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Research Article

Global Stability of Almost Periodic Solution of a Class of Neutral-Type BAM Neural Networks

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A class of BAM neural networks with variable coefficients and neutral delays are investigated. By employing fixed-point theorem, the exponential dichotomy, and differential inequality techniques, we obtain some sufficient conditions to insure the existence and globally exponential stability of almost periodic solution. This is the first time to investigate the almost periodic solution of the BAM neutral neural network and the results of this paper are new, and they extend previously known results.

1. Introduction

Neural networks have been extensively investigated by experts of many areas such as pattern recognition, associative memory, and combinatorial optimization, recently, see [1–10]. Up to now, many results about stability of bidirectional associative memory (BAM) neural networks have been derived. For these BAM systems, periodic oscillatory behavior, almost periodic oscillatory properties, chaos, and bifurcation are their research contents; generally speaking, almost periodic oscillatory property is a common phenomenon in the real world, and in some aspects, it is more actual than other properties, see [11–21].

Time delays cannot be avoided in the hardware implementation of neural networks because of the finite switching speed of amplifiers and the finite signal propagation time in biological networks. The existence of time delay may lead to a system's instability or oscillation, so delay cannot be neglected in modeling. It is known to all that many practical delay systems can be modelled as differential systems of neutral type, whose differential expression concludes not only the derivative term of the current state, but also concludes the derivative of the past state. It means that state's changing at the past time may affect the current state. Practically, such phenomenon always appears in the study of automatic control, population dynamics, and so forth, and it is natural and important that systems will contain some information about the derivative of the past state to further describe and model the dynamics for such complex neural reactions [22]. Authors in [18–29] added neutral delay into the neural networks. In these papers, only [18–20] studied the almost periodic solution of the neutral neural networks. For example, in [19] the following network was studied:

$$\dot{x}_i(t) = -c_i(t)x_i(t) + \sum_{j=1}^{n} a_{ij}(t) f_j\left(x_j\left(t-\tau_{ij}(t)\right)\right) + \sum_{j=1}^{n} b_{ij}(t) g_j\left(\dot{x}_j\left(t-\sigma_{ij}(t)\right)\right) + I_i(t). \tag{1.1}$$

Some sufficient conditions are obtained for the existence and globally exponential stability of almost periodic solution by employing fixed-point theorem and differential inequality techniques. References [21–26] studied the global asymptotic stability of equilibrium point, where [22] investigated the equilibrium point of the following BAM neutral neural network with constant coefficients:

$$\begin{aligned} \dot{u}_i(t) &= -a_i u_i(t) + \sum_{j=1}^{m} w_{1ji} g_j\left(v_j(t-d)\right) + \sum_{j=1}^{n} w_{2ij} \dot{u}_j(t-h) + I_i, \\ \dot{v}_j(t) &= -b_j v_j(t) + \sum_{i=1}^{n} r_{1ij} g_i(u_i(t-h)) + \sum_{i=1}^{m} r_{2ji} \dot{v}_i(t-d) + J_i. \end{aligned} \tag{1.2}$$

By using the Lyapunov method and linear matrix inequality techniques, a new stability criterion was derived. References [27–29] studied the exponential stability of equilibrium point.

It is obviously that men always studied the stability of the equilibrium point of the neutral neural networks, and there is little result for the almost periodic solution of neutral neural networks, especially, for the BAM neutral type neural networks. Besides, in papers [11, 23, 27, 28], time delay must be differentiable, and its derivative is bounded, which we think is a strict condition.

Motivated by the above discussions, in this paper, we consider the almost periodic solution of a class of BAM neural networks with variable coefficients and neutral delays. By fixed-point theorem and differential inequality techniques, we obtain some sufficient conditions to insure the existence and globally exponential stability of almost periodic solution. To the best of the authors' knowledge, this is the first time to investigate the almost periodic solution of the BAM neutral neural network, and we can remove delay's derivable condition, so the results of this paper are new, and they extend previously known results.

2. Preliminaries

In this paper, we consider the following system:

$$\begin{aligned} \dot{x}_i(t) &= -c_i(t)x_i(t) + \sum_{j=1}^{m} a_{ij}(t) f_{1j}\left(y_j\left(t-\tau_{ij}(t)\right)\right) + \sum_{j=1}^{n} b_{ji}(t) f_{2j}\left(\dot{x}_j\left(t-\overline{\delta}_{ji}(t)\right)\right) + I_i(t), \\ \dot{y}_j(t) &= -d_j(t)y_j(t) + \sum_{i=1}^{n} p_{ji}(t) g_{1i}\left(x_i\left(t-\delta_{ji}(t)\right)\right) + \sum_{i=1}^{m} q_{ij}(t) g_{2i}\left(\dot{y}_j\left(t-\overline{\tau}_{ij}(t)\right)\right) + J_j(t), \end{aligned} \tag{2.1}$$

where $i = 1,2,\ldots,n$; $j = 1,2,\ldots,m$. $x_i(t)$, $y_j(t)$ are the states of the ith neuron of X layer and the jth neuron of Y layer, respectively; $a_{ij}(t)$, $p_{ji}(t)$ and $b_{ji}(t)$, $q_{ij}(t)$ are the delayed strengths of connectivity and the neutral delayed strengths of connectivity, respectively; f_{1j}, f_{2j}, g_{1i}, g_{2i} are activation functions; $I_i(t)$, $J_j(t)$ stands for the external inputs; $\tau_{ij}(t), \overline{\tau}_{ij}(t)$, $\delta_{ji}(t)$, and $\overline{\delta}_{ji}(t)$ correspond to the delays, they are nonnegative; $c_i(t)$, $d_j(t) > 0$ represent the rate with which the ith neuron of X layer and the jth neuron of Y layer will reset its potential to the resting state in isolation when disconnected from the networks.

Throughout this paper, we assume the following.

(H_1) $c_i(t)$, $d_j(t)$, $a_{ij}(t)$, $p_{ji}(t)$, $b_{ji}(t)$, $q_{ij}(t)$, $\tau_{ij}(t)$, $\overline{\tau}_{ij}(t)$, $\delta_{ji}(t)$, $\overline{\delta}_{ji}(t)$, $I_i(t)$, and $J_j(t)$ are continuous almost periodic functions. Moreover, we let

$$\begin{gathered} c_i^+ = \sup_{t\in R}\{c_i(t)\}, \qquad c_i^- = \inf_{t\in R}\{c_i(t)\} > 0, \qquad d_j^+ = \sup_{t\in R}\{d_j(t)\}, \qquad d_j^- = \inf_{t\in R}\{d_j(t)\} > 0, \\ a_{ij} = \sup_{t\in R}\{|a_{ij}(t)|\} < \infty, \qquad b_{ji} = \sup_{t\in R}\{|b_{ji}(t)|\} < \infty, \qquad p_{ji} = \sup_{t\in R}\{|p_{ji}(t)|\} < \infty, \\ q_{ij} = \sup_{t\in R}\{|q_{ij}(t)|\} < \infty, \qquad I_i = \sup_{t\in R}\{|I_i(t)|\} < \infty, \qquad J_j = \sup_{t\in R}\{|J_j(t)|\} < \infty. \end{gathered} \tag{2.2}$$

(H_2) f_{1j}, f_{2j}, g_{1i}, and g_{2i} are Lipschitz continuous with the Lipschitz constants F_{1j}, F_{2j}, G_{1i}, G_{2i}, and $f_{1j}(0) = f_{2j}(0) = g_{1i}(0) = g_{2i}(0) = 0$.

(H_3) Consider

$$\begin{aligned} \alpha = \max\Bigg\{ &\max_{1\le i\le n} \max\left\{\frac{1}{c_i^-}, 1 + \frac{c_i^+}{c_i^-}\right\}\left(\sum_{j=1}^{m} a_{ij}F_{1j} + \sum_{j=1}^{n} b_{ji}F_{2j}\right), \\ &\max_{1\le j\le m} \max\left\{\frac{1}{d_j^-}, 1 + \frac{d_j^+}{d_j^-}\right\}\left(\sum_{i=1}^{n} p_{ji}G_{1i} + \sum_{i=1}^{m} q_{ij}G_{2i}\right)\Bigg\} < 1. \end{aligned} \tag{2.3}$$

The initial conditions of system (2.1) are of the following form:

$$\begin{aligned} x_i(t) &= \varphi_i(t), \quad t \in [-\delta, 0], \ \delta = \sup_{t\in R}\max_{i,j}\max\left\{\delta_{ji}(t), \overline{\delta}_{ji}(t)\right\}, \\ y_j(t) &= \phi_j(t), \quad t \in [-\tau, 0], \ \tau = \sup_{t\in R}\max_{i,j}\max\{\tau_{ij}(t), \overline{\tau}_{ij}(t)\}, \end{aligned} \tag{2.4}$$

where $i = 1,2,\ldots,n$; $j = 1,2,\ldots,m$; $\varphi_i(t)$, $\phi_j(t)$ are continuous almost periodic functions.

Let $X = \{\psi \mid \psi = (\varphi_1, \varphi_2, \ldots, \varphi_n, \phi_1, \phi_2, \ldots, \phi_m)^T$, where φ_i, $\phi_j : R \to R$ are continuously differentiable almost periodic functions. For any $\psi \in X$, $\psi(t) = (\varphi_1(t), \varphi_2(t), \ldots, \varphi_n(t), \phi_1(t), \phi_2(t), \ldots, \phi_m(t))^T$. We define $\|\psi(t)\|_1 = \max\{\|\psi(t)\|_0, \|\dot{\psi}(t)\|_0\}$, where $\|\psi(t)\|_0 = \max\{\max_{1\le i\le n}\{|\varphi_i(t)|\}, \max_{1\le j\le m}\{|\phi_i(t)|\}\}$, and $\dot{\psi}(t)$ is the derivative of ψ at t. Let $\|\psi\| = \sup_{t\in R}\|\psi(t)\|_1$, then X is a Banach space.

The following definitions and lemmas will be used in this paper.

Definition 2.1 (see [11]). Let $x(t) : R \to R^n$ be continuous in t. $x(t)$ is said to be almost periodic on R, if for any $\varepsilon > 0$, the set $T(x,\varepsilon) = \{w|x(t+w) - x(t) < \varepsilon, \text{for all } t \in R\}$ is relatively dense, that is, for all $\varepsilon > 0$, it is possible to find a real number $l = l(\varepsilon) > 0$, for any interval length $l(\varepsilon)$, there exists a number $\tau = \tau(\varepsilon)$ in this interval such that $|x(t+\tau) - x(t)| < \varepsilon$, for all $t \in R$.

Definition 2.2 (see [11]). Let $x \in C(R, R^n)$ and $Q(t)$ be $n \times n$ continuous matrix defined on R. The following linear system:

$$\dot{x}(t) = Q(t)x(t) \tag{2.5}$$

is said to admit an exponential dichotomy on R if there exist constants K, α, projection P, and the fundamental solution $X(t)$ of (2.5) satisfying

$$\begin{aligned} \left|X(t)PX^{-1}(s)\right| &\le Ke^{-\alpha(t-s)}, \quad t \ge s, \\ \left|X(t)(I-P)X^{-1}(s)\right| &\le Ke^{-\alpha(s-t)}, \quad t \le s. \end{aligned} \tag{2.6}$$

Definition 2.3. Let $z^*(t) = (x^*(t), y^*(t))^T = (x_1^*(t), \ldots, x_n^*(t), y_1^*(t), \ldots, y_m^*(t))^T$ be a continuously differentiable almost periodic solution of (2.1) with initial value $\psi^* = (\varphi^*, \phi^*)^T = (\varphi_1^*, \ldots, \varphi_n^*, \phi_1^*, \ldots, \phi_m^*)^T$. If there exist constants $\lambda > 0$, $M > 1$ such that for every solution $z(t) = (x(t), y(t))^T = (x_1(t), \ldots, x_n(t), y_1(t), \ldots, y_m(t))^T$ of (2.1) with any initial value $\psi = (\varphi, \phi)^T = (\varphi_1, \ldots, \varphi_n, \phi_1, \ldots, \phi_m)^T$, if

$$\|z(t) - z^*(t)\|_1 \le Me^{\lambda t}\|\psi - \psi^*\|, \quad \text{for } t > 0, \tag{2.7}$$

where $\varphi_i^*(\mathrm{t}), \phi_j^*(t), \varphi_i(t)$, and $\phi_j(t)$ are almost periodic functions. Then $z^*(t)$ is said to be globally exponentially stable.

Lemma 2.4 (see [11]). *If the linear system* (2.5) *admits an exponential dichotomy, then the almost periodic system*

$$\dot{x}(t) = Q(t)x(t) + f(t) \tag{2.8}$$

has a unique almost periodic solution

$$\psi(t) = \int_{-\infty}^{t} X(t)PX^{-1}(s)f(s)ds - \int_{t}^{+\infty} X(t)(I-P)X^{-1}f(s)ds. \tag{2.9}$$

Lemma 2.5 (see [11]). *Let $q_i(t)$ be an almost periodic function on R and*

$$M[q_i] = \lim_{T \to +\infty} \frac{1}{T}\int_{t}^{t+T} q_i(t)ds > 0, \quad i = 1, 2, \ldots, n, \tag{2.10}$$

then the linear system $\dot{z}(t) = \operatorname{diag}\{-q_1(t), \ldots, -q_n(t)\}z(t)$ admits exponential dichotomy on R.

3. Existence and Uniqueness of Almost Periodic Solutions

In this section, we consider the existence and uniqueness of almost periodic solutions by fixed-point theorem.

Theorem 3.1. *Under the assumptions* (H_1) − (H_3)*, the system* (2.1) *has a unique almost periodic solution in the region* $\|\varphi - \varphi_0\| \le \alpha\beta/(1-\alpha)$.

(H_4) *If*

$$\begin{aligned} M[c_i] &= \lim_{T\to+\infty}\frac{1}{T}\int_t^{t+T} c_i(s)ds > 0, \quad i = 1,2,\ldots,n, \\ M\left[d_j\right] &= \lim_{T\to+\infty}\frac{1}{T}\int_t^{t+T} d_j(s)ds > 0, \quad j = 1,2,\ldots,m \end{aligned} \tag{3.1}$$

holds, where

$$\begin{aligned} \beta &= \max\left\{\max_{1\le i\le n}\max\left\{\frac{I_i}{c_i^-}, I_i + \frac{I_i c_i^+}{c_i^-}\right\}, \max_{1\le j\le m}\max\left\{\frac{J_j}{d_j^-}, J_j + \frac{J_j d_j^+}{d_j^-}\right\}\right\}, \\ \varphi_0(t) &= \left(\int_{-\infty}^t e^{-\int_s^t c_1(u)du} I_1(s)ds, \ldots, \int_{-\infty}^t e^{-\int_s^t c_n(u)du} I_n(s)ds,\right. \\ &\qquad \left.\int_{-\infty}^t e^{-\int_s^t d_1(u)du} J_1(s)ds, \ldots, \int_{-\infty}^t e^{-\int_s^t d_m(u)du} J_m(s)ds\right)^T. \end{aligned} \tag{3.2}$$

Proof. For any $(\varphi,\phi)^T = (\varphi_1,\ldots,\varphi_n,\phi_1,\ldots,\phi_m)^T \in X$, we consider the the following system:

$$\begin{aligned} \dot{x}_i(t) &= -c_i(t)x_i(t) + \sum_{j=1}^m a_{ij}(t) f_{1j}\left(\phi_j\left(t-\tau_{ij}(t)\right)\right) + \sum_{j=1}^n b_{ji}(t) f_{2j}\left(\dot{\varphi}_j\left(t-\overline{\delta}_{ji}(t)\right)\right) + I_i(t), \\ \dot{y}_j(t) &= -d_j(t)y_j(t) + \sum_{i=1}^n p_{ji}(t) g_{1i}\left(\varphi_i\left(t-\delta_{ji}(t)\right)\right) + \sum_{i=1}^m q_{ij}(t) g_{2i}\left(\dot{\phi}_i\left(t-\overline{\tau}_{ij}(t)\right)\right) + J_j(t). \end{aligned} \tag{3.3}$$

From (H_4) and Lemma 2.5, we know the following linear system:

$$\begin{aligned} \dot{x}_i(t) &= -c_i(t)x_i(t), \\ \dot{y}_j(t) &= -d_j(t)y_j(t) \end{aligned} \tag{3.4}$$

admits an exponential dichotomy on R. By Lemma 2.4, System (3.3) has an almost periodic solution $z_{(\varphi,\phi)^T}(t)$ which can be expressed as follows:

$$z_{(\varphi,\phi)^T}(t) = \left(\int_{-\infty}^{t} e^{-\int_s^t c_1(u)du}(A_1(s)+I_1(s))ds,\ldots,\int_{-\infty}^{t} e^{-\int_s^t c_n(u)du}(A_n(s)+I_n(s))ds,\right.$$
$$\left.\int_{-\infty}^{t} e^{-\int_s^t d_1(u)du}\left(\overline{A}_1(s)+J_1(s)\right)ds,\ldots,\int_{-\infty}^{t} e^{-\int_s^t d_m(u)du}\left(\overline{A}_m(s)+J_m(s)\right)ds\right)^T, \tag{3.5}$$

where

$$\begin{aligned} A_i(s) &= \sum_{j=1}^{m} a_{ij}(s) f_{1j}(\phi_j(s-\tau_{ij}(s))) + \sum_{j=1}^{n} b_{ji}(s) f_{2j}\left(\dot{\varphi}_j\left(s-\overline{\delta}_{ji}(s)\right)\right), \quad i=1,2,\ldots,n, \\ \overline{A}_j(s) &= \sum_{i=1}^{n} p_{ji}(s) g_{1i}(\varphi_i(s-\delta_{ji}(s))) + \sum_{i=1}^{m} q_{ij}(s) g_{2i}(\dot{\phi}_i(s-\overline{\tau}_{ij}(s))), \quad j=1,2,\ldots,m. \end{aligned} \tag{3.6}$$

So, we can define a mapping $T : X \to X$, by letting

$$T(\varphi,\phi)^T(t) = z_{(\varphi,\phi)^T}(t), \quad \forall(\varphi,\phi)^T \in X. \tag{3.7}$$

Set $X_0 = \{\psi \mid \psi \in X, \|\psi-\psi_0\| \le \alpha\beta/(1-\alpha)\}$; clearly, X_0 is a closed convex subset of X, so we have

$$\begin{aligned} \|\psi_0\| = {} & \max\left\{\sup_{t\in R}\max_{1\le i\le n}\left|\int_{-\infty}^{t} e^{-\int_s^t c_i(u)du} I_i(s)ds\right|, \sup_{t\in R}\max_{1\le i\le n}\left|\left(\int_{-\infty}^{t} e^{-\int_s^t c_i(u)du} I_i(s)ds\right)'\right|,\right. \\ & \left.\sup_{t\in R}\max_{1\le j\le m}\left|\int_{-\infty}^{t} e^{-\int_s^t d_j(u)du} J_j(s)ds\right|, \sup_{t\in R}\max_{1\le j\le m}\left|\left(\int_{-\infty}^{t} e^{-\int_s^t d_j(u)du} J_j(s)ds\right)'\right|\right\} \\ \le {} & \max\left\{\max_{1\le i\le n}\max\left\{\frac{I_i}{c_i^-}, I_i+\frac{I_i c_i^+}{c_i^-}\right\}, \max_{1\le j\le m}\max\left\{\frac{J_j}{d_j^-}, J_j+\frac{J_j d_j^+}{d_j^-}\right\}\right\} = \beta. \end{aligned} \tag{3.8}$$

Therefore,

$$\|\psi\| \le \|\psi-\psi_0\| + \|\psi_0\| \le \frac{\alpha\beta}{1-\alpha} + \beta = \frac{\beta}{1-\alpha}, \quad \forall\psi \in X_0. \tag{3.9}$$

First, we prove that the mapping T is a self-mapping from X_0 to X_0. In fact, for any $\psi = (\overline{\varphi}_1, \ldots, \overline{\varphi}_n, \overline{\phi}_1, \ldots, \overline{\phi}_m)^T \in X_0$, let

$$\begin{aligned} B_i(s) &= \sum_{j=1}^{m} a_{ij}(s) f_{1j}\left(\overline{\phi}_j(s-\tau_{ij}(s))\right) + \sum_{j=1}^{n} b_{ji}(s) f_{2j}\left(\dot{\overline{\varphi}}_j\left(s-\overline{\delta}_{ji}(s)\right)\right), \quad i = 1,2,\ldots,n, \\ \overline{B}_j(s) &= \sum_{i=1}^{n} p_{ji}(s) g_{1i}(\overline{\varphi}_i(s-\delta_{ji}(s))) + \sum_{i=1}^{m} q_{ij}(s) g_{2i}\left(\dot{\overline{\phi}}_i(s-\overline{\tau}_{ij}(s))\right), \quad j = 1,2,\ldots,m. \end{aligned} \tag{3.10}$$

From (H_2) and (H_3), we have

$$\begin{aligned} \|T\psi - \psi_0\| = \max\Bigg\{ & \sup_{t\in R} \max_{1\le i\le n} \left\{ \left| \int_{-\infty}^{t} e^{-\int_s^t c_i(u)du} B_i(s)ds \right| \right\}, \\ & \sup_{t\in R} \max_{1\le i\le n} \left\{ \left| -c_i(t) \int_{-\infty}^{t} e^{-\int_s^t c_i(u)du} B_i(s)ds + B_i(t) \right| \right\}, \\ & \sup_{t\in R} \max_{1\le j\le m} \left\{ \left| \int_{-\infty}^{t} e^{-\int_s^t d_j(u)du} \overline{B}_j(s)ds \right| \right\}, \\ & \sup_{t\in R} \max_{1\le j\le m} \left\{ \left| -d_j(t) \int_{-\infty}^{t} e^{-\int_s^t d_j(u)du} \overline{B}_j(s)ds + \overline{B}_j(t) \right| \right\} \Bigg\} \\ \le \max\Bigg\{ & \sup_{t\in R} \max_{1\le i\le n} \left\{ \int_{-\infty}^{t} e^{c_i^-(s-t)} |B_i(s)| ds \right\}, \\ & \sup_{t\in R} \max_{1\le i\le n} \left\{ c_i^+ \int_{-\infty}^{t} e^{c_i^-(s-t)} |B_i(s)| ds + |B_i(t)| \right\}, \\ & \sup_{t\in R} \max_{1\le j\le m} \left\{ \int_{-\infty}^{t} e^{d_j^-(s-t)} \left|\overline{B}_j(s)\right| ds \right\}, \\ & \sup_{t\in R} \max_{1\le j\le m} \left\{ d_j^+ \int_{-\infty}^{t} e^{d_j^-(s-t)} \left|\overline{B}_j(s)\right| ds + \left|\overline{B}_j(t)\right| \right\} \Bigg\} \\ \le \max\Bigg\{ & \max_{1\le i\le n} \left\{ \frac{1}{c_i^-} \left(\sum_{j=1}^{m} a_{ij} F_{1j} + \sum_{j=1}^{n} b_{ji} F_{2j} \right) \right\}, \\ & \max_{1\le i\le n} \left\{ \left(1 + \frac{c_i^+}{c_i^-}\right) \left(\sum_{j=1}^{m} a_{ij} F_{1j} + \sum_{j=1}^{n} b_{ji} F_{2j} \right) \right\}, \\ & \max_{1\le j\le m} \left\{ \frac{1}{d_j^-} \left(\sum_{i=1}^{n} p_{ji} G_{1i} + \sum_{i=1}^{m} q_{ij} G_{2i} \right) \right\}, \\ & \max_{1\le j\le m} \left\{ \left(1 + \frac{d_j^+}{d_j^-}\right) \left(\sum_{i=1}^{n} p_{ji} G_{1i} + \sum_{i=1}^{m} q_{ij} G_{2i} \right) \right\} \Bigg\} \|\psi\| \end{aligned}$$

$$
\begin{aligned}
&= \max\left\{ \max_{1\le i\le n} \max\left\{ \frac{1}{c_i^-}, 1+\frac{c_i^+}{c_i^-} \right\} \left(\sum_{j=1}^{m} a_{ij}F_{1j} + \sum_{j=1}^{n} b_{ji}F_{2j} \right), \right. \\
&\qquad \left. \max_{1\le j\le m} \max\left\{ \frac{1}{d_j^-}, 1+\frac{d_j^+}{d_j^-} \right\} \left(\sum_{i=1}^{n} p_{ji}G_{1i} + \sum_{i=1}^{m} q_{ij}G_{2i} \right) \right\} \|\varphi\| \\
&= \alpha\|\varphi\| \le \frac{\alpha\beta}{1-\alpha}.
\end{aligned}
\tag{3.11}
$$

This implies that $T(\varphi) \in X_0$, so T is a self-mapping from X_0 to X_0.

Finally, we prove that T is a contraction mapping. In fact, for any $\varphi_1 = (\alpha_1, \dots, \alpha_n, \beta_1, \dots, \beta_m)^T$, $\varphi_2 = (\overline{\alpha}_1, \dots, \overline{\alpha}_n, \overline{\beta}_1, \dots, \overline{\beta}_m)^T \in X_0$. Let

$$
\begin{aligned}
H_i(s) &= \sum_{j=1}^{m} a_{ij}(s)\left[f_{1j}(\beta_j(s-\tau_{ij}(s))) - f_{1j}\left(\overline{\beta}_j(s-\tau_{ij}(s))\right)\right] \\
&\quad + \sum_{j=1}^{n} b_{ji}(s)\left[f_{2j}\left(\dot{\alpha}_j\left(s-\overline{\delta}_{ji}(s)\right)\right) - f_{2j}\left(\dot{\overline{\alpha}}_j\left(s-\overline{\delta}_{ji}(s)\right)\right)\right], \quad i = 1,2,\dots,n, \\
\overline{H}_j(s) &= \sum_{i=1}^{n} p_{ji}(s)\left[g_{1i}(\alpha_i(s-\delta_{ji}(s))) - g_{1i}(\overline{\alpha}_i(s-\delta_{ji}(s)))\right] \\
&\quad + \sum_{i=1}^{m} q_{ij}(s)\left[g_{2i}(\dot{\beta}_i(s-\overline{\tau}_{ij}(s))) - g_{2i}\left(\dot{\overline{\beta}}_i(s-\overline{\tau}_{ij}(s))\right)\right], \quad j = 1,2,\dots,m.
\end{aligned}
\tag{3.12}
$$

We have

$$
\begin{aligned}
\|T\varphi_1 - T\varphi_2\| = \max&\left\{ \sup_{t\in R}\max_{1\le i\le n}\left\{ \left| \int_{-\infty}^{t} e^{-\int_s^t c_i(u)du} H_i(s)ds \right| \right\}, \right. \\
&\sup_{t\in R}\max_{1\le i\le n}\left\{ \left| -c_i(t)\int_{-\infty}^{t} e^{-\int_s^t c_i(u)du} H_i(s)ds + H_i(t) \right| \right\}, \\
&\sup_{t\in R}\max_{1\le j\le m}\left\{ \left| \int_{-\infty}^{t} e^{-\int_s^t d_j(u)du} \overline{H}_j(s)ds \right| \right\}, \\
&\left. \sup_{t\in R}\max_{1\le j\le m}\left\{ \left| -d_j(t)\int_{-\infty}^{t} e^{-\int_s^t d_j(u)du} \overline{H}_j(s)ds + \overline{H}_j(t) \right| \right\} \right\} \\
\le \max&\left\{ \max_{1\le i\le n} \max\left\{ \frac{1}{c_i^-}, 1+\frac{c_i^+}{c_i^-} \right\} \left(\sum_{j=1}^{m} a_{ij}F_{1j} + \sum_{j=1}^{n} b_{ji}F_{2j} \right), \right. \\
&\left. \max_{1\le j\le m} \max\left\{ \frac{1}{d_j^-}, 1+\frac{d_j^+}{d_j^-} \right\} \left(\sum_{i=1}^{n} p_{ji}G_{1i} + \sum_{i=1}^{m} q_{ij}G_{2i} \right) \right\} \|\varphi_1 - \varphi_2\| \\
= \alpha&\|\varphi_1 - \varphi_2\|.
\end{aligned}
\tag{3.13}
$$

Notice that $\alpha < 1$, it means that the mapping T is a contraction mapping. By Banach fixed-point theorem, there exists a unique fixed-point $\varphi^* \in X_0$ such that $T\varphi^* = \varphi^*$, which implies system (2.1) has a unique almost periodic solution. □

4. Global Exponential Stability of the Almost Periodic Solution

In this section, we consider the exponential stability of almost periodic solution, and we give two corollaries.

Theorem 4.1. *Under the assumptions* $(H_1)-(H_4)$*, then system* (2.1) *has a unique almost periodic solution which is global exponentially stable.*

Proof. It follows from Theorem 3.1 that system (2.1) has a unique almost periodic solution $z^*(t) = (x^*(t), y^*(t))^T = (x_1^*(t), \ldots, x_n^*(t), y_1^*(t), \ldots, y_m^*(t))^T$ with the initial value $\varphi^* = (\varphi^*, \phi^*)^T = (\varphi_1^*, \ldots, \varphi_n^*, \phi_1^*, \ldots, \phi_m^*)^T$. Set $z(t) = (x(t), y(t))^T = (x_1(t), \ldots, x_n(t), y_1(t), \ldots, y_m(t))^T$ is an arbitrary solution of system (2.1) with initial value $\varphi = (\varphi, \phi)^T = (\varphi_1, \ldots, \varphi_n, \phi_1, \ldots, \phi_m)^T$. Let $u_i(t) = x_i(t) - x_i^*(t)$, $v_j(t) = y_j(t) - y_j^*(t)$, $\Psi_i = \varphi_i - \varphi_i^*$, $\Phi_j = \phi_j - \phi_j^*$. Then $z(t) - z^*(t) = (u_1(t), \ldots, u_n(t), v_1(t), \ldots, v_m(t))^T$, where $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$. Then system (2.1) is equivalent to the following system:

$$\begin{aligned} \dot{u}_i(s) + c_i(s)u_i(s) &= F_i(s), \quad s > 0, \\ \dot{v}_j(s) + d_j(s)v_j(s) &= \overline{F}_j(s), \quad s > 0, \end{aligned} \tag{4.1}$$

with the initial value

$$\begin{aligned} \Psi_i(s) &= \varphi_i(s) - \varphi_i^*(s), \quad s \in [-\delta, 0], \\ \Phi_j(s) &= \phi_j(s) - \phi_j^*(s), \quad s \in [-\tau, 0], \end{aligned} \tag{4.2}$$

where

$$\begin{aligned} F_i(s) &= \sum_{j=1}^{m} a_{ij}(s)\Big[f_{1j}\Big(y_j^*(s-\tau_{ij}(s)) + v_j(s-\tau_{ij}(s))\Big) - f_{1j}\Big(y_j^*(s-\tau_{ij}(s))\Big)\Big] \\ &\quad + \sum_{j=1}^{n} b_{ji}(s)\Big[f_{2j}\Big(\dot{x}_j^*\Big(s-\overline{\delta}_{ji}(s)\Big) + \dot{u}_j\Big(s-\overline{\delta}_{ji}(s)\Big)\Big) - f_{2j}\Big(\dot{x}_j^*\Big(s-\overline{\delta}_{ji}(s)\Big)\Big)\Big], \\ \overline{F}_j(s) &= \sum_{i=1}^{n} p_{ji}(s)\left[g_{1i}(x_i^*(s-\delta_{ji}(s)) + u_i(s-\delta_{ji}(s))) - g_{1i}(x_i^*(s-\delta_{ji}(s)))\right] \\ &\quad + \sum_{i=1}^{m} q_{ij}(s)\left[g_{2i}(\dot{y}_i^*(s-\overline{\tau}_{ij}(s)) + \dot{v}_i(s-\overline{\tau}_{ij}(s))) - g_{2i}(\dot{y}_i^*(s-\overline{\tau}_{ij}(s)))\right]. \end{aligned} \tag{4.3}$$

Let

$$\Gamma_i(\xi_i) = c_i^- - \xi_i - \sum_{j=1}^{m} a_{ij} F_{1j} e^{\tau \xi_i} - \sum_{j=1}^{n} b_{ji} F_{2j} e^{\delta \xi_i},$$
$$\overline{\Gamma}_i\left(\overline{\xi}_i\right) = c_i^- - \overline{\xi}_i - \left(c_i^+ + c_i^-\right)\left(\sum_{j=1}^{m} a_{ij} F_{1j} e^{\tau \overline{\xi}_i} + \sum_{j=1}^{n} b_{ji} F_{2j} e^{\delta \overline{\xi}_i}\right), \tag{4.4}$$

where $\xi_i, \overline{\xi}_i \geq 0$, $i = 1, 2, \ldots, n$. From (H_3), we know $\Gamma_i(0) > 0$, $\overline{\Gamma}_i(0) > 0$. Since $\Gamma_i(\cdot)$ and $\overline{\Gamma}_i(\cdot)$ are continuous on $[0, \infty]$ and $\Gamma_i(\xi_i), \overline{\Gamma}_i(\overline{\xi}_i) \to -\infty$ as $\xi_i, \overline{\xi}_i \to +\infty$, so there exist $\xi_i^*, \overline{\xi}_i^* > 0$ such that $\Gamma_i(\xi_i^*) = \overline{\Gamma}_i(\overline{\xi}_i^*) = 0$ and $\Gamma_i(\xi_i) > 0$ for $\xi_i \in (0, \xi_i^*)$, $\overline{\Gamma}_i(\overline{\xi}_i) > 0$ for $\overline{\xi}_i \in (0, \overline{\xi}_i^*)$. By choosing $\xi = \min\{\xi_1^*, \ldots, \xi_n^*, \overline{\xi}_1^*, \ldots, \overline{\xi}_n^*\}$, we obtain $\Gamma_i(\xi), \overline{\Gamma}_i(\xi) \geq 0$. So we can choose a positive constant λ_1, $0 < \lambda_1 < \min\{\xi, c_i^-, \ldots, c_n^-\}$ such that $\Gamma_i(\lambda_1), \overline{\Gamma}_i(\lambda_1) > 0$. For the same reason, we define

$$G_j(\eta_j) = d_j^- - \eta_j - \sum_{i=1}^{n} p_{ji} G_{1i} e^{\delta \eta_j} - \sum_{i=1}^{m} q_{ij} G_{2i} e^{\tau \eta_j},$$
$$\overline{G}_j\left(\overline{\eta}_j\right) = d_j^- - \overline{\eta}_j - \left(d_j^- + d_j^+\right)\left(\sum_{i=1}^{n} p_{ji} G_{1i} e^{\delta \overline{\eta}_j} + \sum_{i=1}^{m} q_{ij} G_{2i} e^{\tau \overline{\eta}_j}\right). \tag{4.5}$$

There exists λ_2, $0 < \lambda_2 < d_j^-$, $j = 1, 2, \ldots, m$, such that $G_j(\lambda_2), \overline{G}_j(\lambda_2) > 0$. Taking $\lambda = \min\{\lambda_1, \lambda_2\}$, since $\Gamma_i(\cdot)$, $\overline{\Gamma}_i(\cdot)$, $G_j(\cdot)$, and $\overline{G}_j(\cdot)$ are strictly monotonous decrease functions, therefore, $\Gamma_i(\lambda), \overline{\Gamma}_i(\lambda), G_j(\lambda), \overline{G}_j(\lambda) > 0$, which implies

$$r_i := \frac{1}{c_i^- - \lambda}\left(\sum_{j=1}^{m} a_{ij} F_{1j} e^{\tau\lambda} + \sum_{j=1}^{n} b_{ji} F_{2j} e^{\delta\lambda}\right) < 1,$$
$$\overline{r}_i := \left(1 + \frac{c_i^+}{c_i^- - \lambda}\right)\left(\sum_{j=1}^{m} a_{ij} F_{1j} e^{\tau\lambda} + \sum_{j=1}^{n} b_{ji} F_{2j} e^{\delta\lambda}\right) < 1, \quad i = 1, 2, \ldots, n;$$
$$\frac{1}{d_j^- - \lambda}\left(\sum_{i=1}^{n} p_{ji} G_{1i} e^{\delta\lambda} + \sum_{i=1}^{m} q_{ij} G_{2i} e^{\tau\lambda}\right) < 1,$$
$$\left(1 + \frac{d_j^+}{d_j^- - \lambda}\right)\left(\sum_{i=1}^{n} p_{ji} G_{1i} e^{\delta\lambda} + \sum_{i=1}^{m} q_{ij} G_{2i} e^{\tau\lambda}\right) < 1, \quad j = 1, 2, \ldots, m. \tag{4.6}$$

Multiplying the two equations of system (4.1) by $e^{\int_0^s c_i(u)du}$ and $e^{\int_0^s d_j(u)du}$, respectively, and integrating on $[0,t]$, we get

$$\begin{aligned} u_i(t) &= u_i(0)e^{-\int_0^t c_i(u)du} + \int_0^t e^{-\int_s^t c_i(u)du} F_i(s)ds, \\ v_j(t) &= v_j(0)e^{-\int_0^t d_j(u)du} + \int_0^t e^{-\int_s^t d_j(u)du} \overline{F}_j(s)ds. \end{aligned} \tag{4.7}$$

Taking

$$M = \max\left\{ \max_{1\le i\le n} \frac{c_i^-}{\sum_{j=1}^m a_{ij}F_{1j} + \sum_{j=1}^n b_{ji}F_{2j}}, \max_{1\le j\le m} \frac{d_j^-}{\sum_{i=1}^n p_{ji}G_{1i} + \sum_{i=1}^m q_{ij}G_{2i}} \right\}, \tag{4.8}$$

then $M > 1$, thus

$$\|z(t) - z^*(t)\|_1 = \left\|\varphi(t) - \varphi^*(t)\right\|_1 \le \left\|\varphi - \varphi^*\right\| \le M\left\|\varphi - \varphi^*\right\|e^{\lambda t}, \quad t \le 0, \tag{4.9}$$

where $\lambda > 0$ as in (4.6). We claim that

$$\|z(t) - z^*(t)\|_1 \le M\left\|\varphi - \varphi^*\right\|e^{\lambda t}, \quad t > 0. \tag{4.10}$$

To prove (4.10), we first show for any $p > 1$, the following inequality holds:

$$\|z(t) - z^*(t)\|_1 < pM\left\|\varphi - \varphi^*\right\|e^{\lambda t}, \quad t > 0. \tag{4.11}$$

If (4.11) is false, then there must be some $t_1 > 0$ and some $i, l \in \{1,2,\ldots,n\}$, $j,k \in \{1,2,\ldots,m\}$, such that

$$\begin{aligned} \|z(t_1) - z^*(t_1)\|_1 &= \max\left\{|u_i(t_1)|, |\dot{u}_l(t_1)|, \left|v_j(t_1)\right|, |\dot{v}_k(t_1)|\right\} \\ &= pM\left\|\varphi - \varphi^*\right\|e^{\lambda t_1}, \end{aligned} \tag{4.12}$$

$$\|z(t) - z^*(t)\|_1 < pM\left\|\varphi - \varphi^*\right\|e^{\lambda t}, \quad 0 < t < t_1. \tag{4.13}$$

By (4.3)–(4.8), (4.12), and (4.13), we have

$$
\begin{aligned}
|u_i(t_1)| &= \left| u_i(0)e^{-\int_0^{t_1} c_i(u)du} + \int_0^{t_1} e^{-\int_s^{t_1} c_i(u)du} F_i(s)ds \right| \\
&\le e^{-c_i^- t_1}\|\varphi - \varphi^*\| + \int_0^{t_1} e^{-c_i^-(t_1-s)}|F_i(s)|ds \\
&\le e^{-c_i^- t_1}\|\varphi - \varphi^*\| + \int_0^{t_1} e^{-c_i^-(t_1-s)} \left(\sum_{j=1}^m a_{ij}F_{1j}pM\|\varphi - \varphi^*\|e^{-\lambda(s-\tau_{ij}(s))} \right. \\
&\qquad \left. + \sum_{j=1}^n b_{ji}F_{2j}pM\|\varphi - \varphi^*\|e^{-\lambda(s-\overline{\delta}_{ji}(s))} \right) ds \\
&< pM\|\varphi - \varphi^*\|e^{-\lambda t_1} \left[\frac{e^{t_1(\lambda - c_i^-)}}{M} + \frac{1 - e^{t_1(\lambda - c_i^-)}}{c_i^- - \lambda} \left(\sum_{j=1}^m a_{ij}F_{1j}e^{\lambda\tau} + \sum_{j=1}^n b_{ji}F_{2j}e^{\lambda\delta} \right) \right] \\
&= pM\|\varphi - \varphi^*\|e^{-\lambda t_1} \left[\left(\frac{1}{M} - r_i \right) e^{t_1(\lambda - c_i^-)} + r_i \right] \\
&< pM\|\varphi - \varphi^*\|e^{-\lambda t_1}; \\
|\dot{u}_l(t_1)| &= \left| -c_l(t_1)u_l(0)e^{-\int_0^{t_1} c_l(u)du} - c_l(t_1)\int_0^{t_1} e^{-\int_s^{t_1} c_l(u)du} F_l(s)ds + F_l(t_1) \right| \\
&\le c_l^+ e^{-c_l^- t_1}\|\varphi - \varphi^*\| + c_l^+ \int_0^{t_1} e^{-c_l^-(t_1-s)}|F_l(s)|ds + |F_l(t_1)| \\
&\le c_l^+ e^{-c_l^- t_1}\|\varphi - \varphi^*\| + c_l^+ \int_0^{t_1} e^{-c_l^-(t_1-s)} \left(\sum_{j=1}^m a_{lj}F_{1j}pM\|\varphi - \varphi^*\|e^{-\lambda(s-\tau_{lj}(s))} \right. \\
&\qquad \left. + \sum_{j=1}^n b_{jl}F_{2j}pM\|\varphi - \varphi^*\|e^{-\lambda(s-\overline{\delta}_{jl}(s))} \right) ds \\
&\quad + \sum_{j=1}^m a_{lj}F_{1j}pM\|\varphi - \varphi^*\|e^{-\lambda(t_1-\tau_{lj}(t_1))} + \sum_{j=1}^n b_{jl}F_{2j}pM\|\varphi - \varphi^*\|e^{-\lambda(t_1-\overline{\delta}_{jl}(t_1))} \\
&< pM\|\varphi - \varphi^*\|e^{-\lambda t_1} \left[\left(\frac{1}{M} - r_l \right) e^{t_1(\lambda - c_l^-)} + \overline{r}_l \right] \\
&< pM\|\varphi - \varphi^*\|e^{-\lambda t_1}.
\end{aligned}
\tag{4.14}
$$

We also can get

$$
\begin{aligned}
|v_j(t_1)| &< pM\|\psi - \psi^*\|e^{-\lambda t_1}, \\
|\dot{v}_k(t_1)| &< pM\|\psi - \psi^*\|e^{-\lambda t_1}.
\end{aligned}
\tag{4.15}
$$

From (4.14)–(4.15), we have

$$\|z(t_1) - z^*(t_1)\|_1 = \max\{|u_i(t_1)|, |\dot{u}_l(t_1)|, \left|v_j(t_1)\right|, |\dot{v}_k(t_1)|\} < pM\left\|\varphi - \varphi^*\right\|e^{-\lambda t_1}, \tag{4.16}$$

which contradicts the equality (4.12), so (4.11) holds. Letting $p \rightarrow 1$, then (4.10) holds. The almost periodic solution of system (2.1) is globally exponentially stable. □

Corollary 4.2. *Let $b_{ji}(t) = q_{ij}(t) = 0$. Under assumptions (H_1), (H_2), and (H_4), if, (H_5)*

$$\alpha_1 = \max\left\{\max_{1\leq i\leq n}\left\{\frac{1}{c_i^-}\sum_{j=1}^{m}a_{ij}F_{1j}\right\}, \max_{1\leq j\leq m}\left\{\frac{1}{d_j^-}\sum_{i=1}^{n}p_{ji}G_{1i}\right\}\right\} < 1 \tag{4.17}$$

holds, then system

$$\begin{aligned}\dot{x}_i(t) &= -c_i(t)x_i(t) + \sum_{j=1}^{m}a_{ij}(t)f_{1j}\left(y_j\left(t-\tau_{ij}(t)\right)\right) + I_i(t),\\ \dot{y}_j(t) &= -d_j(t)y_j(t) + \sum_{i=1}^{n}p_{ji}(t)g_{1i}\left(x_i\left(t-\delta_{ji}(t)\right)\right) + J_j(t)\end{aligned} \tag{4.18}$$

has a unique almost periodic solution in the region $\|\varphi - \varphi_0\| \leq \alpha_1\beta/(1-\alpha_1)$, which is global exponentially stable.

In fact, Zhang and Si [11, 16] and Chen et al. [17] studied system (4.18). This Corollary 4.2 is the Theorem 3.1 in [11], Theorem 1.1 in [16], and Theorem 1 in [17]. Especially, in [17], authors let

(H_5')

$$\overline{\alpha}_1 = \max_{1\leq i\leq n}\left\{\frac{1}{c_i^-}\sum_{j=1}^{m}a_{ij}F_{1j}\right\} + \max_{1\leq j\leq m}\left\{\frac{1}{d_j^-}\sum_{i=1}^{n}p_{ji}G_{1i}\right\} < 1. \tag{4.19}$$

Therefore, we extend and improve previously known results.

Remark 4.3. Let $c_i(t) = d_j(t)$, $a_{ij}(t) = p_{ji}(t)$, $b_{ji} = q_{ij}(t)$, $I_i(t) = J_j(t)$, $\tau_{ij}(t) = \delta_{ji}(t)$, $\overline{\tau}_{ij}(t) = \overline{\delta}_{ji}(t)$, $n = m$. Then system (2.1) is reduced to be system (1.1), hence we have the following.

Corollary 4.4. *Under assumptions* (H_1), (H_2), *and* (H_4), *if* (H_6)

$$\alpha_2 = \max_{1\le i\le n} \max\left\{1+\frac{c_i^+}{c_i^-}\right\}\sum_{j=1}^{m}\left(a_{ij}F_{1j}+b_{ji}F_{2j}\right)<1, \tag{4.20}$$

holds, then system (1.1) *has a unique almost periodic solution in the region* $\|\varphi-\varphi_0\|\le\alpha_2\beta/(1-\alpha_2)$, *which is global exponentially stable.*

This Corollary 4.4 is the result of [19].

5. An Example

In this section, we give an example to illustrate the effectiveness of our results.

Let $n = m = 2$, $f_1(y_1) = y_1/10$, $f_2(y_2) = \sin y_2/10$, $g_1(x_1) = x_1/12$, $g_2(x_2) = |x_2|/8$, $\tau_{ij}(t) = \overline{\tau}_{ij}(t) = \cos^2 t$, $\delta_{ji}(t) = \overline{\delta}_{ji}(t) = 0.5$, $I_1(t) = 1+\sin^2(t)$, $I_2(t) = 1+\cos^2 t$, $J_1(t) = 1+|\sin t|$, and $J_2(t) = \sin 2t + 0.5$, then we consider the following almost periodic system:

$$\begin{aligned}
\dot{x}_1(t) &= -c_1(t)x_1(t)+\sum_{j=1}^{2}a_{1j}(t)f_j\left(y_j\left(t-\cos^2 t\right)\right)+\sum_{j=1}^{2}b_{j1}(t)\dot{x}_j(t-0.5)+I_1(t),\\
\dot{x}_2(t) &= -c_2(t)x_2(t)+\sum_{j=1}^{2}a_{2j}(t)f_j\left(y_j\left(t-\cos^2 t\right)\right)+\sum_{j=1}^{2}b_{j2}(t)\dot{x}_j(t-0.5)+I_2(t),\\
\dot{y}_1(t) &= -d_1(t)y_1(t)+\sum_{i=1}^{2}p_{1i}(t)g_i(x_i(t-0.5))+\sum_{i=1}^{2}q_{i1}(t)\dot{y}_i\left(t-\cos^2 t\right)+J_1(t),\\
\dot{y}_2(t) &= -d_2(t)y_2(t)+\sum_{i=1}^{2}p_{2i}(t)g_i(x_i(t-0.5))+\sum_{i=1}^{2}q_{i2}(t)\dot{y}_i\left(t-\cos^2 t\right)+J_2(t),
\end{aligned} \tag{5.1}$$

where $c_1(t) = 1+\cos^2 t$, $c_2(t) = 1+\sin^2 t$, $d_1(t) = 1+|\cos t|$, $d_2(t) = 1+|\sin t|$, $a_{11}(t) = |\sin t|/4$, $a_{12}(t) = \cos^2 t/8$, $a_{21}(t) = \cos^2 t/6$, $a_{22}(t) = |\sin t|/4$, $b_{11}(t) = \cos 2t/8$, $b_{12}(t) = 0$, $b_{21}(t) = 0$, $b_{22}(t) = \sin 2t/10$, $p_{11}(t) = \cos 2t/4$, $p_{12}(t) = \sin 2t/9$, $p_{21}(t) = \sin^2 t/8$, $p_{22}(t) = |\cos t|/6$, $q_{11}(t) = \cos t/8$, $q_{12}(t) = 0$, $q_{21}(t) = 0$, and $q_{22}(t) = \cos^2 t/10$. By simple calculation, we obtain $\alpha = \max\{39/80, 51/120, 69/144, 63/160\} < 1$, hence this system has a unique almost periodic solution, which is global exponentially stable by Theorem 4.1. Figure 1 depicts the time responses of state variables of $x_1(t)$, $x_2(t)$, $y_1(t)$, and $y_2(t)$ with step $h = 0.005$ and initial states $[-0.2, 0.2, -0.3, 0.4]^T$ for $t \in [-1, 0]$, and Figures 2, 3, and 4 depict the phase orbits of $x_1(t)$ and $y_1(t)$, $x_1(t)$, and $x_2(t)$, $y_1(t)$ and $y_2(t)$. It confirms that our results are effective for (5.1).

6. Conclusions

In this paper, a class of BAM neural networks with variable coefficients and neutral time-varying delays are investigated. By employing Banach fixed-point theorem, the exponential

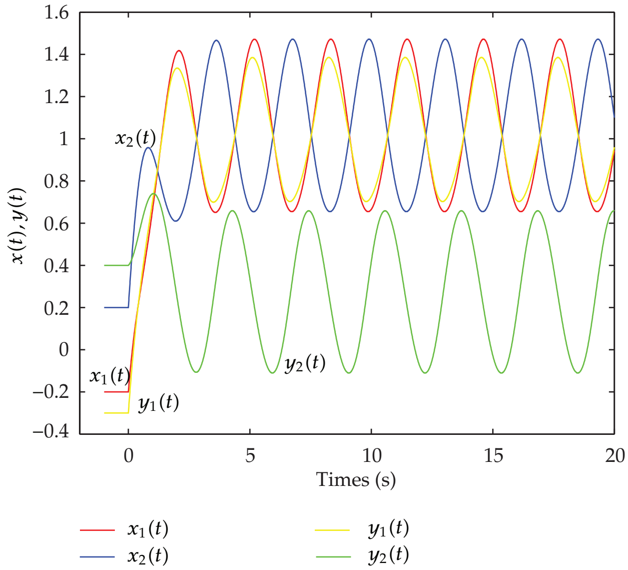

Figure 1: Transient response of state variable $x_1(t)$, $x_2(t)$, $y_1(t)$ and $y_2(t)$.

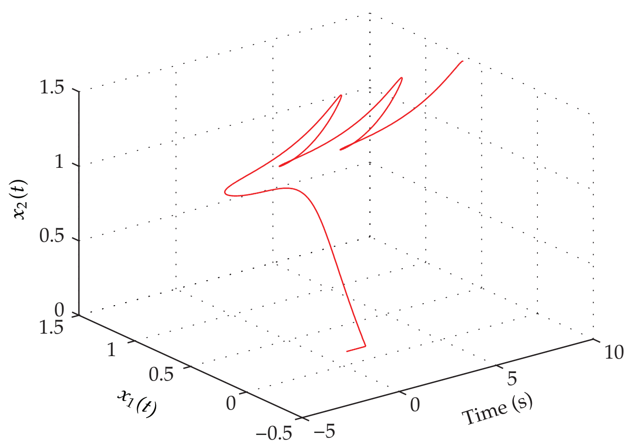

Figure 2: Phase response of state variable $x_1(t)$ and $x_2(t)$.

dichotomy and differential inequality techniques, some sufficient conditions are obtained to ensure the existence, uniqueness, and stability of the almost periodic solution. As is known to all, neural networks with neutral delays are studied rarely, and most authors solve these problems by linear matrix inequality techniques. In addition, BAM neural networks are much more complicated than the one-layer neural network. In a word, this paper is original, and novel. It also extends and improves other previously known results (see [11, 16, 17, 19]).

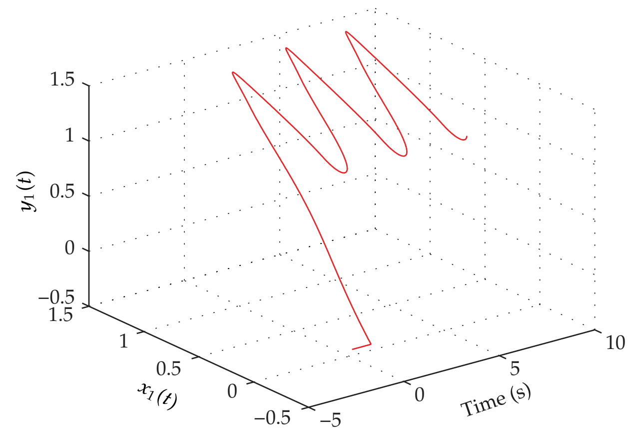

Figure 3: Phase response of state variable $x_1(t)$ and $y_1(t)$.

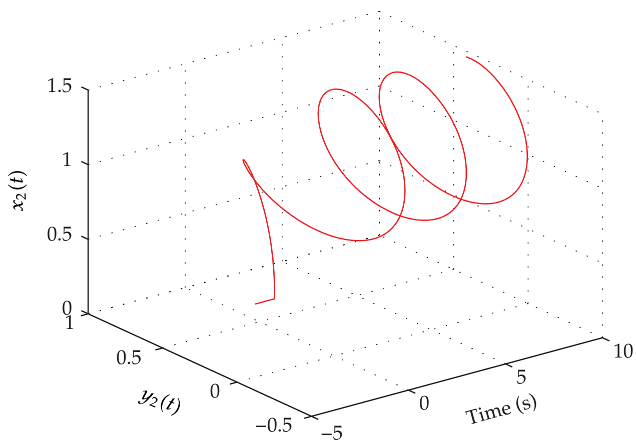

Figure 4: Phase response of state variable $x_2(t)$ and $y_2(t)$.

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