrix} -1 & 0.5 \\ 0.5 & -1 \end{bmatrix}. \tag{5.2}$$

It is not difficult to verify that (H1) holds and there exists a vector $\xi = [1, 1.5]^T$ such that $\mathcal{A}_s \xi \prec 0$ for $s = 1, 2$. Therefore, by Theorem 3.1, we know that system (2.1) is asymptotically stable under arbitrary switching signal. Since system (2.1) is not positive, Theorem 2 in [22] is invalid for this case. The state of the system is given in Figure 1.

It is not difficult to work out an example of Theorem 4.1. We omit it here due to the similarity with the above example.

6. Conclusion

In this paper, we investigate the delay-independent stability of the nonpositive switched linear systems with time-varying delays. By using a new method that is different from those methods to positive systems, we show that the stability of the system is also independent of delays if the switched system shares a common linear copositive Lyapunov function. We also apply this method to a class of time-varying switched linear systems with mixed delays, which generalizes some existing results in the literature.

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