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Research Article

Higher Ring Derivation and Intuitionistic Fuzzy Stability

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We take account of the stability of higher ring derivation in intuitionistic fuzzy Banach algebra associated to the Jensen type functional equation. In addition, we deal with the superstability of higher ring derivation in intuitionistic fuzzy Banach algebra with unit.

1. Introduction and Preliminaries

The stability problem of functional equations has originally been formulated by Ulam [1]: *under what condition does there exist a homomorphism near an approximate homomorphism?* Hyers [2] answered the problem of Ulam under the assumption that the groups are Banach spaces. A generalized version of the theorem of Hyers for approximately additive mappings was given by Aoki [3] and for approximately linear mappings was presented by Rassias [4] by considering an unbounded Cauchy difference. The paper work of Rassias [4] has had a lot of influence in the development of what is called the *generalized Hyers-Ulam stability* of functional equations. Since then, more generalizations and applications of the generalized Hyers-Ulam stability to a number of functional equations and mappings have been investigated (e.g., [5–7]). In particular, Badora [8] gave a generalization of the Bourgin's result [9], and he also dealt with the stability and the Bourgin-type superstability of derivations in [10]. Recently, fuzzy version is discussed in [11, 12]. Quite recently, the intuitionistic fuzzy stability problem for Jensen functional equation and cubic functional equation is considered in [13–15], respectively, while the idea of intuitionistic fuzzy normed space was introduced in [16], and there are some recent and important results which are directly related to the central theme of this paper, that is, intuitionistic fuzziness (see e.g., [17–20]).

In this paper, we establish the stability of higher ring derivation in intuitionistic fuzzy Banach algebra associated to the Jensen type functional equation $lf(x + y/l) = f(x) + f(y)$. Moreover, we consider the superstability of higher ring derivation in intuitionistic fuzzy Banach algebra with unit.

We now recall some notations and basic definitions used in this paper.

Definition 1.1 (see [5]). Let $\mathcal{A}$ and $\mathcal{B}$ be algebras over the real or complex field $\mathbb{F}$. Let $\mathbb{N}$ be the set of the natural numbers. From $m \in \mathbb{N} \cup \{0\}$, a sequence $H = \{h_0, h_1, \ldots, h_m\}$ (resp., $H = \{h_0, h_1, \ldots, h_k, \ldots\}$) of additive operators from $\mathcal{A}$ into $\mathcal{B}$ is called a *higher ring derivation* of rank m (resp., infinite rank) if the functional equation $h_k(xy) = \sum_{i=0}^{k} h_i(x)h_{k-i}(y)$ holds for each $k = 0, 1, \ldots, m$ (resp., $k = 0, 1, \ldots$) and for all $x, y \in \mathcal{A}$. A higher ring derivation H of additive operators on $\mathcal{A}$, particularly, is called *strong* if h_0 is an identity operator.

Of course, a higher ring derivation of rank 0 from $\mathcal{A}$ into $\mathcal{B}$ (resp., a strong higher ring derivation of rank 1 on $\mathcal{A}$) is a ring homomorphism (resp., a ring derivation). Note that a higher ring derivation is a generalization of both a ring homomorphism and a ring derivation.

Definition 1.2. A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is said to be a *continuous t-norm* if it satisfies the following conditions:

(1) $*$ is associative and commutative, (2) $*$ is continuous, (3) $a * 1 = a$ for all $a \in [0,1]$, and (4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for each $a, b, c, d \in [0,1]$.

Definition 1.3. A binary operation $\diamond : [0,1] \times [0,1] \to [0,1]$ is said to be a *continuous t-conorm* if it satisfies the following conditions:

(1) $\diamond$ is associative and commutative, (2) $\diamond$ is continuous, (3) $a \diamond 0 = a$ for all $a \in [0,1]$, and (4) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$ for each $a, b, c, d \in [0,1]$.

Using the notions of continuous t-norm and t-conorm, Saadati and Park [16] have recently introduced the concept of intuitionistic fuzzy normed space as follows.

Definition 1.4. The five-tuple $(\mathcal{X}, \mu, \nu, *, \diamond)$ is said to be an *intuitionistic fuzzy normed space* if $\mathcal{X}$ is a vector space, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm, and μ, ν are fuzzy sets on $\mathcal{X} \times (0, \infty)$ satisfying the following conditions. For every $x, y \in \mathcal{X}$ and $s, t > 0$, (1) $\mu(x,t) + \nu(x,t) \leq 1$, (2) $\mu(x,t) > 0$, (3) $\mu(x,t) = 1$ if and only if $x = 0$, (4) $\mu(\alpha x, t) = \mu(x, t/|\alpha|)$ for each $\alpha \neq 0$, (5) $\mu(x,t) * \mu(y,s) \leq \mu(x+y, t+s)$, (6) $\mu(x,\cdot) : (0,\infty) \to [0,1]$ is continuous, (7) $\lim_{t\to\infty}\mu(x,t) = 1$ and $\lim_{t\to 0}\mu(x,t) = 0$, (8) $\nu(x,t) < 1$, (9) $\nu(x,t) = 0$ if and only if $x = 0$, (10) $\nu(\alpha x, t) = \nu(x, t/|\alpha|)$ for each $\alpha \neq 0$, (11) $\nu(x,t) \diamond \mu(y,s) \geq \nu(x+y, t+s)$, (12) $\nu(x,\cdot) : (0,\infty) \to [0,1]$ is continuous, (13) $\lim_{t\to\infty}\nu(x,t) = 0$ and $\lim_{t\to 0}\nu(x,t) = 1$.

In this case, (μ, ν) is called an intuitionistic fuzzy norm.

Example 1.5. Let $(\mathcal{X}, \|\cdot\|)$ be a normed space, $a * b = ab$, and $a \diamond b = \min\{a + b, 1\}$ for all $a, b \in [0,1]$. For all $x \in \mathcal{X}$ and every $t > 0$, consider

$$\mu(x,t) = \begin{cases} 1, & \text{if } t > \|x\|, \\ 0, & \text{if } t \leq \|x\|, \end{cases} \qquad \nu(x,t) = \begin{cases} 0, & \text{if } t > \|x\|, \\ 1, & \text{if } t \leq \|x\|. \end{cases} \tag{1.1}$$

Then $(\mathcal{X}, \mu, \nu, *, \diamond)$ is an intuitionistic fuzzy normed space.

Example 1.6. Let $(\mathcal{X}, \|\cdot\|)$ be a normed space, $a * b = ab$, and $a \diamond b = \min\{a+b, 1\}$ for all $a, b \in [0,1]$. For all $x \in \mathcal{X}$ and every $t > 0$ and $k = 1, 2$, consider

$$\mu(x,t) = \begin{cases} \dfrac{t}{t + \|x\|}, & \text{if } t > 0, \\ 0, & \text{if } t \leq 0, \end{cases} \qquad \nu(x,t) = \begin{cases} \dfrac{k\|x\|}{t + k\|x\|}, & \text{if } t > 0, \\ 0, & \text{if } t \leq 0. \end{cases} \tag{1.2}$$

Then $(\mathcal{X}, \mu, \nu, *, \diamond)$ is an intuitionistic fuzzy normed space.

Definition 1.7 (see [21]). The five-tuple $(\mathcal{X}, \mu, \nu, *, \diamond)$ is said to be an *intuitionistic fuzzy normed algebra* if $\mathcal{X}$ is an algebra, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm, and μ, ν are fuzzy sets on $\mathcal{X} \times (0, \infty)$ satisfying the conditions (1)–(13) of the Definition 1.4. Furthermore, for every $x, y \in \mathcal{X}$ and $s, t > 0$, (14) $\max\{\mu(x,t), \mu(y,s)\} \leq \mu(xy, t+s)$, (15) $\min\{\nu(x,t), \nu(y,s)\} \geq \nu(xy, t+s)$.

For an intuitionistic fuzzy normed algebra $(\mathcal{X}, \mu, \nu, *, \diamond)$, we further assume that (16) $a * a = a$ and $a \diamond a = a$ for all $a \in [0,1]$.

The concepts of convergence and Cauchy sequences in an intuitionistic fuzzy normed space are studied in [16]. Let $(\mathcal{X}, \mu, \nu, *, \diamond)$ be an intuitionistic fuzzy normed space or intuitionistic fuzzy normed algebra. A sequence $x = \{x_k\}$ is said to be *intuitionistic fuzzy convergent* to $L \in \mathcal{X}$ if $\lim_{k\to\infty}\mu(x_k - L, t) = 1$ and $\lim_{k\to\infty}\nu(x_k - L, t) = 0$ for all $t > 0$. In this case, we write $(\mu,\nu) - \lim_{k\to\infty} x_k = L$ or $x_k \overset{IF}{\to} L$ as $k \to \infty$. A sequence $x = \{x_k\}$ in $(\mathcal{X}, \mu, \nu, *, \diamond)$ is said to be *intuitionistic fuzzy Cauchy sequence* if $\lim_{k\to\infty}\mu(x_{k+p} - x_k, t) = 1$ and $\lim_{k\to\infty}\nu(x_{k+p} - x_k, t) = 0$ for all $t > 0$ and $p = 1, 2, \ldots$. An intuitionistic fuzzy normed space (resp., intuitionistic fuzzy normed algebra) $(\mathcal{X}, \mu, \nu, *, \diamond)$ is said to be *complete* if every intuitionistic fuzzy Cauchy sequence in $(\mathcal{X}, \mu, \nu, *, \diamond)$ is intuitionistic fuzzy convergent in $(\mathcal{X}, \mu, \nu, *, \diamond)$. A complete intuitionistic fuzzy normed space (resp., intuitionistic fuzzy normed algebra) is also called an *intuitionistic fuzzy Banach space* (resp., *intuitionistic fuzzy Banach algebra*).

2. Stability of Higher Ring Derivation in Intuitionistic Fuzzy Banach Algebra

As a matter of convenience in this paper, we use the following abbreviation:

$$\prod_{j=0}^{n} a_j := a_1 * a_2 * \cdots * a_n, \qquad \prod_{j=0}^{\infty} a_j := a_1 * a_2 * \cdots . \tag{2.1}$$

In addition,

$$\coprod_{j=0}^{n} a_j := a_1 \diamond a_2 \diamond \cdots \diamond a_n, \qquad \coprod_{j=0}^{\infty} a_j := a_1 \diamond a_2 \diamond \cdots . \tag{2.2}$$

We begin with a generalized Hyers-Ulam theorem in intuitionistic fuzzy Banach space for the Jensen type functional equation. The following result is also the generalization of the theorem introduced in [13].

Theorem 2.1. *Let $\mathcal{A}$ be a vector space, and let f be a mapping from $\mathcal{A}$ to an intuitionistic fuzzy Banach space $(\mathcal{B},\mu,\nu,*,\diamond)$ with $f(0)=0$. Suppose that φ is a function from $\mathcal{A}$ to an intuitionistic fuzzy normed space $(C,\mu',\nu',*,\diamond)$ such that*

$$\mu\left(lf\left(\frac{x+y}{l}\right)-f(x)-f(y),t+s\right)\geq\mu'(\varphi(x),t)*\mu'(\varphi(y),s), \tag{2.3}$$

$$\nu\left(lf\left(\frac{x+y}{l}\right)-f(x)-f(y),t+s\right)\leq\nu'(\varphi(x),t)\diamond\nu'(\varphi(y),s) \tag{2.4}$$

for all $x,y\in\mathcal{A}\setminus\{0\}$, $t>0$ and $s>0$. If $l>1$ is a fixed integer, and $\varphi((l+1)x)=\alpha\varphi(x)$ for some real number α with $0<|\alpha|<l+1$, then there exists a unique additive mapping $\mathcal{L}:\mathcal{A}\to\mathcal{B}$ such that $\mathcal{L}(x):=(\mu,\nu)-\lim_{n\to\infty}(f((l+1)^n x)/(l+1)^n)$,

$$\begin{aligned}\mu(\mathcal{L}(x)-f(x),t)&\geq\prod_{j=0}^{\infty}M\left(x,\frac{((l+1)-\alpha)t}{2(l+1)}\right),\\ \nu(\mathcal{L}(x)-f(x),t)&\leq\coprod_{j=0}^{\infty}N\left(x,\frac{((l+1)-\alpha)t}{2(l+1)}\right)\end{aligned} \tag{2.5}$$

for all $x\in\mathcal{A}$ and $t>0$, where

$$\begin{aligned}M(x,t)&:=\mu'\left(\varphi(x),\frac{l+1}{4}t\right)*\mu'\left(\varphi(-x),\frac{l+1}{4}t\right)*\mu'\left(\varphi(-x),\frac{l+1}{4}t\right)*\mu'\left(\varphi((l+1)x),\frac{l+1}{4}t\right),\\ N(x,t)&:=\nu'\left(\varphi(x),\frac{l+1}{4}t\right)\diamond\nu'\left(\varphi(-x),\frac{l+1}{4}t\right)\diamond\nu'\left(\varphi(-x),\frac{l+1}{4}t\right)\diamond\nu'\left(\varphi((l+1)x),\frac{l+1}{4}t\right).\end{aligned} \tag{2.6}$$

Proof. Without loss of generality, we assume that $0<\alpha<l+1$. From (2.3) and (2.4), we get

$$\begin{aligned}\mu(f(x)+f(-x),lt)&\geq\mu'\left(\varphi(x),\frac{l}{2}t\right)*\mu'\left(\varphi(-x),\frac{l}{2}t\right),\\ \nu(f(x)+f(-x),lt)&\leq\nu'\left(\varphi(x),\frac{l}{2}t\right)\diamond\left(\varphi(-x),\frac{l}{2}t\right)\end{aligned} \tag{2.7}$$

for all $x\in\mathcal{A}$ and $t>0$. Again, by (2.3) and (2.4), we obtain

$$\begin{aligned}\mu(lf(x)-f(-x)-f((l+1)x),lt)&\geq\mu'\left(\varphi(-x),\frac{l}{2}t\right)*\mu'\left(\varphi((l+1)x),\frac{l}{2}t\right),\\ \nu(lf(x)-f(-x)-f((l+1)x),lt)&\leq\nu'\left(\varphi(-x),\frac{l}{2}t\right)\diamond\nu'\left(\varphi((l+1)x),\frac{l}{2}t\right)\end{aligned} \tag{2.8}$$

for all $x \in \mathcal{A}$ and $t > 0$. Combining (2.7) and (2.8), we arrive at

$$\begin{aligned} \mu((l+1)f(x) - f((l+1)x), 2lt) &\geq \mu'\left(\varphi(x), \frac{l}{2}t\right) * \mu'\left(\varphi(-x), \frac{l}{2}t\right) * \mu'\left(\varphi(-x), \frac{l}{2}t\right) \\ &\quad * \mu'\left(\varphi((l+1)x), \frac{l}{2}t\right), \\ \nu((l+1)f(x) - f((l+1)x), 2lt) &\leq \nu'\left(\varphi(x), \frac{l}{2}t\right) \diamond \nu'\left(\varphi(-x), \frac{l}{2}t\right) \diamond \nu'\left(\varphi(-x), \frac{l}{2}t\right) \\ &\quad \diamond \nu'\left(\varphi((l+1)x), \frac{l}{2}t\right), \end{aligned} \tag{2.9}$$

for all $x \in \mathcal{A}$ and $t > 0$. This implies that

$$\begin{aligned} \mu\left(f(x) - \frac{f((l+1)x)}{(l+1)}, t\right) &\geq \mu'\left(\varphi(x), \frac{l+1}{4}t\right) * \mu'\left(\varphi(-x), \frac{l+1}{4}t\right) * \mu'\left(\varphi(-x), \frac{l+1}{4}t\right) \\ &\quad * \mu'\left(\varphi((l+1)x), \frac{l+1}{4}t\right), \\ \nu\left(f(x) - \frac{f((l+1)x)}{(l+1)}, t\right) &\leq \nu'\left(\varphi(x), \frac{l+1}{4}t\right) \diamond \nu'\left(\varphi(-x), \frac{l+1}{4}t\right) \diamond \nu'\left(\varphi(-x), \frac{l+1}{4}t\right) \\ &\quad \diamond \nu'\left(\varphi((l+1)x), \frac{l+1}{4}t\right), \end{aligned} \tag{2.10}$$

for all $x \in \mathcal{A}$ and $t > 0$. Now we define

$$\begin{aligned} M(x,t) &:= \mu'\left(\varphi(x), \frac{l+1}{4}t\right) * \mu'\left(\varphi(-x), \frac{l+1}{4}t\right) * \mu'\left(\varphi(-x), \frac{l+1}{4}t\right) * \mu'\left(\varphi((l+1)x), \frac{l+1}{4}t\right), \\ N(x,t) &:= \nu'\left(\varphi(x), \frac{l+1}{4}t\right) \diamond \nu'\left(\varphi(-x), \frac{l+1}{4}t\right) \diamond \nu'\left(\varphi(-x), \frac{l+1}{4}t\right) \diamond \nu'\left(\varphi((l+1)x), \frac{l+1}{4}t\right), \end{aligned} \tag{2.11}$$

for all $x \in \mathcal{A}$ and $t > 0$. Then we have by assumption

$$M((l+1)x, t) = M\left(x, \frac{t}{\alpha}\right), \qquad N((l+1)x, t) = N\left(x, \frac{t}{\alpha}\right), \tag{2.12}$$

for all $x \in \mathcal{A}$ and $t > 0$. Using (2.10) and (2.12), we get

$$\begin{aligned} \mu\left(\frac{f((l+1)^n x)}{(l+1)^n} - \frac{f\left((l+1)^{n+1}x\right)}{(l+1)^{n+1}}, \frac{\alpha^n t}{(l+1)^n}\right) &= \mu\left(f((l+1)^n x) - \frac{f\left((l+1)^{n+1}x\right)}{l+1}, \alpha^n t\right) \\ &\geq M((l+1)^n x, \alpha^n t) = M(x,t), \end{aligned}$$

$$\nu\left(\frac{f((l+1)^n x)}{(l+1)^n} - \frac{f\left((l+1)^{n+1} x\right)}{(l+1)^{n+1}}, \frac{\alpha^n t}{(l+1)^n}\right) = \nu\left(f((l+1)^n x) - \frac{f\left((l+1)^{n+1} x\right)}{l+1}, \alpha^n t\right)$$
$$\leq N((l+1)^n x, \alpha^n t) = N(x,t), \tag{2.13}$$

for all $x \in \mathcal{A}$ and $t > 0$. Therefore, for all $n > m$, we have

$$\begin{aligned}
&\mu\left(\frac{f((l+1)^m x)}{(l+1)^m} - \frac{f((l+1)^n x)}{(l+1)^n}, \sum_{j=m}^{n-1}\frac{\alpha^j t}{(l+1)^j}\right) \\
&\quad= \mu\left(\sum_{j=m}^{n-1}\left[\frac{f\left((l+1)^j x\right)}{(l+1)^j} - \frac{f\left((l+1)^{j+1} x\right)}{(l+1)^{j+1}}\right], \sum_{j=m}^{n-1}\frac{\alpha^j t}{(l+1)^j}\right) \\
&\quad\geq \prod_{j=m}^{n-1}\mu\left(\frac{f\left((l+1)^j x\right)}{(l+1)^j} - \frac{f\left((l+1)^{j+1} x\right)}{(l+1)^{j+1}}, \frac{\alpha^j t}{(l+1)^j}\right) \geq \prod_{j=m}^{n-1} M(x,t), \\
&\nu\left(\frac{f((l+1)^m x)}{(l+1)^m} - \frac{f((l+1)^n x)}{(l+1)^n}, \sum_{j=m}^{n-1}\frac{\alpha^j t}{(l+1)^j}\right) \\
&\quad= \nu\left(\sum_{j=m}^{n-1}\left[\frac{f\left((l+1)^j x\right)}{(l+1)^j} - \frac{f\left((l+1)^{j+1} x\right)}{(l+1)^{j+1}}\right], \sum_{j=m}^{n-1}\frac{\alpha^j t}{(l+1)^j}\right) \\
&\quad\leq \coprod_{j=m}^{n-1}\nu\left(\frac{f\left((l+1)^j x\right)}{(l+1)^j} - \frac{f\left((l+1)^{j+1} x\right)}{(l+1)^{j+1}}, \frac{\alpha^j t}{(l+1)^j}\right) \leq \coprod_{j=m}^{n-1} N(x,t),
\end{aligned} \tag{2.14}$$

for all $x \in \mathcal{A}$ and $t > 0$. Let $\varepsilon > 0$ and $\delta > 0$ be given. Since $\lim_{t\to\infty}\prod_{j=m}^{n-1} M(x,t) = 1$ and $\lim_{t\to\infty}\coprod_{j=m}^{n-1} N(x,t) = 0$, there exists some t_0 such that $\prod_{j=m}^{n-1} M(x,t_0) > 1-\varepsilon$, $\coprod_{j=m}^{n-1} N(x,t_0) < \varepsilon$. Since $\sum_{j=0}^{\infty}(\alpha^j t/(l+1)^j) < \infty$, there exists a positive integer n_0 such that $\sum_{j=m}^{n-1}(\alpha^j t/(l+1)^j) < \delta$ for all $n > m \geq n_0$.

Then

$$\mu\left(\frac{f((l+1)^m x)}{(l+1)^m} - \frac{f((l+1)^n x)}{(l+1)^n}, \delta\right) \geq \mu\left(\frac{f((l+1)^m x)}{(l+1)^m} - \frac{f((l+1)^n x)}{(l+1)^n}, \sum_{j=m}^{n-1}\frac{\alpha^j t_0}{(l+1)^j}\right)$$
$$\geq \prod_{j=m}^{n-1} M(x,t_0) > 1-\varepsilon,$$

$$\nu\left(\frac{f((l+1)^m x)}{(l+1)^m}-\frac{f((l+1)^n x)}{(l+1)^n},\delta\right)\leq \nu\left(\frac{f((l+1)^m x)}{(l+1)^m}-\frac{f((l+1)^n x)}{(l+1)^n},\sum_{j=m}^{n-1}\frac{\alpha^j t_0}{(l+1)^j}\right)$$
$$\leq \coprod_{j=m}^{n-1} N(x,t_0)<\varepsilon. \tag{2.15}$$

This shows that $\{(f((l+1)^n x))/((l+1)^n)\}$ is a Cauchy sequence in $(\mathcal{B},\mu',\nu',*,\diamond)$. Since $\mathcal{B}$ is complete, we can define a mapping $\mathcal{L}$ by $\mathcal{L}(x):=(\mu,\nu)-\lim_{n\to\infty}(f((l+1)^n x)/(l+1)^n)$ for all $x\in\mathcal{A}$. Moreover, if we let $m=0$ in (2.14), then we get

$$\mu\left(\frac{f((l+1)^n x)}{(l+1)^n}-f(x),\sum_{j=0}^{n-1}\frac{\alpha^j t}{(l+1)^j}\right)\geq \prod_{j=0}^{n-1} M(x,t),$$
$$\nu\left(\frac{f((l+1)^n x)}{(l+1)^n}-f(x),\sum_{j=0}^{n-1}\frac{\alpha^j t}{(l+1)^j}\right)\leq \coprod_{j=0}^{n-1} N(x,t), \tag{2.16}$$

for all $x\in\mathcal{A}$ and $t>0$. Therefore, we find that

$$\mu\left(\frac{f((l+1)^n x)}{(l+1)^n}-f(x),t\right)\geq \prod_{j=0}^{n-1} M\left(x,\frac{t}{\sum_{j=0}^{n-1}\left(\alpha^j/(l+1)^j\right)}\right),$$
$$\nu\left(\frac{f((l+1)^n x)}{(l+1)^n}-f(x),t\right)\leq \coprod_{j=0}^{n-1} N\left(x,\frac{t}{\sum_{j=0}^{n-1}\left(\alpha^j/(l+1)^j\right)}\right). \tag{2.17}$$

Next, we will show that $\mathcal{L}$ is additive mapping. Note that

$$\mu\left(l\mathcal{L}\left(\frac{x+y}{l}\right)-\mathcal{L}(x)-\mathcal{L}(y),t\right)\geq \mu\left(l\mathcal{L}\left(\frac{x+y}{l}\right)-\frac{lf(((l+1)^n(x+y))/l)}{(l+1)^n},\frac{t}{4}\right)$$
$$*\,\mu\left(\frac{f((l+1)^n x)}{(l+1)^n}-\mathcal{L}(x),\frac{t}{4}\right)*\mu\left(\frac{f((l+1)^n y)}{(l+1)^n}-\mathcal{L}(y),\frac{t}{4}\right)$$
$$*\,\mu\left(\frac{lf(((l+1)^n(x+y))/(l))}{(l+1)^n}-\frac{f((l+1)^n x)}{(l+1)^n}-\frac{f((l+1)^n y)}{(l+1)^n},\frac{t}{4}\right),$$

$$\nu\left(l\mathcal{L}\left(\frac{x+y}{l}\right)-\mathcal{L}(x)-\mathcal{L}(y),t\right)\leq\nu\left(l\mathcal{L}\left(\frac{x+y}{l}\right)-\frac{lf(((l+1)^n(x+y))/l)}{(l+1)^n},\frac{t}{4}\right)$$
$$\diamond\,\nu\left(\frac{f((l+1)^n x)}{(l+1)^n}-\mathcal{L}(x),\frac{t}{4}\right)\diamond\,\nu\left(\frac{f((l+1)^n y)}{(l+1)^n}-\mathcal{L}(y),\frac{t}{4}\right)$$
$$\diamond\,\nu\left(\frac{lf(((l+1)^n(x+y))/(l))}{(l+1)^n}-\frac{f((l+1)^n x)}{(l+1)^n}-\frac{f((l+1)^n y)}{(l+1)^n},\frac{t}{4}\right). \tag{2.18}$$

On the other hand, (2.3) and (2.4) give the following:

$$\mu\left(\frac{lf(((l+1)^n(x+y))/l)}{(l+1)^n}-\frac{f((l+1)^n x)}{(l+1)^n}-\frac{f((l+1)^n y)}{(l+1)^n},\frac{t}{4}\right)$$
$$\geq\mu'\left(\varphi(x),\left(\frac{l+1}{\alpha}\right)^n\frac{t}{8}\right)*\mu'\left(\varphi(y),\left(\frac{l+1}{\alpha}\right)^n\frac{t}{8}\right),$$
$$\nu\left(\frac{lf(((l+1)^n(x+y))/l)}{(l+1)^n}-\frac{f((l+1)^n x)}{(l+1)^n}-\frac{f((l+1)^n y)}{(l+1)^n},\frac{t}{4}\right)$$
$$\leq\nu'\left(\varphi(x),\left(\frac{l+1}{\alpha}\right)^n\frac{t}{8}\right)\diamond\,\nu'\left(\varphi(y),\left(\frac{l+1}{\alpha}\right)^n\frac{t}{8}\right). \tag{2.19}$$

Letting $n\to\infty$ in (2.18) and (2.19), we yield

$$\mu\left(l\mathcal{L}\left(\frac{x+y}{l}\right)-\mathcal{L}(x)-\mathcal{L}(y),t\right)=1,\qquad\nu\left(l\mathcal{L}\left(\frac{x+y}{l}\right)-\mathcal{L}(x)-\mathcal{L}(y),t\right)=0. \tag{2.20}$$

So we see that $\mathcal{L}$ is additive mapping.

Now, we approximate the difference between f and $\mathcal{L}$ in an intuitionistic fuzzy sense. By (2.17), we get

$$\mu(\mathcal{L}(x)-f(x),t)\geq\mu\left(\mathcal{L}(x)-\frac{f((l+1)^n x)}{(l+1)^n},\frac{t}{2}\right)*\mu\left(\frac{f((l+1)^n x)}{(l+1)^n}-f(x),\frac{t}{2}\right)$$
$$\geq\prod_{j=0}^{\infty}M\left(x,\frac{((l+1)-\alpha)t}{2(l+1)}\right),$$
$$\nu(\mathcal{L}(x)-f(x),t)\leq\nu\left(\mathcal{L}(x)-\frac{f((l+1)^n x)}{(l+1)^n},\frac{t}{2}\right)\diamond\,\nu\left(\frac{f((l+1)^n x)}{(l+1)^n}-f(x),\frac{t}{2}\right)$$
$$\leq\coprod_{j=0}^{\infty}N\left(x,\frac{((l+1)-\alpha)t}{2(l+1)}\right), \tag{2.21}$$

for all $x\in\mathcal{A}$ and $t>0$ and sufficiently large n.

In order to prove the uniqueness of $\mathcal{L}$, we assume that T is another additive mapping from $\mathcal{A}$ to $\mathcal{B}$, which satisfies the inequality (2.5). Then

$$\begin{aligned}
\mu(\mathcal{L}(x)-T(x),t) &\geq \mu\left(\mathcal{L}(x)-f(x),\frac{t}{2}\right)*\mu\left(T(x)-f(x),\frac{t}{2}\right) \\
&\geq \prod_{j=0}^{\infty} M\left(x,\frac{((l+1)-\alpha)t}{4(l+1)}\right), \\
\nu(\mathcal{L}(x)-T(x),t) &\leq \nu\left(\mathcal{L}(x)-f(x),\frac{t}{2}\right)\diamond\nu\left(T(x)-f(x),\frac{t}{2}\right) \\
&\leq \coprod_{j=0}^{\infty} N\left(x,\frac{((l+1)-\alpha)t}{4(l+1)}\right),
\end{aligned} \tag{2.22}$$

for all $x\in\mathcal{A}$ and $t>0$. Therefore, due to the additivity of $\mathcal{L}$ and T, we obtain that

$$\begin{aligned}
\mu(\mathcal{L}(x)-T(x),t) &= \mu\left(\mathcal{L}\left((l+1)^n x\right)-T\left((l+1)^n x\right),(l+1)^n t\right) \\
&\geq \prod_{j=0}^{\infty} M\left(x,\left(\frac{l+1}{\alpha}\right)^n\frac{((l+1)-\alpha)t}{4(l+1)}\right), \\
\nu(\mathcal{L}(x)-T(x),t) &= \nu\left(\mathcal{L}\left((l+1)^n x\right)-T\left((l+1)^n x\right),(l+1)^n t\right) \\
&\leq \coprod_{j=0}^{\infty} M\left(x,\left(\frac{l+1}{\alpha}\right)^n\frac{((l+1)-\alpha)t}{4(l+1)}\right).
\end{aligned} \tag{2.23}$$

Since $0<\alpha<l+1$, $\lim_{n\to\infty}((l+1)/\alpha)^n=\infty$, and we get

$$\lim_{n\to\infty} M\left(x,\left(\frac{l+1}{\alpha}\right)^n\frac{((l+1)-\alpha)t}{4(l+1)}\right)=1, \qquad \lim_{n\to\infty} N\left(x,\left(\frac{l+1}{\alpha}\right)^n\frac{((l+1)-\alpha)t}{4(l+1)}\right)=0, \tag{2.24}$$

that is, $\mu(\mathcal{L}(x)-T(x),t)=1$ and $\nu(\mathcal{L}(x)-T(x),t)=0$ for all $x\in\mathcal{A}$, $t>0$. So $\mathcal{L}=T$, which completes the proof. □

In particular, we can prove the preceding result for the case when $\alpha>l+1$. In this case, the mapping $\mathcal{L}(x):=(\mu,\nu)-\lim_{n\to\infty}(l+1)^n f((l+1)^{-n}x)$. We now establish a generalized Hyers-Ulam stability in intuitionistic fuzzy Banach algebra for the higher ring derivation.

Theorem 2.2. *Let $\mathcal{A}$ be an algebra, and let $F=\{f_0,f_1,\ldots,f_k,\ldots\}$ be a sequence of mappings from $\mathcal{A}$ to an intuitionistic fuzzy Banach algebra $(\mathcal{B},\mu,\nu,*,\diamond)$ with $f_k(0)=0$ for each $k=0,1,\ldots$. Suppose* *that* φ *is a function from* $\mathcal{A}$ *to an intuitionistic fuzzy normed algebra* $(C, \mu', \nu', *, \diamond)$ *such that for each* $k = 0, 1, \ldots,$

$$\mu\left(lf_k\left(\frac{x+y}{l}\right) - f_k(x) - f_k(y), t+s\right) \geq \mu'(\varphi(x), t) * \mu'(\varphi(y), s),$$
$$\nu\left(lf_k\left(\frac{x+y}{l}\right) - f_k(x) - f_k(y), t+s\right) \leq \nu'(\varphi(x), t) \diamond \nu'(\varphi(y), s) \tag{2.25}$$

for all $x, y \in \mathcal{A} \setminus \{0\}, t > 0$ *and* $s > 0$*, and that* Φ *is a function from* $\mathcal{A}$ *to an intuitionistic fuzzy normed space* $(D, \mu'', \nu'', *, \diamond)$ *such that for each* $k = 0, 1, \ldots,$

$$\mu\left(f_k(xy) - \sum_{i=0}^{k} f_i(x) f_{k-i}(y), t+s\right) \geq \max\{\mu''(\Phi(x), t), \mu''(\Phi(y), s)\},$$
$$\nu\left(f_k(xy) - \sum_{i=0}^{k} f_i(x) f_{k-i}(y), t+s\right) \leq \min\{\nu''(\Phi(x), t), \nu''(\Phi(y), s)\} \tag{2.26}$$

for all $x, y \in \mathcal{A}, t > 0$*, and* $s > 0$*. If* $l > 1$ *is a fixed integer,* $\varphi((l+1)x) = \alpha\varphi(x)$*, and* $\Phi((l+1)x) = \beta\Phi(x)$ *for some real numbers* α *and* β *with* $0 < |\alpha| < l+1$ *and* $0 < |\beta| < l+1$*, then there exists a unique higher ring derivation* $H = \{\mathcal{L}_0, \mathcal{L}_1, \ldots, \mathcal{L}_k, \ldots\}$ *of any rank such that for each* $k = 0, 1, \ldots,$

$$\mu(\mathcal{L}_k(x) - f_k(x), t) \geq M\left(x, \frac{((l+1)-\alpha)t}{2(l+1)}\right),$$
$$\nu(\mathcal{L}_k(x) - f_k(x), t) \leq N\left(x, \frac{((l+1)-\alpha)t}{2(l+1)}\right), \tag{2.27}$$

for all $x \in \mathcal{A}$ *and* $t > 0$*. In this case,*

$$M(x,t) := \mu'\left(\varphi(x), \frac{l+1}{4}t\right) * \mu'\left(\varphi(-x), \frac{l+1}{4}t\right) * \mu'\left(\varphi((l+1)x), \frac{l+1}{4}t\right),$$
$$N(x,t) := \nu'\left(\varphi(x), \frac{l+1}{4}t\right) \diamond \nu'\left(\varphi(-x), \frac{l+1}{4}t\right) \diamond \nu'\left(\varphi((l+1)x), \frac{l+1}{4}t\right). \tag{2.28}$$

Moreover, the identity

$$\sum_{i=0}^{k} \mathcal{L}_i(y)\{\mathcal{L}_{k-i}(y) - f_{k-i}(y)\} = 0 \tag{2.29}$$

holds for each $k = 0, 1, \ldots$ *and all* $x, y \in \mathcal{A}$*.*

Proof. It follows by Theorem 2.1 that for each $k = 0, 1, \ldots$ and all $x \in \mathcal{A}$, there exists a unique additive mapping $\mathcal{L}_k : \mathcal{A} \to \mathcal{B}$ given by

$$\mathcal{L}_k(x) := (\mu, \nu) - \lim_{n\to\infty} \frac{f_k((l+1)^n x)}{(l+1)^n}, \tag{2.30}$$

satisfying (2.27) since $(C, \mu', \nu', *, \diamond)$ is an intuitionistic fuzzy normed algebra.

Without loss of generality, we suppose that $0 < \beta < l + 1$. Now, we need to prove that the sequence $H = \{\mathcal{L}_0, \mathcal{L}_1, \ldots, \mathcal{L}_k, \ldots\}$ satisfies the identity $\mathcal{L}_k(xy) = \sum_{i=0}^{k} \mathcal{L}_i(x)\mathcal{L}_{k-i}(y)$ for each $k = 0, 1, \ldots$ and all $x \in \mathcal{A}$. It is observed that for each $k = 0, 1, \ldots$,

$$\begin{aligned}
&\mu\left(\mathcal{L}_k(xy) - \sum_{i=0}^{k} \mathcal{L}_i(x) f_{k-i}(y), t\right) \\
&\quad \geq \mu\left(\mathcal{L}_k(xy) - \frac{f_k((l+1)^n xy)}{(l+1)^n}, \frac{t}{3}\right) * \mu\left(\frac{f_k((l+1)^n xy)}{(l+1)^n} - \sum_{i=0}^{k} \frac{f_i((l+1)^n x)}{(l+1)^n} f_{k-i}(y), \frac{t}{3}\right) \\
&\qquad * \mu\left(\sum_{i=0}^{k} \frac{f_i((l+1)^n x)}{(l+1)^n} f_{k-i}(y) - \sum_{i=0}^{k} \mathcal{L}_i(x) f_{k-i}(y), \frac{t}{3}\right), \\
&\nu\left(\mathcal{L}_k(xy) - \sum_{i=0}^{k} \mathcal{L}_i(x) f_{k-i}(y), t\right) \\
&\quad \leq \nu\left(\mathcal{L}_k(xy) - \frac{f_k((l+1)^n xy)}{(l+1)^n}, \frac{t}{3}\right) \diamond \nu\left(\frac{f_k((l+1)^n xy)}{(l+1)^n} - \sum_{i=0}^{k} \frac{f_i((l+1)^n x)}{(l+1)^n} f_{k-i}(y), \frac{t}{3}\right) \\
&\qquad \diamond \nu\left(\sum_{i=0}^{k} \frac{f_i((l+1)^n x)}{(l+1)^n} f_{k-i}(y) - \sum_{i=0}^{k} \mathcal{L}_{\mathrm{i}}(x) f_{k-i}(y), \frac{t}{3}\right)
\end{aligned} \tag{2.31}$$

for all $x, y \in \mathcal{A}$ and $t > 0$. On account of (2.26), we see that for each $k = 0, 1, \ldots$,

$$\begin{aligned}
&\mu\left(\frac{f_k((l+1)^n x \cdot y)}{(l+1)^n} - \sum_{i=0}^{k} \frac{f_i((l+1)^n x)}{(l+1)^n} f_{k-i}(y), \frac{t}{3}\right) \\
&\quad = \mu\left(f_k((l+1)^n x \cdot y) - \sum_{i=0}^{k} f_i((l+1)^n x) f_{k-i}(y), \frac{(l+1)^n t}{3}\right) \\
&\quad \geq \max\left\{\mu''\left(\Phi(x), \left(\frac{l+1}{\beta}\right)^n \frac{t}{6}\right), \mu''\left(\Phi(y), \frac{(l+1)^n t}{6}\right)\right\},
\end{aligned}$$

$$\begin{aligned}
&\nu\left(\frac{f_k((l+1)^n x\cdot y)}{(l+1)^n}-\sum_{i=0}^{k}\frac{f_i((l+1)^n x)}{(l+1)^n}f_{k-i}(y),\frac{(l+1)^n t}{3}\right)\\
&\quad=\nu\left(f_k((l+1)^n x\cdot y)-\sum_{i=0}^{k}f_i((l+1)^n x)f_{k-i}(y),\frac{(l+1)^n t}{3}\right)\\
&\quad\le\min\left\{\mu''\left(\Phi(x),\left(\frac{l+1}{\beta}\right)^n\frac{t}{6}\right),\nu''\left(\Phi(y),\frac{(l+1)^n t}{6}\right)\right\},
\end{aligned}\tag{2.32}$$

for all $x, y \in \mathcal{A}$ and $t > 0$. Due to additivity of $\mathcal{L}_k$, for each $k = 0, 1, \ldots$,

$$\begin{aligned}
&\mu\left(\sum_{i=0}^{k}\frac{f_i((l+1)^n x)}{(l+1)^n}f_{k-i}(y)-\sum_{i=0}^{k}\mathcal{L}_i(x)f_{k-i}(y),\frac{t}{3}\right)\\
&\quad\ge\prod_{i=0}^{k}\mu\left(f_i((l+1)^n x)f_{k-i}(y)-(l+1)^n\mathcal{L}_i(x)f_{k-i}(y),\frac{(l+1)^n t}{3(k+1)}\right),\\
&\nu\left(\sum_{i=0}^{k}\frac{f_i((l+1)^n x)}{(l+1)^n}f_{k-i}(y)-\sum_{i=0}^{k}\mathcal{L}_i(x)f_{k-i}(y),\frac{t}{3}\right)\\
&\quad\le\coprod_{i=0}^{k}\nu\left(f_i((l+1)^n x)f_{k-i}(y)-(l+1)^n\mathcal{L}_i(x)f_{k-i}(y),\frac{(l+1)^n t}{3(k+1)}\right)
\end{aligned}\tag{2.33}$$

for all $x, y \in \mathcal{A}$ and $t > 0$. In addition, we feel that

$$\begin{aligned}
&\mu\left(f_i((l+1)^n x)f_{k-i}(y)-(l+1)^n\mathcal{L}_i(x)f_{k-i}(y),\frac{(l+1)^n t}{3(k+1)}\right)\\
&\quad\ge\max\left\{\mu\left(f_i((l+1)^n x)-(l+1)^n\mathcal{L}_i(x),\frac{(l+1)^n t}{6(k+1)}\right),\mu\left(f_{k-i}(y),\frac{(l+1)^n t}{6(k+1)}\right)\right\},\\
&\nu\left(f_i((l+1)^n x)f_{k-i}(y)-(l+1)^n\mathcal{L}_i(x)f_{k-i}(y),\frac{(l+1)^n t}{3(k+1)}\right)\\
&\quad\le\min\left\{\nu\left(f_i((l+1)^n x)-(l+1)^n\mathcal{L}_i(x),\frac{(l+1)^n t}{6(k+1)}\right),\nu\left(f_{k-i}(y),\frac{(l+1)^n t}{6(k+1)}\right)\right\}.
\end{aligned}\tag{2.34}$$

Letting $n \to \infty$ in (2.31), (2.32), (2.33), and (2.34), we get $\mu(\mathcal{L}_k(xy)-\sum_{i=0}^{k}\mathcal{L}_i(x)f_{k-i}(y),t)=1$ and $\nu(\mathcal{L}_k(xy)-\sum_{i=0}^{k}\mathcal{L}_i(x)f_{k-i}(y),t)=0$. This implies that

$$\mathcal{L}_k(xy)=\sum_{i=0}^{k}\mathcal{L}_i(x)f_{k-i}(y),\tag{2.35}$$

for each $k = 0, 1, \ldots$ and all $x, y \in \mathcal{A}$.

Using additivity of $\mathcal{L}_k$ and (2.35), we find that

$$(l+1)^n \sum_{i=0}^{k} \mathcal{L}_i(x) f_{k-i}(y) = \mathcal{L}_k\left((l+1)^n x \cdot y\right) = \mathcal{L}_k\left(x \cdot (l+1)^n y\right) = \sum_{i=0}^{k} \mathcal{L}(x) f\left((l+1)^n y\right). \tag{2.36}$$

So we obtain $\sum_{i=0}^{k} \mathcal{L}_i(x) f_{k-i}(y) = \sum_{i=0}^{k} \mathcal{L}_i(x)(f_{k-i}((l+1)^n y)/(l+1)^n)$. Hence for each $k = 0, 1, \ldots,$

$$\mu\left(\sum_{i=0}^{k} \mathcal{L}_i(x) f_{k-i}(y) - \sum_{i=0}^{k} \mathcal{L}_i(x) \frac{f_{k-i}\left((l+1)^n y\right)}{(l+1)^n}, t\right) = 1,$$
$$\nu\left(\sum_{i=0}^{k} \mathcal{L}_i(x) f_{k-i}(y) - \sum_{i=0}^{k} \mathcal{L}_i(x) \frac{f_{k-i}\left((l+1)^n y\right)}{(l+1)^n}, t\right) = 0, \tag{2.37}$$

for all $x, y \in \mathcal{A}$ and $t > 0$. This relation yields that for each $k = 0, 1, \ldots,$

$$\begin{aligned}
&\mu\left(\sum_{i=0}^{k} \mathcal{L}_i(x) \mathcal{L}_{k-i}(y) - \sum_{i=0}^{k} \mathcal{L}_i(x) f_{k-i}(y), t\right) \\
&\quad \geq \mu\left(\sum_{i=0}^{k} \mathcal{L}_i(x) \mathcal{L}_{k-i}(y) - \sum_{i=0}^{k} \mathcal{L}_i(x) \frac{f_{k-i}\left((l+1)^n y\right)}{(l+1)^n}, \frac{t}{2}\right) \\
&\qquad * \mu\left(\sum_{i=0}^{k} \mathcal{L}_i(x) \frac{f_{k-i}\left((l+1)^n y\right)}{(l+1)^n} - \sum_{i=0}^{k} \mathcal{L}_i(x) f_{k-i}(y), \frac{t}{2}\right) \\
&\quad \geq \prod_{i=0}^{k} \mu\left(\mathcal{L}_i(x) \mathcal{L}_{k-i}(y) - \mathcal{L}_i(x) \frac{f_{k-i}\left((l+1)^n y\right)}{(l+1)^n}, \frac{t}{2(k+1)}\right),
\end{aligned} \tag{2.38}$$

$$\begin{aligned}
&\nu\left(\sum_{i=0}^{k} \mathcal{L}_i(x) \mathcal{L}_{k-i}(y) - \sum_{i=0}^{k} \mathcal{L}_i(x) f_{k-i}(y), t\right) \\
&\quad \leq \nu\left(\sum_{i=0}^{k} \mathcal{L}_i(x) \mathcal{L}_{k-i}(y) - \sum_{i=0}^{k} \mathcal{L}_i(x) \frac{f_{k-i}\left((l+1)^n y\right)}{(l+1)^n}, \frac{t}{2}\right) \\
&\qquad \diamond \nu\left(\sum_{i=0}^{k} \mathcal{L}_i(x) \frac{f_{k-i}\left((l+1)^n y\right)}{(l+1)^n} - \sum_{i=0}^{k} \mathcal{L}_i(x) f_{k-i}(y), \frac{t}{2}\right) \\
&\quad \leq \coprod_{i=0}^{k} \nu\left(\mathcal{L}_i(x) \mathcal{L}_{k-i}(y) - \mathcal{L}_i(x) \frac{f_{k-i}\left((l+1)^n y\right)}{(l+1)^n}, \frac{t}{2(k+1)}\right),
\end{aligned} \tag{2.39}$$

for all $x, y \in \mathcal{A}$ and $t > 0$. On the other hand, we see that

$$\begin{aligned}
&\mu\left(\mathcal{L}_i(x)\mathcal{L}_{k-i}(y) - \mathcal{L}_i(x)\frac{f_{k-i}((l+1)^n y)}{(l+1)^n}, \frac{t}{2(k+1)}\right) \\
&\quad \geq \max\left\{\mu\left(\mathcal{L}_i(x), \frac{(l+1)^n t}{4(k+1)}\right), \mu\left(\mathcal{L}_{k-i}(y) - \frac{f_{k-i}((l+1)^n y)}{(l+1)^n}, \frac{t}{4(k+1)}\right)\right\}, \\
&\mu\left(\mathcal{L}_i(x)\mathcal{L}_{k-i}(y) - \mathcal{L}_i(x)\frac{f_{k-i}((l+1)^n y)}{(l+1)^n}, \frac{t}{2(k+1)}\right) \\
&\quad \leq \min\left\{\nu\left(\mathcal{L}_i(x), \frac{(l+1)^n t}{4(k+1)}\right), \nu\left(\mathcal{L}_{k-i}(y) - \frac{f_{k-i}((l+1)^n y)}{(l+1)^n}, \frac{t}{4(k+1)}\right)\right\}.
\end{aligned} \tag{2.40}$$

Sending $n \to \infty$ in (2.38) and (2.40), we have that for each $k = 0, 1, \ldots,$

$$\begin{aligned}
&\mu\left(\sum_{i=0}^{k}\mathcal{L}_i(x)\mathcal{L}_{k-i}(y) - \sum_{i=0}^{k}\mathcal{L}_i(x)f_{k-i}(y), t\right) = 1, \\
&\nu\left(\sum_{i=0}^{k}\mathcal{L}_i(x)\mathcal{L}_{k-i}(y) - \sum_{i=0}^{k}\mathcal{L}_i(x)f_{k-i}(y), t\right) = 0,
\end{aligned} \tag{2.41}$$

for all $x, y \in \mathcal{A}$ and $t > 0$. Thus, we conclude that

$$\sum_{i=0}^{k}\mathcal{L}_i(x)\mathcal{L}_{k-i}(y) = \sum_{i=0}^{k}\mathcal{L}_i(x)f_{k-i}(y), \tag{2.42}$$

for each $k = 0, 1, \ldots$ and all $x, y \in \mathcal{A}$.

Therefore, by combining (2.35) and (2.42), we get the required result, which completes the proof. □

As a consequence of Theorem 2.2, we get the following superstability.

Corollary 2.3. *Let $(\mathcal{B}, \mu, \nu, *, \diamond)$ be an intuitionistic fuzzy Banach algebra with unit, and let a sequence of operators $F = \{f_0, f_1, \ldots, f_k, \ldots\}$ on $\mathcal{A}$ satisfy $f_k(0) = 0$ for each $k = 0, 1, \ldots$, where f_0 is an identity operator. Suppose that φ is a function from $\mathcal{A}$ to an intuitionistic fuzzy normed algebra $(C, \mu', \nu', *, \diamond)$ satisfying* (2.25) *and (2.14) and that Φ is a function from $\mathcal{A}$ to an intuitionistic fuzzy normed space $(D, \mu'', \nu'', *, \diamond)$ satisfying* (2.26)*. If $l > 1$ is a fixed integer, $\varphi((l+1)x) = \alpha\varphi(x)$, and $\Phi((l+1)x) = \beta\Phi(x)$ for some real numbers α and β with $0 < |\alpha| < l+1$ and $0 < |\beta| < l+1$, then F is a strong higher ring derivation on $\mathcal{A}$.*

Proof. According to (2.30), we have $\mathcal{L}_0(x) = x$ for all $x \in \mathcal{A}$, and so $\mathcal{L}_0(=f_0)$ is an identity operator on $\mathcal{A}$. By induction, we get the conclusion. If $k = 1$, then it follows from (2.29) that $f_1(x) = \mathcal{L}_1(x)$ holds for all $x \in \mathcal{A}$ since $\mathcal{A}$ contains the unit element. Let us assume that $f_m(x) = \mathcal{L}_m(x)$ is valid for all $x \in \mathcal{A}$ and $m < k$. Then (2.29) implies that $x\{\mathcal{L}_m(y) - f_m(y)\} = 0$ for all $x, y \in \mathcal{A}$. Since $\mathcal{A}$ has the unit element, $f_k(y) = \mathcal{L}_k(y)$ for all $x \in \mathcal{A}$. Hence we conclude that $f_k(y) = \mathcal{L}_k(y)$ for each $k = 0, 1, 2, \ldots$ and all $x \in \mathcal{A}$. So this tells us that F is a higher ring derivation of any rank from $\mathcal{A}$ and $\mathcal{B}$. The proof of the corollary is complete. □

We remark that we can prove the preceding result for the case when $\alpha > l + 1$ and $\beta > l + 1$.

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