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Research Article

Exponential Decay to Thermoelastic Systems over Noncylindrical Domains

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This paper is concerned with linear thermoelastic systems defined in domains with moving boundary. The uniform rate of decay of the energy associated is proved.

1. Introduction

In the study of asymptotic behavior for thermoelastic systems, a pioneering work is the one by Dafermos [1] concerned with the classical linear thermoelasticity for inhomogeneous and anisotropic materials, where the existence of a unique global solution and asymptotic stability of the system were proved. The existence of solution and asymptotic behavior to thermoelastic systems has been investigated extensively in the literature. For example, Muñoz Rivera [2] showed that the energy of the linear thermoelastic system (on cylindrical domain) decays to zero exponentially as $t \to \infty$. In [3], Burns et al. proved the energy decay for a linear thermoelastic bar. The asymptotic behaviour of a semigroup of the thermoelasticity was established in [4]. Concerning nonlinear thermoelasticity we can cite [5–7].

In the last two decades, several well-known evolution partial differential equations were extended to domains with moving boundary, which is also called noncylindrical problems. See, for instance, [8–10] and the references therein. In this work we studied the linear thermoelastic system in a noncylindrical domain with Dirichlet boundary conditions. This problem was early considered by Caldas et al. [11], which concluded that the energy associated to the system decreases inversely proportional to the growth of the functions that describes the noncylindrical domain. However they did not establish a rate of decay. The goal in the present work is to provide a uniform rate of decay for this noncylindrical problem.

Let us consider noncylindrical domains $\widehat{Q} \subset \mathbb{R}^2$ of the form

$$\widehat{Q} = \left\{ (x,t) \in \mathbb{R}^2;\ x = K(t)y,\ y \in (-1,1),\ t \in (0,T) \right\}, \tag{1.1}$$

with lateral boundary

$$\widehat{\Sigma} = \bigcup_{0<t<T} \{\{-K(t) \times \{t\}\} \cup \{K(t) \times \{t\}\}\}, \tag{1.2}$$

where $K : [0,T] \to \mathbb{R}^+$ is a given C^2 function. Then our problem is

$$u_{tt} - u_{xx} + \alpha\theta_x = 0 \quad \text{in } \widehat{Q}, \tag{1.3}$$

$$\theta_t - k\theta_{xx} + \beta u_{xt} = 0 \quad \text{in } \widehat{Q}, \tag{1.4}$$

with initial conditions

$$u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \quad \theta(x,0) = \theta_0(x), \quad -K(0) < x < K(0), \tag{1.5}$$

and boundary conditions

$$u(-K(t),t) = u(K(t),t) = 0, \quad \theta_x(-K(t),t) = \theta(K(t),t) = 0, \quad 0 < t < T, \tag{1.6}$$

where α, β, and k are positive real constants.

The function $K(t)$ and the constants α, β, and k satisfy the following conditions.

(H1) $K \in C^2([0,T], \mathbb{R}^+)$ and

$$K_0 = \min_{0\le t\le T} K(t) > 0. \tag{1.7}$$

(H2) There exists a positive constant K_1 such that

$$1 - \left(K'(t)y\right)^2 > K_1. \tag{1.8}$$

Problem (1.3)–(1.6) is slightly different from the one of [11] with respect to condition (1.6). Indeed, they assumed that $\theta(-K(t),t) = \theta(K(t),t) = 0$, for all $t \in [0,T]$. Because of this mixed boundary condition in (1.6), we are able to construct a suitable Liapunov functional to derive decay rates of the energy. This is sufficient to provide a uniform rate of decay for this noncylindrical problem.

The existence and uniqueness of global solutions are derived by the arguments of [11] step by step, that is, to prove that the result of existence and uniqueness is based on transforming the system (1.3)–(1.6) into another initial boundary-value problem defined over a cylindrical domain whose sections are not time-dependent. This is done using a suitable change of variable. Then to show the existence and uniqueness for this equivalent system using Galerkin Methods and the existence result on noncylindrical domains will follows using the inverse of the transformation.

Therefore, we have the following result.

Theorem 1.1. *Let Ω_t and Ω_0 be the intervals $(-K(t), K(t))$, $0 < t < T$, and $(-K(0), K(0))$, respectively. Then, given $u_0, \theta_0 \in H_0^1(\Omega_0) \cap H^2(\Omega_0)$ and $u_1 \in H_0^1(\Omega_0)$, there exist unique functions*

$$u : \widehat{Q} \longrightarrow \mathbb{R}, \qquad \theta : \widehat{Q} \longrightarrow \mathbb{R} \tag{1.9}$$

satisfying the following conditions:

$$\begin{gathered} u \in L^\infty\left(0,T; H_0^1(\Omega_t) \cap H^2(\Omega_t)\right), \qquad u_t \in L^\infty\left(0,T; H_0^1(\Omega_t)\right), \\ u_{tt} \in L^\infty\left(0,T; L^2(\Omega_t)\right), \qquad \theta \in L^2\left(0,T, H^2(\Omega_t)\right), \qquad \theta_t \in L^2\left(0,T; H_0^1(\Omega_t)\right), \end{gathered} \tag{1.10}$$

which are solutions of (1.3)–(1.6) *in* $\widehat{Q}$.

2. Energy Decay

In [11] the authors proved that the energy associated with (1.3)–(1.6) decays at the rate $1/[K(t)]^{\gamma_1}$ with $\gamma_1 > 0$; that is, the energy is decreasing inverserly proportional to the increase of sections of $\widehat{Q}$. We make a slightly difference from the one of [11] with respect to the hypotheses about K; we are able to construct a suitable Liapunov functional to derive decay rates of the energy. This is done with the thermal dissipation only. More specifically, in this section we prove that the energy associated with (1.3)–(1.6) decays exponentially. Instead considering an auxiliary problem, we work directly on the original problem (1.3)-(1.4) in its noncylindrical domain.

In order to decay rates of the energy let us suppose the following hypotheses.

(H3) There exist positive constants δ_0 and δ_1 such that

$$0 < \delta_0 \le K'(t) \le \delta_1 < 1, \quad t \ge 0. \tag{2.1}$$

(H4) There exists a positive constant δ_2 such that

$$0 < K(t)K'(t) \le \delta_2, \quad t \ge 0. \tag{2.2}$$

Let us introduce the energy functional

$$E(t) = E(t; u, \theta) = \frac{1}{2}\int_{-K(t)}^{K(t)} \left(|u_t|^2 + |u_x|^2 + \frac{\alpha}{\beta}|\theta|^2\right) dx. \tag{2.3}$$

Our main result is the following.

Theorem 2.1. *Under the hypotheses (H1)–(H4), there exist positive constants $\widetilde{C}$ and γ such that*

$$E(t;u,\theta) \leq \widetilde{C}E(0;u,\theta)e^{-\gamma t}. \tag{2.4}$$

The proof of Theorem 2.1 is given by using multipliers techniques. The notations and function spaces used here are standard and can be found, for instance, in the book by Lions [8].

Lemma 2.2. *Let (u,θ) be solution of (1.3)–(1.5) given by Theorem 1.1; then one obtains*

$$\frac{d}{dt}E(t;u,\theta) \leq -\frac{k\alpha}{\beta}\int_{-K(t)}^{K(t)}|\theta_x|^2dx - C_0\Big[|u_x(K(t),t)|^2 + |u_x(-K(t),t)|^2\Big], \tag{2.5}$$

where $C_0 = (\delta_0/2)(1-\delta_1^2) > 0$.

Proof. From hypothesis $u(K(t),t) = 0 = u(-K(t),t)$ it follows that

$$u_t(K(t),t) = -K'(t)u_x(K(t),t)eu_t(-K(t),t) = K'(t)u_x(-K(t),t). \tag{2.6}$$

Multiplying (1.3) by u_t, integrating in the variable x, and from (2.6) we obtain

$$\begin{aligned}\int_{-K(t)}^{K(t)} u_{tt}u_t\,dx &= \frac{1}{2}\left[\frac{d}{dt}\int_{-K(t)}^{K(t)}|u_t|^2dx - K'(t)|u_t(K(t),t)|^2 - K'(t)|u_t(-K(t),t)|^2\right]\\ &= \frac{1}{2}\left[\frac{d}{dt}\int_{-K(t)}^{K(t)}|u_t|^2dx - K'(t)^3|u_x(K(t),t)|^2 - K'(t)^3|u_x(-K(t),t)|^2\right].\end{aligned} \tag{2.7}$$

Now, applying integration by parts and using (2.6) it follows that

$$\begin{aligned}\int_{-K(t)}^{K(t)} u_{xx}u_t\,dx &= -u_x(K(t),t)u_t(K(t),t) + u_x(-K(t),t)u_t(-K(t),t) + \int_{-K(t)}^{K(t)} u_xu_{tx}\,dx\\ &= \frac{K'(t)}{2}|u_x(K(t),t)|^2 + \frac{K'(t)}{2}|u_x(-K(t),t)|^2 + \frac{1}{2}\frac{d}{dt}\int_{-K(t)}^{K(t)}|u_x|^2dx.\end{aligned} \tag{2.8}$$

Thus, from inequalities (2.7) and (2.8) we have

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\int_{-K(t)}^{K(t)}\left(|u_t|^2 + |u_x|^2\right)dx &= \frac{1}{2}K'(t)^3\left[|u_x(K(t),t)|^2 + |u_x(-K(t),t)|^2\right]\\ &\quad - \frac{K'(t)}{2}\left[|u_x(K(t),t)|^2 + |u_x(-K(t),t)|^2\right]\\ &\quad - \alpha\int_{-K(t)}^{K(t)}\theta_xu_t\,dx.\end{aligned} \tag{2.9}$$

Multiplying (1.4) by θ and integrating in the variable x and using (2.6) we obtain

$$\frac{1}{2}\frac{d}{dt}\int_{-K(t)}^{K(t)}|\theta|^2dx = -k\int_{-K(t)}^{K(t)}|\theta_x|^2dx + \beta\int_{-K(t)}^{K(t)}u_t\theta_x\,dx. \tag{2.10}$$

Multiplying (2.10) by α/β and summing with (2.9) it follows that

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\int_{-K(t)}^{K(t)}\left[|u_t|^2+|u_x|^2+\frac{\alpha}{\beta}|\theta|^2\right]dx = &-\frac{k\alpha}{\beta}\int_{-K(t)}^{K(t)}|\theta_x|^2dx\\ &+\frac{K'(t)^3}{2}\left[|u_x(K(t),t)|^2+|u_x(-K(t),t)|^2\right]\\ &-\frac{K'(t)}{2}\left[|u_x(K(t),t)|^2+|u_x(-K(t),t)|^2\right].\end{aligned} \tag{2.11}$$

Thus, following the hypothesis (H3),

$$\begin{aligned}\frac{1}{2}\frac{d}{dt}\int_{-K(t)}^{K(t)}\left[|u_t|^2+|u_x|^2+\frac{\alpha}{\beta}|\theta|^2\right]dx \le &-\frac{k\alpha}{\beta}\int_{-K(t)}^{K(t)}|\theta_x|^2dx\\ &-\frac{K'(t)}{2}\left(1-\delta_1^2\right)\left[|u_x(K(t),t)|^2+|u_x(-K(t),t)|^2\right]\\ \le &-\frac{k\alpha}{\beta}\int_{-K(t)}^{K(t)}|\theta_x|^2dx\\ &-\frac{\delta_0}{2}\left(1-\delta_1^2\right)\left[|u_x(K(t),t)|^2+|u_x(-K(t),t)|^2\right],\end{aligned} \tag{2.12}$$

which concludes the demonstration. □

To estimate the term $\int_{-K(t)}^{K(t)}|u_x|^2dx$ of the energy we use the following lemma.

Lemma 2.3. *With the same hypothesis of Lemma 2.2, one gets*

$$\frac{d}{dt}\int_{-K(t)}^{K(t)}u_tu\,dx \le \int_{-K(t)}^{K(t)}|u_t|^2dx - \frac{1}{2}\int_{-K(t)}^{K(t)}|u_x|^2dx + \frac{C_p\alpha^2}{2}\int_{-K(t)}^{K(t)}|\theta_x|^2dx, \tag{2.13}$$

where C_p is Poincare's constant.

Proof. From the outline condition $u(-K(t),t) = u(K(t),t) = 0$ follows that

$$\frac{d}{dt}\int_{-K(t)}^{K(t)}u_tu\,dx = \int_{-K(t)}^{K(t)}|u_t|^2dx + \int_{-K(t)}^{K(t)}uu_{tt}\,dx. \tag{2.14}$$

Replacing $u_{tt} = u_{xx} - \alpha\theta_x$ in the derivative above we get

$$\frac{d}{dt}\int_{-K(t)}^{K(t)} u_t u\,dx = \int_{-K(t)}^{K(t)} |u_t|^2 dx - \int_{-K(t)}^{K(t)} |u_x|^2 dx + \alpha\int_{-K(t)}^{K(t)} u_x\theta\,dx. \tag{2.15}$$

Applying Cauchy-Schwartz's inequality, Young's inequality, and Poincare's inequality in (2.15) we have

$$\frac{d}{dt}\int_{-K(t)}^{K(t)} u_t u\,dx \le \int_{-K(t)}^{K(t)} |u_t|^2 dx - \frac{1}{2}\int_{-K(t)}^{K(t)} |u_x|^2 dx + \frac{C_p\alpha^2}{2}\int_{-K(t)}^{K(t)} |\theta_x|^2 dx. \tag{2.16}$$

Therefore our conclusion follows. □

To estimate the term $\int_{-K(t)}^{K(t)} |u_t|^2 dx$ of the energy we introduce the function $q = \int_{-K(t)}^{x} \theta\,ds$. By these conditions we have the following lemma.

Lemma 2.4. *With the same hypothesis of Lemma 2.2, there are positive constants C_1 and C_2 such that*

$$\begin{aligned}\frac{d}{dt}\int_{-K(t)}^{K(t)} u_t q\,dx \le{}& C_1\int_{-K(t)}^{K(t)} |\theta_x|^2 dx + \frac{\beta}{32}\int_{-K(t)}^{K(t)} |u_x|^2 dx - \frac{\beta}{4}\int_{-K(t)}^{K(t)} |u_t|^2 dx \\ &+ \varepsilon K(t)|u_x(K(t),t)|^2 + C_2\Big[|u_x(K(t),t)|^2 + |u_x(-K(t),t)|^2\Big],\end{aligned} \tag{2.17}$$

where $C_1 = ((C_p/2\delta_0) + \alpha C_p + (k^2/2\beta) + (C_p/2) + (8/\beta))$ *and* $C_2 = \delta_2(1 + 2\beta\delta_1 + \delta_1^3)$.

Proof. Calculate the derivative

$$\begin{aligned}\frac{d}{dt}\int_{-K(t)}^{K(t)} u_t q\,dx ={}& \int_{-K(t)}^{K(t)} \frac{\partial}{\partial t}(u_t q)dx + K'(t)u_t(K(t),t)q(K(t),t) \\ &+ K'(t)u_t(-K(t),t)q(-K(t),t) \\ ={}& \int_{-K(t)}^{K(t)} u_{tt} q\,dx + \int_{-K(t)}^{K(t)} u_t q_t\,dx + K'(t)u_t(K(t),t)q(K(t),t).\end{aligned} \tag{2.18}$$

From (1.3) and recording that $q = \int_{-K(t)}^{x} \theta\,ds$, we get

$$\begin{aligned}I_1 =:{}& u_x(K(t),t)q(K(t),t) - \int_{-K(t)}^{K(t)} u_x q_x\,dx + \int_{-K(t)}^{K(t)} \alpha\theta\left[\frac{d}{dx}\int_{-K(t)}^{x} \theta\,ds\right]dx \\ &+ \int_{-K(t)}^{K(t)} u_t\,dx\left[\int_{-K(t)}^{x} \theta_t\,ds\right] + K'(t)u_t(K(t),t)q(K(t),t).\end{aligned} \tag{2.19}$$

As $q_x = \theta$ and $q_t = \int_{-K(t)}^{x} \theta_t \, ds$ we obtain

$$
\begin{aligned}
I_1 = u_x(K(t),t) \int_{-K(t)}^{K(t)} \theta \, dx - \int_{-K(t)}^{K(t)} u_x \theta \, dx + \alpha \int_{-K(t)}^{K(t)} |\theta|^2 dx \\
+ \int_{-K(t)}^{K(t)} u_t q_t \, dx + K'(t) u_t(K(t),t) \int_{-K(t)}^{K(t)} \theta \, dx.
\end{aligned}
\tag{2.20}
$$

Now, integrating (1.4) from $-K(t)$ to x, multiplying by u_t, and after integrating from $-K(t)$ to $K(t)$, it follows that

$$
\begin{aligned}
I_2 &=: \int_{-K(t)}^{K(t)} u_t q_t \, dx \\
&= k \int_{-K(t)}^{K(t)} \theta_x u_t \, dx - \beta \int_{-K(t)}^{K(t)} |u_t|^2 dx + \beta u_t(-K(t),t) \int_{-K(t)}^{K(t)} u_t \, dx.
\end{aligned}
\tag{2.21}
$$

Replacing (I_2) in (I_1) and from (2.6) we get

$$
\begin{aligned}
I_1 &= \frac{d}{dt} \int_{-K(t)}^{K(t)} u_t q \, dx \\
&= u_x(K(t),t) \int_{-K(t)}^{K(t)} \theta \, dx - \int_{-K(t)}^{K(t)} u_x \theta \, dx \\
&\quad + \alpha \int_{-K(t)}^{K(t)} |\theta|^2 dx + k \int_{-K(t)}^{K(t)} \theta_x u_t \, dx - \beta \int_{-K(t)}^{K(t)} |u_t|^2 dx \\
&\quad + \beta K'(t) u_x(-K(t),t) \int_{-K(t)}^{K(t)} u_t \, dx - \left[K'(t)\right]^2 u_x(K(t),t) \int_{-K(t)}^{K(t)} \theta \, dx.
\end{aligned}
\tag{2.22}
$$

Estimating some terms of (2.22) we obtain

$$
\begin{aligned}
\frac{d}{dt} \int_{-K(t)}^{K(t)} u_t q \, dx \le \alpha \int_{-K(t)}^{K(t)} |\theta|^2 dx + K'(t) K(t) |u_x(K(t),t)|^2 + \frac{C_p}{2K'(t)} \int_{-K(t)}^{K(t)} |\theta_x|^2 dx \\
- \int_{-K(t)}^{K(t)} u_x \theta \, dx - \frac{\beta}{4} \int_{-K(t)}^{K(t)} |u_t|^2 dx + \left(\frac{k^2}{2\beta} + \frac{C_p}{2} \right) \int_{-K(t)}^{K(t)} |\theta_x|^2 dx \\
+ 2\beta K(t) K'(t)^2 |u_x(-K(t),t)|^2 + K(t) K'(t)^4 |u_x(K(t),t)|^2.
\end{aligned}
\tag{2.23}
$$

Applying Poincare's inequality in the first term of the previous inequality, using the hypothesis (H3), and grouping the common terms, we obtain

$$\begin{aligned}\frac{d}{dt}\int_{-K(t)}^{K(t)} u_t q\,dx \le \left(\alpha C_p + \frac{C_p}{2\delta_0} + \frac{k^2}{2\beta} + \frac{C_p}{2}\right)\int_{-K(t)}^{K(t)} |\theta_x|^2 dx - \int_{-K(t)}^{K(t)} u_x\theta\,dx - \frac{\beta}{4}\int_{-K(t)}^{K(t)} |u_t|^2 dx \\ + K(t)K'(t)\left(1 + 2\beta\delta_1 + \delta_1^3\right)\left[|u_x(-K(t),t)|^2 + |u_x(K(t),t)|^2\right].\end{aligned} \tag{2.24}$$

From hypothesis (H4) we have

$$\begin{aligned}\frac{d}{dt}\int_{-K(t)}^{K(t)} u_t q\,dx \le C_1\int_{-K(t)}^{K(t)} |\theta_x|^2 dx + \frac{\beta}{32}\int_{-K(t)}^{K(t)} |u_x|^2 dx - \frac{\beta}{4}\int_{-K(t)}^{K(t)} |u_t|^2 dx \\ + C_2\left[|u_x(K(t),t)|^2 + |u_x(-K(t),t)|^2\right].\end{aligned} \tag{2.25}$$

where C_1 and C_2 are positive constants. This concludes the demonstration of the lemma. □

Now we use the above auxiliary lemmas to conclude the proof of Theorem 2.1.

Proof of Theorem 2.1. Consider the functional

$$\mathcal{F}(t) = \int_{-K(t)}^{K(t)} \left(\frac{\beta}{8}u_t u + u_t q\right) dx. \tag{2.26}$$

From Lemmas 2.3 and 2.4 we obtain

$$\begin{aligned}\frac{d}{dt}\mathcal{F}(t) \le -\frac{\beta}{32}\int_{-K(t)}^{K(t)} |u_x|^2 dx - \frac{\beta}{8}\int_{-K(t)}^{K(t)} |u_t|^2 dx \\ + \left(C_1 + \frac{\alpha^2\beta C_p}{16}\right)\int_{-K(t)}^{K(t)} |\theta_x|^2 dx + C_2\left[|u_x(K(t),t)|^2 + |u_x(-K(t),t)|^2\right].\end{aligned} \tag{2.27}$$

Finally we introduce the functional

$$\mathcal{L}(t) = \mathcal{F}(t) + NE(t), \tag{2.28}$$

where $N \in \mathbb{N}$ will be chosen later.

From Lemma 2.2 and from (2.27) it follows that

$$\begin{aligned}\frac{d}{dt}\mathcal{L}(t) \le& -\left(Nk - \frac{\alpha C_1}{\beta} - \frac{\alpha\beta^2 C_p}{16}\right)\frac{\alpha}{\beta}\int_{-K(t)}^{K(t)} |\theta_x|^2 dx \\ &- \left(\frac{N\delta_0}{2} - C_2\right)\left[|u_x(K(t),t)|^2 + |u_x(-K(t),t)|^2\right] \\ &- \frac{\beta}{32}\int_{-K(t)}^{K(t)} |u_x|^2 dx - \frac{\beta}{8}\int_{-K(t)}^{K(t)} |u_t|^2 dx.\end{aligned} \tag{2.29}$$

Taking N sufficiently large we find that there is a positive constant C_3 such that

$$\frac{d}{dt}\mathcal{L}(t) \le -C_3\left[\int_{-K(t)}^{K(t)} |u_x|^2 dx + \int_{-K(t)}^{K(t)} |u_t|^2 dx + \frac{\alpha}{\beta}\int_{-K(t)}^{K(t)} |\theta_x|^2 dx\right]. \tag{2.30}$$

Observe that $\mathcal{L}(t)$ and $E(t)$ are equivalents, that is, there exists positive constant C_4 satisfying

$$\frac{N}{2}E(t;u,\theta) \le \mathcal{L}(t) \le C_4 E(t;u,\theta). \tag{2.31}$$

Therefore,

$$\frac{d}{dt}\mathcal{L}(t) \le -\frac{C_3}{C_4}L(t). \tag{2.32}$$

Now, from equivalence (2.31) it follows that

$$E(t) \le \tilde{C}E(0)e^{-\gamma t}, \tag{2.33}$$

where $\tilde{C} = 2C_4/N$ and $\gamma = C_3/C_4$. The proof is now complete. □

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