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Research Article

A Coupled System of Nonlinear Fractional Differential Equations with Multipoint Fractional Boundary Conditions on an Unbounded Domain

Guotao Wang,[1] Bashir Ahmad,[2] and Lihong Zhang[1]

[1] *School of Mathematics and Computer Science, Shanxi Normal University, Linfen, Shanxi 041004, China*
[2] *Department of Mathematics, Faculty of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia*

Correspondence should be addressed to Lihong Zhang, zhanglih149@126.com

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This paper investigates the existence of solutions for a coupled system of nonlinear fractional differential equations with m-point fractional boundary conditions on an unbounded domain. Some standard fixed point theorems are applied to obtain the main results. The paper concludes with two illustrative examples.

1. Introduction

In the last few decades, the subject of fractional calculus has gained considerable popularity and importance as it finds its applications in numerous fields of science and engineering. Some of the areas of recent applications of fractional models include fluid mechanics, solute transport or dynamical processes in porous media, material viscoelastic theory, dynamics of earthquakes, control theory of dynamical systems, and biomathematics. In the aforementioned areas, there are phenomena with estrange kinetics involving microscopic complex dynamical behaviour that cannot be characterized by classical derivative models. It has been learnt through experimentation that most of the processes associated with complex systems have nonlocal dynamics possessing long-memory in time, and the integral and derivative operators of fractional order do have some of these characteristics. Thus, due to the modeling capabilities of fractional integrals and derivatives for complex phenomena, the fractional modelling has emerged as a powerful tool and has accounted for the rapid development of the theory of fractional differential equations. Fractional differential equations also serve as an excellent tool for the description of hereditary properties of various materials and processes [1]. The presence of memory term in such models not only takes into account the history of the process involved but also carries its impact to present and future development of the process. For more details and applications, we refer the reader to the books [2–6]. For some recent work on the topic, see [7–27] and references therein.

The study of coupled systems involving fractional differential equations is also important as such systems occur in various problems of applied nature. For some recent results on systems of fractional differential equations, see [28–35].

Much of the work on fractional differential equations has been considered on finite domain and there are few papers dealing with infinite domain [36–43]. In this paper, we discuss the existence and uniqueness of the solutions of a coupled system of nonlinear fractional differential equations with m-point boundary conditions on an unbounded domain. Precisely, we consider the following problem:

$$\begin{gathered}
D^p u(t) + f(t, v(t)) = 0, \quad 2 < p < 3, \\
D^q v(t) + g(t, u(t)) = 0, \quad 2 < q < 3, \\
u(0) = u'(0) = 0, \qquad D^{p-1} u(+\infty) = \sum_{i=1}^{m-2} \beta_i u(\xi_i), \\
v(0) = v'(0) = 0, \qquad D^{q-1} v(+\infty) = \sum_{i=1}^{m-2} \gamma_i v(\xi_i),
\end{gathered} \tag{1.1}$$

where $t \in J = [0, +\infty)$, $f, g \in C(J \times \mathbb{R}, \mathbb{R})$, $0 < \xi_1 < \xi_2 < \cdots < \xi_{m-2} < +\infty$, D^p and D^q denote Riemann-Liouville fractional derivatives of order p and q, respectively, and $\beta_i > 0$, and $\gamma_i > 0$ are such that $0 < \sum_{i=1}^{m-2} \beta_i \xi_i^{p-1} < \Gamma(p)$ and $0 < \sum_{i=1}^{m-2} \gamma_i \xi_i^{q-1} < \Gamma(q)$.

2. Preliminaries

For the convenience of the readers, in this section we first present some useful definitions and lemmas.

Definition 2.1 (see [5]). The Riemann-Liouville fractional derivative of order δ for a continuous function f is defined by

$$D^\delta f(t) = \frac{1}{\Gamma(n-\delta)} \left(\frac{d}{dt}\right)^n \int_0^t (t-s)^{n-\delta-1} f(s) ds, \quad n = [\delta] + 1, \tag{2.1}$$

provided that the right-hand side is pointwise defined on $(0, \infty)$.

Definition 2.2 (see [5]). The Riemann-Liouville fractional integral of order δ for a function f is defined as

$$I^\delta f(t) = \frac{1}{\Gamma(\delta)} \int_0^t (t-s)^{\delta-1} f(s) ds, \quad \delta > 0, \tag{2.2}$$

provided that such integral exists.

For the forthcoming analysis, we define the spaces

$$X = \left\{ u \in C[0,+\infty) : \sup_{t\in J} \frac{|u(t)|}{1+t^{p-1}} < +\infty \right\},$$
$$Y = \left\{ v \in C[0,+\infty) : \sup_{t\in J} \frac{|v(t)|}{1+t^{q-1}} < +\infty \right\} \tag{2.3}$$

equipped with the norms

$$\|u\|_X = \sup_{t\in J} \frac{|u(t)|}{1+t^{p-1}}, \tag{2.4}$$

$$\|v\|_Y = \sup_{t\in J} \frac{|v(t)|}{1+t^{q-1}}. \tag{2.5}$$

Obviously X and Y are Banach spaces.

Lemma 2.3 (see [38]). *Let $h \in C([0,+\infty))$. For $2 < \alpha < 3$, the fractional boundary value problem*

$$D^\alpha u(t) + h(t) = 0,$$
$$u(0) = u'(0) = 0, \qquad D^{\alpha-1}u(+\infty) = \sum_{i=1}^{m-2} \beta_i u(\xi_i) \tag{2.6}$$

has a unique solution

$$u(t) = \int_0^{+\infty} G(t,s)h(s)ds, \tag{2.7}$$

where

$$G(t,s) = G^*(t,s) + G^{**}(t,s), \tag{2.8}$$

with

$$G^*(t,s) = \frac{1}{\Gamma(\alpha)} \begin{cases} t^{\alpha-1} - (t-s)^{\alpha-1}, & 0 \le s \le t < +\infty, \\ t^{\alpha-1}, & 0 \le t \le s < +\infty. \end{cases} \tag{2.9}$$

$$G^{**}(t,s) = \frac{\sum_{i=1}^{m-2} \beta_i t^{\alpha-1}}{\Gamma(\alpha) - \sum_{i=1}^{m-2} \beta_i \xi_i^{\alpha-1}} G^*(\xi_i, s). \tag{2.10}$$

Lemma 2.4 (see [38]). *For $(s,t) \in [0,+\infty) \times [0,+\infty)$, $G(t,s)/1 + t^{\alpha-1} \le L_1$, where*

$$L_1 = \frac{1}{\Gamma(\alpha)} + \frac{\sum_{i=1}^{m-2} \beta_i \xi_{m-2}^{\alpha-1}}{\Gamma(\alpha)\left(\Gamma(\alpha) - \sum_{i=1}^{m-2} \beta_i \xi_i^{\alpha-1}\right)}. \tag{2.11}$$

3. Main Results

This section is devoted to some existence and uniqueness results for problem (1.1).

Define the space

$$Z = \{(u,v) \mid u \in X,\ v \in Y\} \tag{3.1}$$

equipped with the norm

$$\|(u,v)\|_Z = \max\{\|u\|_X, \|v\|_Y\}. \tag{3.2}$$

Clearly Z is a Banach space.

Let an operator $Q : Z \to Z$ be defined by

$$\begin{aligned} Q(u,v) &= (Q_1(v), Q_2(u)) \\ &= \left(\int_0^{+\infty} G_1(t,s) f(t,v(s))ds, \int_0^{+\infty} G_2(t,s) g(t,u(s))ds\right), \end{aligned} \tag{3.3}$$

where $G_1(t,s) = G_{11}(t,s) + G_{12}(t,s),\ G_2(t,s) = G_{21}(t,s) + G_{22}(t,s)$, with

$$\begin{aligned} G_{11}(t,s) &= \frac{1}{\Gamma(p)} \begin{cases} t^{p-1} - (t-s)^{p-1}, & 0 \le s \le t < +\infty, \\ t^{p-1}, & 0 \le t \le s < +\infty, \end{cases} \\ G_{12}(t,s) &= \frac{\sum_{i=1}^{m-2} \beta_i t^{p-1}}{\Gamma(p) - \sum_{i=1}^{m-2} \beta_i \xi_i^{p-1}} G_{11}(\xi_i, s), \\ G_{21}(t,s) &= \frac{1}{\Gamma(q)} \begin{cases} t^{q-1} - (t-s)^{q-1}, & 0 \le s \le t < +\infty, \\ t^{q-1}, & 0 \le t \le s < +\infty, \end{cases} \\ G_{22}(t,s) &= \frac{\sum_{i=1}^{m-2} \gamma_i t^{q-1}}{\Gamma(q) - \sum_{i=1}^{m-2} \gamma_i \xi_i^{q-1}} G_{21}(\xi_i, s). \end{aligned} \tag{3.4}$$

Observe that the problem (1.1) has a solution if and only if the operator Q defined by (3.3) has a fixed point.

Lemma 3.1. *For $(s,t) \in [0,+\infty) \times [0,+\infty)$, one has*

$$\frac{G_1(t,s)}{1+t^{p-1}} \le L, \qquad \frac{G_2(t,s)}{1+t^{q-1}} \le L, \tag{3.5}$$

where

$$L = \max\left\{ \frac{1}{\Gamma(p)} + \frac{\sum_{i=1}^{m-2} \beta_i \xi_{m-2}^{p-1}}{\Gamma(p)\left(\Gamma(p) - \sum_{i=1}^{m-2} \beta_i \xi_i^{p-1}\right)}, \frac{1}{\Gamma(q)} + \frac{\sum_{i=1}^{m-2} \gamma_i \xi_{m-2}^{q-1}}{\Gamma(q)\left(\Gamma(q) - \sum_{i=1}^{m-2} \gamma_i \xi_i^{q-1}\right)} \right\}. \tag{3.6}$$

Theorem 3.2. *Assume that*

(H_1) *there exist nonnegative functions* $a(t), b(t) \in C[0,+\infty)$ *such that*

$$\begin{gathered} |f(t,x)| \leq a(t)|x| + b(t), \quad t \in [0,+\infty), \\ \int_0^{+\infty} \left(1+t^{q-1}\right) a(t)dt < \frac{1}{L}, \qquad \int_0^{+\infty} b(t)dt < +\infty; \end{gathered} \tag{3.7}$$

(H_2) *there exist nonnegative functions* $c(t), d(t) \in C[0,+\infty)$ *such that*

$$\begin{gathered} |g(t,y)| \leq c(t)|y| + d(t), \quad t \in [0,+\infty), \\ \int_0^{+\infty} \left(1+t^{p-1}\right) c(t)dt < \frac{1}{L}, \qquad \int_0^{+\infty} d(t)dt < +\infty. \end{gathered} \tag{3.8}$$

Then the system (1.1) *has a solution.*

Proof. Let us take

$$R > \max\left\{ \frac{L\int_0^{+\infty} b(s)ds}{1 - L\int_0^{+\infty}(1+s^{q-1})a(s)ds}, \frac{L\int_0^{+\infty} d(s)ds}{1 - L\int_0^{+\infty}(1+s^{p-1})c(s)ds} \right\}, \tag{3.9}$$

and define

$$B_R = \{(u,v) \in Z \mid \|(u,v)\|_Z \leq R\}. \tag{3.10}$$

Obviously, B_R is a bounded closed and convex set of Z.

As a first step, we show that the operator Q is $B_R \to B_R$.

For any $(u,v) \in B_R$, we have

$$\begin{aligned} \|Q_1 v\|_X &= \sup_{t\in J} \frac{1}{1+t^{p-1}} \left| \int_0^{+\infty} G_1(t,s) f(s,v(s))ds \right| \\ &\leq \sup_{t\in J} \frac{1}{1+t^{p-1}} \int_0^{+\infty} G_1(t,s)(a(s)|v(s)| + b(s))ds \\ &\leq L\int_0^{+\infty} \left(1+s^{q-1}\right) a(s)ds\|v\|_Y + L\int_0^{+\infty} b(s)ds \\ &< \frac{L\int_0^{+\infty} b(s)ds}{1 - L\int_0^{+\infty}(1+t^{q-1})a(s)ds} \\ &< R. \end{aligned} \tag{3.11}$$

Similarly, we can get

$$\begin{aligned}
\|Q_2u\|_Y &= \sup_{t\in J}\frac{1}{1+t^{q-1}}\left|\int_0^{+\infty} G_2(t,s)g(s,u(s))ds\right| \\
&\le L\int_0^{+\infty}\left(1+s^{p-1}\right)c(s)ds\|u\|_X + L\int_0^{+\infty} d(s)ds \\
&< \frac{L\int_0^{+\infty} d(s)ds}{1-L\int_0^{+\infty}\left(1+s^{p-1}\right)c(s)ds} \\
&< R.
\end{aligned} \tag{3.12}$$

That is, $\|Q(u,v)\|_Z \le R$. Thus, $QB_R \subset B_R$.

Next, we show that Q is completely continuous. By continuity of f, g, G_1, and G_2, it follows that Q is continuous. On the other hand, by a similar process used in [38], we can easily prove that the operators Q_1 and Q_2 are equicontinuous. Therefore it follows that QB_R is an equicontinuous set. Also, it is uniformly bounded as $QB_R \subset B_R$. Thus, we conclude that Q is a completely continuous operator. Hence, by Schauder fixed point theorem, there exists a solution of (1.1). This completes the proof. □

Theorem 3.3. *Assume that*

(H_3) *there exist* $0<\rho_1<1$ *and nonnegative functions* $a_1(t), b_1(t) \in C[0,+\infty)$ *such that*

$$\begin{gathered}
\left|f(t,x)\right| \le a_1(t)|x|^{\rho_1} + b_1(t), \quad t\in[0,+\infty), \\
\int_0^{+\infty}\left(1+t^{q-1}\right)a_1(t)dt < +\infty, \qquad \int_0^{+\infty} b_1(t)dt < +\infty.
\end{gathered} \tag{3.13}$$

(H_4) *there exist* $0<\rho_2<1$ *and nonnegative functions* $c_1(t), d_1(t) \in C[0,+\infty)$ *such that*

$$\begin{gathered}
\left|g(t,y)\right| \le c_1(t)\left|y\right|^{\rho_2} + d_1(t), \quad t\in[0,+\infty), \\
\int_0^{+\infty}\left(1+t^{p-1}\right)c_1(t)dt < +\infty, \qquad \int_0^{+\infty} d_1(t)dt < +\infty.
\end{gathered} \tag{3.14}$$

Then the system (1.1) *has a solution.*

Proof. In this case, we take

$$\begin{aligned}
R > \max\Bigg\{ & 2L\int_0^{+\infty} b_1(s)ds, \left(2L\int_0^{+\infty}\left(1+s^{q-1}\right)a_1(s)ds\right)^{1/(1-\rho_1)}, \\
& 2L\int_0^{+\infty} d_1(s)ds, \left(2L\int_0^{+\infty}\left(1+s^{p-1}\right)c_1(s)ds\right)^{1/(1-\rho_2)}\Bigg\}.
\end{aligned} \tag{3.15}$$

The rest of the proof is similar to that of Theorem 3.2. So we omit it. □

Remark 3.4. By taking $\rho_1, \rho_2 > 1$ (instead of $0 < \rho_1 < 1$, $0 < \rho_2 < 1$) in (H_3) and (H_4), one can show that (1.1) has a solution.

Theorem 3.5. *Assume that*

(H_5) the functions f and g satisfy Lipschitz condition; that is, there exist nonnegative functions $K_1(t)$ and $K_2(t)$ such that

$$\begin{aligned} \left|f(t,x)-f(t,y)\right| &\leq K_1(t)\left|x-y\right|, \quad t\in[0,+\infty), \\ \left|g(t,x)-g(t,y)\right| &\leq K_2(t)\left|x-y\right|, \quad t\in[0,+\infty). \end{aligned} \tag{3.16}$$

Then the problem (1.1) *has a unique solution if*

$$\mu = L\int_0^{+\infty} K_1(s)\left(1+s^{q-1}\right)ds < 1, \qquad \tau = L\int_0^{+\infty} K_2(s)\left(1+s^{p-1}\right)ds < 1. \tag{3.17}$$

Proof. For any $(u_1, v_1), (u_2, v_2) \in Z$, we have

$$\begin{aligned} \|Q_1 v_2 - Q_1 v_1\|_X &= \sup_{t\in J}\frac{1}{1+t^{p-1}}\left|\int_0^{+\infty} G_1(t,s)\left[f(s,v_2(s)) - f(s,v_1(s))\right]ds\right| \\ &\leq \sup_{t\in J}\int_0^{+\infty}\frac{G_1(t,s)}{1+t^{p-1}}K_1(s)|(v_2-v_1)(s)|ds \\ &\leq L\int_0^{+\infty} K_1(s)\left(1+s^{q-1}\right)ds\|v_2-v_1\|_Y \\ &= \mu\|v_2-v_1\|_Y. \end{aligned} \tag{3.18}$$

Similarly, it can be shown that

$$\begin{aligned} \|Q_2 u_2 - Q_2 u_1\|_Y &= \sup_{t\in J}\frac{1}{1+t^{q-1}}\left|\int_0^{+\infty} G_1(t,s)\left(g(s,u_2(s)) - f(s,u_2(s))\right)ds\right| \\ &\leq L\int_0^{+\infty} K_2(s)\left(1+s^{p-1}\right)ds\|u_2-u_1\|_X \\ &= \tau\|u_2-u_1\|_X. \end{aligned} \tag{3.19}$$

Thus, we get

$$\|Q(u_2,v_2) - Q(u_1,v_1)\|_Z \leq \max\{\mu,\tau\}\|(u_2,v_2)-(u_1,v_1)\|_Z. \tag{3.20}$$

Obviously, Q is a contraction. Thus, the conclusion of the theorem follows by the contraction mapping principle. This completes the proof. □

4. Example

Example 4.1. Consider the following multipoint boundary value problem on an unbounded domain:

$$\begin{gathered}
D^{9/4}u(t) + \frac{\sin t \ln(1+|v(t)|)}{(1+t^{7/4})(2+t)^2} + (1+\cos 2t)e^{-t} = 0, \\
D^{11/4}v(t) + \frac{e^{-5t}\sin|u(t)|}{3(1+t^{5/4})(1+t)^2} + \frac{4}{(t+4)^2} = 0, \\
u(0) = u'(0) = 0, \qquad D^{5/4}u(+\infty) = \frac{2}{5}u\left(\frac{1}{4}\right) + \frac{1}{10}u(1), \\
v(0) = v'(0) = 0, \qquad D^{7/4}v(+\infty) = \frac{3}{10}u\left(\frac{1}{4}\right) + \frac{1}{5}u(1).
\end{gathered} \tag{4.1}$$

Here $t \in [0,+\infty)$, $p = 9/4$, $q = 11/4$, $\xi_1 = 1/4$, $\xi_2 = 1$, $\beta_1 = 2/5$, $\beta_2 = 1/10$, $\gamma_1 = 3/10$, and $\gamma_2 = 1/5$. One has

$$f(t,v(t)) = \frac{\sin t\ \ln(1+|v(t)|)}{(1+t^{7/4})(2+t)^2} + (1+\cos 2t)e^{-t}, \qquad g(t,u(t)) = \frac{e^{-5t}\sin|u(t)|}{3(1+t^{5/4})(1+t)^2} + \frac{4}{(t+4)^2}. \tag{4.2}$$

For $a(t) = 1/(1+t^{7/4})(2+t)^2$, $b(t) = 2e^{-t}$, $c(t) = 1/3(1+t^{5/4})(1+t)^2$, $d(t) = 4/(t+4)^2$, by direct calculation we find that

$$\begin{aligned}
L &= \max\left\{ \frac{1}{\Gamma(p)} + \frac{\sum_{i=1}^{m-2}\beta_i\xi_{m-2}^{p-1}}{\Gamma(p)\left(\Gamma(p) - \sum_{i=1}^{m-2}\beta_i\xi_i^{p-1}\right)}, \frac{1}{\Gamma(q)} \right. \\
&\quad \left. + \frac{\sum_{i=1}^{m-2}\gamma_i\xi_{m-2}^{q-1}}{\Gamma(q)\left(\Gamma(q) - \sum_{i=1}^{m-2}\gamma_i\xi_i^{q-1}\right)} \right\} \\
&= \max\left\{ \frac{1}{\Gamma(9/4)} + \frac{(2/5)+(1/10)}{\Gamma(9/4)\left(\Gamma(9/4) - 2/5(1/4)^{5/4} - (1/10)\right)}, \frac{1}{\Gamma(11/4)} \right. \\
&\quad \left. + \frac{(3/10)+(1/5)}{\Gamma(11/4)\left(\Gamma(11/4) - 3/10(1/4)^{7/4} - (1/5)\right)} \right\} \\
&= 1.341213, \\
&\left|f(t,x)\right| \le a(t)|x| + b(t), \qquad \left|g(t,y)\right| \le c(t)\left|y\right| + d(t), \qquad t \in [0,+\infty), \\
&\int_0^{+\infty}\left(1+t^{q-1}\right)a(t)dt = \frac{1}{2} < \frac{1}{L} = 0.745594, \qquad \int_0^{+\infty} b(t)dt = 2 < +\infty, \\
&\int_0^{+\infty}\left(1+t^{p-1}\right)c(t)dt = \frac{1}{3} < \frac{1}{L} = 0.745594, \qquad \int_0^{+\infty} d(t)dt = 1 < +\infty.
\end{aligned} \tag{4.3}$$

Thus all conditions of Theorem 3.2 are satisfied. Therefore, by Theorem 3.2, the couple system of nonlinear fractional differential (4.1) has at least one solution.

Example 4.2. Consider the following problem on an unbounded domain:

$$\begin{aligned} &D^p u(t) + M_1(t)\sin v(t) + N_1(t) = 0, \\ &D^q v(t) + \frac{M_2(t)}{1+u^2(t)} + N_2(t) = 0, \\ &u(0) = u'(0) = 0, \qquad D^{p-1}u(+\infty) = \frac{2}{5}u\left(\frac{1}{4}\right) + \frac{1}{10}u(1), \\ &v(0) = v'(0) = 0, \qquad D^{q-1}v(+\infty) = \frac{3}{10}u\left(\frac{1}{4}\right) + \frac{1}{5}u(1). \end{aligned} \tag{4.4}$$

Here $t \in [0,+\infty)$, $2 < p, q < 3$, $\xi_1 = 1/4$, $\xi_2 = 1$, $\beta_1 = 2/5$, $\beta_2 = 1/10$, $\gamma_1 = 3/10$, and $\gamma_2 = 1/5$, $M_1(t),\ M_2(t),\ N_1(t),\ N_2(t) \in C([0,+\infty),\mathbb{R})$.

With

$$f(t, v(t)) = M_1(t)\sin v(t) + N_1(t), \qquad g(t,u(t)) = \frac{M_2(t)}{1+u^2(t)} + N_2(t), \tag{4.5}$$

we have

$$\begin{aligned} \left|f(t,x) - f(t,y)\right| &= |M_1(t)|\left|\sin x - \sin y\right| \le |M_1(t)|\left|x-y\right|, \quad t \in [0,+\infty), \\ \left|g(t,x) - g(t,y)\right| &= |M_2(t)|\left|\frac{1}{1+x^2} - \frac{1}{1+y^2}\right| \le |M_2(t)|\left|x-y\right|, \quad t \in [0,+\infty), \end{aligned} \tag{4.6}$$

where $K_1(t) = |M_1(t)|$, $K_2(t) = |M_2(t)|$. So, the condition (H_5) holds. Let us assume that

$$\mu = L\int_0^{+\infty} |M_1(s)|\left(1+s^{q-1}\right)ds < 1, \qquad \tau = L\int_0^{+\infty} |M_2(s)|\left(1+s^{p-1}\right)ds < 1. \tag{4.7}$$

For example, condition (4.7) holds if we take

$$p = \frac{9}{4}, \qquad q = \frac{11}{4}, \qquad M_1(t) = \frac{1}{(1+t^{7/4})(2+t)^2}, \qquad M_2(t) = \frac{1}{3(1+t^{5/4})(1+t)^2}. \tag{4.8}$$

Thus all the conditions of Theorem 3.5 are satisfied. Therefore, by the conclusion of Theorem 3.5, the coupled system (4.4) has a unique solution.

5. Conclusion

We have shown the existence and uniqueness of solutions for a coupled system of nonlinear fractional differential equations with multipoint fractional boundary conditions on a semi-infinite domain. Our existence results are based on Schauder's fixed point theorem, while the uniqueness result is obtained by applying Banach's contraction mapping principle. The existence of solutions for (1.1) has been addressed for different kinds of growth conditions. Our approach is simple and can easily be applied to a variety of problems. This has been demonstrated by solving two examples.

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