Correction to "Separatrices at singular points of planar vector fields"

by

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Professor M. E. Sagalovich has kindly provided us a detailed explanation of his examples, published in [2], of singular points of degree d, $d\geq 3$, with $4d-2$ separatrices. We had been aware of these examples, but had erroneously concluded that they had fewer separatrices. These examples show that Theorem 3.13 of [3], which asserts that a singular point of degree d, $d\geq 3$, can have at most $4d-4$ separatrices, is wrong. The correct bound is $4d-2$, as had already been proved by Sagalovich in [1].

The error in our proof occurs at the bottom of p. 75. Under discussion there are Dumortier pictures in which the singularities on Γ, the homeomorph of S^1 that represents the original singularity, are (1) two saddles, each of which has two of its separatrices lying within Γ; (2) some corners; (3) singularities resulting from the blow-up of a single special singularity. See [3] for definitions; also see Figure 18 of [3]. In our argument we implicitly assume that each of the two arcs into which Γ is divided by the two saddles must contain a subarc resulting from the blow-up of the special singularity. This is the case in Figure 18 of [3], but it need not be true. It is not true in Sagalovich's examples.

Our argument for Theorem 3.13 in fact demonstrates the following: *Suppose a singularity of degree d has* $4d-2$ *separatrices. Then its tree* $\mathcal{T}$ *has a subtree* $\mathcal{T}''$ *whose terminal vertices are* (1) *one vertex* W_1, *also terminal in* $\mathcal{T}$, *that represents two saddles in the Dumortier picture, each of which has two separatrices lying within* Γ; (2) *some corners, also terminal in* $\mathcal{T}$, *whose separatrices lie within* Γ; (3) *degree one saddles* $V_1, \ldots, V_{d-1}$, *the successors of a single nonterminal special vertex* V. (As remarked on p. 75 of [3], $V_1, \ldots, V_{d-1}$ need not be terminal in $\mathcal{T}$. In Sagalovich's examples, they are not.) *In the Dumortier picture associated with* $\mathcal{T}''$, *one of the two arcs into which* Γ *is divided by the two saddles corresponding to* W_1 *does not contain a subarc resulting from the blow-up of* V.

References

[1] SAGALOVICH, M. E., Topological structure of the neighborhood of a critical point of a differential equation. *Differential Equations,* 11 (1975), 1498–1503.

[2] — Classes of local topological structures of an equilibrium state. *Differential Equations,* 15 (1979), 253–255.

[3] SCHECTER, S. & SINGER, M. F., Separatrices at singular points of planar vector fields. *Acta Math.,* 145 (1980), 47–78.

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