Topological **M**ethods in **N**onlinear **A**nalysis
Journal of the Juliusz Schauder Center
Volume 7, 1996, 261–270

A MULTIPLICITY RESULT FOR THE GENERALIZED KADOMTSEV–PETVIASHVILI EQUATION

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Dedicated to Louis Nirenberg

1. Introduction

We consider the existence and multiplicity of solitary waves of the generalized Kadomtsev–Petviashvili equation

$$\omega_t + \omega_{xxx} + (f(\omega))_x = D_x^{-1}\omega_{yy}, \tag{1}$$

where

$$D_x^{-1}h(x,y) := \int_{-\infty}^{x} h(s,y)\,ds.$$

See [5] for references concerning this equation. A *solitary wave* is a solution of the form

$$\omega(t,x,y) = u(x-ct,y),$$

where $c > 0$ is fixed. Substituting in (1), we obtain

$$-cu_x + u_{xxx} + (f(u))_x = D_x^{-1}u_{yy}$$

or

$$(-u_{xx} + D_x^{-2}u_{yy} + cu - f(u))_x = 0.$$

Existence results have been established by de Bouard and Saut ([3, 4]) for pure power nonlinearities using a minimization method, and by Willem ([10]) for more

1991 *Mathematics Subject Classification.* 35Q20, 58E05, 58G20, 58E15.

general nonlinearities including nonhomogeneous ones using the Ambrosetti–Rabinowitz mountain-pass theorem. As observed in [4], a physical example of a nonhomogeneous nonlinearity is contained in [8].

In this note, we shall consider multiplicity of solitary waves. To state our results, we first give some preliminaries.

In this section, $c > 0$ is fixed.

Definition. On $Y := \{g_x : g \in \mathcal{D}(\mathbb{R}^2)\}$ we define the inner product

$$(u, v) := \int_{\mathbb{R}^2} [u_x v_x + D_x^{-1} u_y D_x^{-1} v_y + cuv] \tag{2}$$

and the corresponding norm

$$\|u\| := \left(\int_{\mathbb{R}^2} [u_x^2 + (D_x^{-1} u_y)^2 + cu^2] \right)^{1/2}. \tag{3}$$

A function $u : \mathbb{R}^2 \to \mathbb{R}$ belongs to X if there exists $(u_n) \subset Y$ such that

(a) $u_n \to u$ a.e. on $\mathbb{R}^2$,
(b) $\|u_j - u_k\| \to 0$ as $j, k \to \infty$.

The space X with inner product (2) and norm (3) is a Hilbert space.

Now consider the problem

$$(-u_{xx} + D_x^{-2} u_{yy} + cu - f(u))_x = 0, \quad u \in X. \tag{$\mathcal{P}$}$$

We assume

(f$_1$) $f \in C^1(\mathbb{R}, \mathbb{R})$ and for some $2 < p < 6$ and $c_0 > 0$,

$$|f'(u)| \leq c_0 |u|^{p-2},$$

(f$_2$) there exists $2 < \alpha < p$ such that, for every $u \in \mathbb{R} \backslash \{0\}$,

$$0 < \alpha F(u) \leq u f(u)$$

where

$$F(u) := \int_0^u f(s)\, ds,$$

(f$_3$) for every $u \in \mathbb{R} \backslash \{0\}$, $f(u)u < f'(u)u^2$,

(f$_4$) there exist $0 < a < b$ such that, for every $u \in \mathbb{R}$,

$$a|u|^p \leq F(u) \leq b|u|^p.$$

The *weak solutions* of ($\mathcal{P}$) are the critical points of the functional φ defined on X by

$$\varphi(u) := \int_{\mathbb{R}^2} \left[\tfrac{1}{2} (u_x^2 + (D_x^{-1} u_y)^2 + cu^2) - F(u) \right]. \tag{4}$$

In order to obtain multiplicity results, we shall reformulate the problem to one defined on the unit sphere in X. For $u \in S$, where S is the unit sphere in X, and $\lambda > 0$, one finds

$$\begin{aligned} \varphi(\lambda u) &= \frac{\lambda^2}{2} - \int_{\mathbb{R}^2} F(\lambda u), \\ \frac{d}{d\lambda}\varphi(\lambda u) &= \lambda - \int_{\mathbb{R}^2} f(\lambda u)u, \\ \frac{d^2}{d\lambda^2}\varphi(\lambda u) &= 1 - \int_{\mathbb{R}^2} f'(\lambda u)u^2. \end{aligned}$$

As in [1], it is easy to verify that, for every $u \in S$, there exists a unique $\lambda(u) > 0$ such that

$$\frac{d}{d\lambda}\varphi(\lambda u)\bigg|_{\lambda=\lambda(u)} = 0 \quad \text{and} \quad \varphi(\lambda(u)u) = \max_{\lambda \geq 0} \varphi(\lambda u).$$

We define a new functional on S by

$$\psi(u) := \varphi(\lambda(u)u). \tag{5}$$

LEMMA 1. *Under assumptions* (f$_1$)–(f$_3$), *if $u \in S$ is a critical point of ψ, then $\lambda(u)u$ is a critical point of φ.*

If we replace $f(u)$ by the nonlinear term $d|u|^{p-2}u$, where $d > 0$, we obtain the associated functionals φ_d defined on X and ψ_d defined on S. We shall prove that the infima

$$m := \inf_{u \in S} \psi(u), \quad m_d := \inf_{u \in S} \psi_d(u) \tag{6}$$

are always achieved and positive. We shall use the following notations:

$$\begin{aligned} &K(\psi) := \{u \in S \mid \psi'(u) = 0\}, \\ &\psi^{-1}((\alpha, \beta)) := \{u \in S \mid \alpha < \psi(u) < \beta\}, \\ &\psi^c := \{u \in S \mid \psi(u) \leq c\}. \end{aligned}$$

For any set $A \subset X$ invariant with respect to translations, we denote by $A/\mathbb{R}^2$ the quotient of A with respect to translations.

Our main assumption is

($*$) there exists γ satisfying $0 < \gamma \leq m_b$ such that

$$\psi_b^{-1}((m_b, m_b + \gamma)) \cap K(\psi_b) = \emptyset$$

and that $\psi_b^{m_b}/\mathbb{R}^2$ contains only isolated points.

THEOREM 1. *Under assumptions* (f$_1$)–(f$_4$) *and* ($*$), *if*

$$b/a < (1 + \gamma/m_b)^{(p-2)/2},$$

then ($\mathcal{P}$) *has at least two geometrically distinct weak solutions.*

2. A compactness condition

In this section, we shall give a characterization of all (PS) sequences for φ (defined in (4)) in X. Similar results were obtained in [6] for Hamiltonian systems.

Lemma 2. (i) *The following imbeddings are continuous:*

$$X \subset L^p(\mathbb{R}^2), \quad 2 \le p \le 6.$$

(ii) *The following imbeddings are compact:*

$$X \subset L^p_{\mathrm{loc}}(\mathbb{R}^2), \quad 1 \le p < 6.$$

Proof. For (i), see [2], p. 323. For (ii), see [4], Lemma 3.3. □

Lemma 3. *If (u_n) is bounded in X and if for some $r > 0$,*

$$\sup_{(x,y)\in\mathbb{R}^2} \int_{B_r(x,y)} |u_n|^2 \to 0 \quad \text{as } n \to \infty,$$

then $u_n \to 0$ in $L^p(\mathbb{R}^2)$ for $2 < p < 6$.

Proof. See [10], Lemma 4. □

Lemma 4. *There exists $c_1 > 0$ such that $\varphi(u) \ge c_1$ for all $u \in K(\varphi) \setminus \{0\}$.*

Proof. Note first that 0 is an isolated critical point of φ. If there is $\{u_n\} \subset K(\varphi) \setminus \{0\}$ such that $\lim_{n\to\infty} \varphi(u_n) \le 0$, we get

$$\lim_{n\to\infty} \left(\frac{1}{2}\|u_n\|^2 - \int F(u_n) \right) \le 0$$

and

$$\|u_n\|^2 - \int f(u_n)u_n = 0.$$

Hence

$$\lim_{n\to\infty} (\alpha/2 - 1)\|u_n\|^2 \le 0,$$

which is a contradiction. □

Lemma 5. *Let $\{u_n\} \subset X$ be such that $\varphi(u_n) \to c \ne 0$ and $\varphi'(u_n) \to 0$ as $n \to \infty$. Then there are $\ell \in \mathbb{N}$ (depending on c), $v_1, \ldots, v_\ell \in K(\varphi) \setminus \{0\}$, a subsequence of $\{u_n\}$ and corresponding $\{(x_n^i, y_n^i)\} \subset \mathbb{R}^2$ for $i = 1, \ldots, \ell$ such that*

$$\Big\| u_n - \sum_{i=1}^{\ell} v_i(\,\cdot + x_n^i, \cdot + y_n^i) \Big\| \to 0 \quad \text{as } n \to \infty, \tag{7}$$

$$\sum_{i=1}^{\ell} \varphi(v_i) = c, \tag{8}$$

and

$$(x_n^i - x_n^j)^2 + (y_n^i - y_n^j)^2 \to \infty \quad \text{as } n \to \infty, i \neq j.$$

PROOF. First, by (f_2) for n large,

$$c + 1 + \frac{1}{\alpha}\|u_n\| \geq \varphi(u_n) - \frac{1}{\alpha}\langle \varphi'(u_n), u_n\rangle \geq \left(\frac{1}{2} - \frac{1}{\alpha}\right)\|u_n\|^2.$$

Hence, u_n is bounded in X. By Lemma 3, we may assume there exist $\delta > 0$, $\nu > 0$ and $(x_n^1, y_n^1) \in \mathbb{R}^2$ such that

$$\int_{B_r(x_n^1, y_n^1)} |u_n|^2 \geq \delta.$$

Define $u_n^1(x,y) = u_n(x + x_n^1, y + y_n^1)$ and $B_r = B_r(0,0)$. Then

$$\|u_n^1\|_{L^2(B_r)} \geq \delta \tag{10}$$

and

$$\varphi(u_n^1) = \varphi(u_n), \quad \|\varphi'(u_n^1)\| = \|\varphi'(u_n)\|, \quad \|u_n^1\| = \|u_n\|.$$

Therefore going if necessary to a subsequence, $\{u_n^1\}$ converges to v_1 both weakly in X and strongly in $L^p_{\mathrm{loc}}(\mathbb{R}^2)$ for $2 \leq p < 6$. By (10),

$$\|v_1\|_{L^2(B_r)} \geq \delta$$

and $v_1 \neq 0$.

Next, we show that v_1 is a critical point of φ. For every $w \in Y$, we have

$$\langle \varphi'(v_1), w\rangle = \lim_{n\to\infty} \langle \varphi'(u_n^1), w\rangle = 0.$$

By Lemma 4, $\varphi(v_1) = c_1 > 0$.

Next, we consider the new sequence $u_n^2 = u_n^1 - v_1$ and we shall show

$$\varphi(u_n^2) \to c - \varphi(v_1) \tag{11}$$

and

$$\varphi'(u_n^2) \to 0. \tag{12}$$

Therefore, we may repeat the proof above finishing the proof of the lemma.

First,

$$\begin{aligned}\varphi(u_n^1) = \varphi(u_n^2 + v_1) &= \varphi(u_n^2) + \varphi(v_1) + (u_n^2, v_1) \\ &\quad - \int_{\mathbb{R}^2} (F(u_n^2 + v_1) - F(u_n^2) - F(v_1)).\end{aligned} \tag{13}$$

Note that $(u_n^2, v_1) \to 0$ as $n \to \infty$. So it suffices to show that the last integral in (13) tends to zero as $n \to \infty$. For any $\varepsilon > 0$, we may choose $R > 0$ such that

$$\int_{\mathbb{R}^2 \setminus B_R} F(v_1) \leq \varepsilon \quad \text{and} \quad \int_{\mathbb{R}^2 \setminus B_R} |v_1|^2 < \varepsilon. \tag{14}$$

In the following c denotes various constants independent of u. By (f_1),

$$
\begin{aligned}
\int_{\mathbb{R}^2\setminus B_R} &|F(u_n^2+v_1)-F(u_n^2)| \\
&\leq \int_{\mathbb{R}^2\setminus B_R} |f(u_n^2+\xi v_1)|\cdot|v_1| \\
&\leq \int_{\mathbb{R}^2\setminus B_R} \{|u_n^2|+|v_1|+c(|u_n^2|+|v_1|)^{p-1}\}|v_1| \\
&\leq \bigg(\int_{\mathbb{R}^2\setminus B_R} |u_n^2|^2\bigg)^{1/2}\bigg(\int_{\mathbb{R}^2\setminus B_R} |v_1|^2\bigg)^{1/2} \\
&\quad + \int_{\mathbb{R}^2\setminus B_R} |v_1|^2 + c\bigg(\int_{\mathbb{R}^2\setminus B_R} (|u_n^2|+|v_1|)^p\bigg)^{(p-1)/p}\bigg(\int_{\mathbb{R}^2\setminus B_R} |v_1|^p\bigg)^{1/p} \\
&= O(\varepsilon).
\end{aligned}
$$

Combining this with the fact that $u_n^2 \to 0$ in $L^p_{\mathrm{loc}}(\mathbb{R}^2)$ for any $2 \leq p < 6$, we get (11). To show (12), let $\omega \in Y$. Then

$$
\langle \varphi'(u_n^2),\omega\rangle = \langle \varphi'(u_n^1),\omega\rangle - \int_{\mathbb{R}^2} (f(u_n^2)-f(u_n^1)+f(v_1))\omega.
$$

Since $\varphi'(u_n^1)\to 0$, it suffices to show

$$
\sup_{\|\omega\|\leq 1}\bigg|\int_{\mathbb{R}^2} (f(u_n^2)-f(u_n^1)+f(v_1))\omega\bigg| \to 0 \quad \text{as } n\to\infty.
$$

Let $\varepsilon > 0$, and choose $R > 0$ again such that (14) holds. Then

$$
\begin{aligned}
\bigg|\int_{\mathbb{R}^2\setminus B_R} f(v_1)\omega\bigg| &\leq \int_{\mathbb{R}^2\setminus B_R} (|v_1|+c|v_1|^{p-1})|\omega| \\
&\leq \varepsilon\|\omega\| + C\varepsilon\|\omega\|.
\end{aligned}
$$

And

$$
\begin{aligned}
\bigg|\int_{\mathbb{R}^2\setminus B_R} (f(u_n^2)-f(u_n^2+v_1))\omega| &\leq \int_{\mathbb{R}^2\setminus B_R} |f'(u_n^2+\xi v_1)|\cdot|v_1|\cdot|\omega| \\
&\leq \int_{\mathbb{R}^2\setminus B_R} C(|u_n^2|+|v_1|)^{p-2}|v_1|\cdot|\omega| \leq O(\varepsilon)\|\omega\|.
\end{aligned}
$$

Using the convergence of $u_n^2 \to 0$ in $L^p_{\mathrm{loc}}(\mathbb{R}^2)$ again, we get (15). □

Since there is a one-to-one correspondence between the critical points of φ in X and the critical points of ψ on S, the following lemma is a consequence of Lemma 5.

LEMMA 6. *Let* $\{u_n\} \subset S$ *be such that* $\psi(u_n) \to c \in [m, 2m)$ *and* $\psi'(u_n) \to 0$ *as* $n \to \infty$. *Then there exist* $(x_n, y_n) \in \mathbb{R}^2$ *such that* $u_n(\cdot + x_n, \cdot + y_n)$ *(up to a subsequence) converges to* $u_0 \in S$, *and* $\psi'(u_0) = 0$, $\psi(u_0) \in [m, 2m)$.

Recall that the *least energy* for ψ on S is defined by

$$m = \inf_{u \in S} \psi(u).$$

THEOREM 2. *Under assumptions* (f_1)–(f_3), *the least energy* m *is always achieved and therefore* $(\mathcal{P})$ *has a nontrivial weak solution. If we further assume* f *to be odd in* u, *then* $(\mathcal{P})$ *has a pair of nontrivial geometrically distinct weak solutions.*

PROOF. It is easy to see that Lemma 6 implies that m is attained.

If f is odd, ψ is even on S. Then it suffices to show that for $u \neq 0$, $-u$ cannot be a translation of u. Indeed, if for some $(x_0, y_0) \in \mathbb{R}^2$,

$$-u(x, y) = u(x + x_0, y + y_0), \quad \forall (x, y) \in \mathbb{R}^2,$$

then

$$u(x + 2x_0, y + 2y_0) = -u(x + x_0, y + y_0) = u(x, y), \quad \forall (x, y) \in \mathbb{R}^2,$$

i.e., u is a periodic function, which is impossible. □

REMARK. A weak convergence argument was used in [10] by Willem to show the existence of solutions of $(\mathcal{P})$, which allows weaker assumptions on f.

3. Multiplicity results

To prove our main results, we follow the approach used in [1] where multiplicity results for homoclinic solutions were proved for a class of autonomous Hamiltonian systems. The basic tool is the Lyusternik–Schnirelman category theory.

LEMMA 7. *For any* $c \in [m, 2m)$, ψ *has at least* $\mathrm{cat}(\psi^c)$ *critical points in* ψ^c.

PROOF. If the standard (PS) condition were satisfied in ψ^c, this would be just a special case of the Lyusternik–Schnirelman theory. Though (PS) is not satisfied by ψ in ψ^c, the following property (usually called *property* (C)) is satisfied: For any $c \in [m, 2m)$, if c is the only critical value of ψ in $[c - \varepsilon, c + \varepsilon]$ for some $\varepsilon > 0$ and U is a neighbourhood of $K(\psi) \cap \psi^{-1}(c)$, then there exists $\delta > 0$ such that for all $u \in \psi^{-1}([c - \varepsilon, c + \varepsilon]) \setminus U$, $\|\psi'(u)\| \geq \delta$. As was noted in [1] this property is enough to establish the Lyusternik–Schnirelman theory in ψ^c for $c \in [m, 2m)$. □

Our main theorem will be proved if for some $c \in [m, 2m)$, we can get

$$\mathrm{cat}(\psi^c) \geq 2,$$

because if ψ has only one critical point modulo translations the category of this point together with its translations is 1.

To estimate the category of the level sets for ψ, we shall compare them with the ones of ψ_a and of ψ_b. First, some preliminaries.

For $u \in X$, we define $[u] = \{u(\,\cdot\, + x_0, \,\cdot\, + y_0) \mid (x_0, y_0) \in \mathbb{R}^2\}$. We may abuse the notation denoting by $[u]$ a point in $X/\mathbb{R}^2$.

Lemma 8. *Let $A \subset X$ be such that $A/\mathbb{R}^2$ is an isolated set. Then for any $u \in A$, there exists an open set U_u in X such that*

(1) $[u] \subset U_u$.
(2) *If $v \in [u]$, then $U_v \equiv U_u$, i.e., U_u is translation-invariant.*
(3) *$U_u \cap U_v = \emptyset$ if $u, v \in A, [u] \neq [v]$.*
(4) *$[u]$ is a deformation retract of $\overline{U}_u$.*

Proof. For any $u \in A$, consider $[u] \in A/\mathbb{R}^2$. Then there is an ε-neighbourhood $V_{[u]}$ in $X/\mathbb{R}^2$. By the fact that $A/\mathbb{R}^2$ is isolated, we may choose $V_{[u]}$ such that $V_{[u]} \cap V_{[v]} = \emptyset$ for $[u], [v] \in A/\mathbb{R}^2$, $[u] \neq [v]$. Then consider the projection map $\pi : X \to X/\mathbb{R}^2$, which is continuous. Define

$$U_u = \pi^{-1}(V_{[u]}).$$

Then it is obvious that (1)–(3) are satisfied. For (4), note that $\overline{V}_{[u]}$ is contractible to $[u]$ and therefore $\overline{U}_u$ is contractible to $[u]$ in X. □

Lemma 9. *Let* (f$_1$)–(f$_4$) *and* $(*)$ *be satisfied. Then there exists $\varepsilon_0 > 0$ satisfying $\gamma/m_b > \varepsilon_0 > 0$ such that setting $\delta = \delta(\gamma, m_b, \varepsilon_0, c) = (\gamma/m_b - \varepsilon_0)c$, we have*

$$\psi_b^c \subset \psi^{c+\delta} \subset \psi_b^{c+\delta}.$$

Proof. By (f$_4$), for every $u \in X$,

$$\varphi_b(u) \leq \varphi(u) \leq \varphi_a(u)$$

and thus

$$\psi_b(u) \leq \psi(u) \leq \psi_a(u), \quad \forall u \in S.$$

This proves the second inclusion for any c and δ.

Next, we choose $\varepsilon_0 > 0$ such that $b/a = (1 + \gamma/m_b - \varepsilon_0)^{(p-2)/2}$. Since $b > a$, we have $0 < \varepsilon_0 < \gamma/m_b$. Then for all $0 < \varepsilon \leq \varepsilon_0$, if $\psi_b(u) \leq c$,

$$\begin{aligned}
\psi(u) \leq \psi_a(u) &= \frac{p-2}{2p} a^{-2/(p-2)} \|u\|_{L^p(\mathbb{R}^2)}^{2p/(p-2)} \\
&\leq \frac{p-2}{2p} b^{-2/(p-2)} (1 + \gamma/m_b - \varepsilon) \|u\|_{L^p(\mathbb{R}^2)}^{2p/(p-2)} \\
&= (1 + \gamma/m_b - \varepsilon)\psi_b(u) \\
&\leq c + (\gamma/m_b - \varepsilon)c.
\end{aligned}$$

□

Lemma 10. *Let $A \subset B \subset C$. Assume A is a deformation retract of C. Then* $\mathrm{cat}(B) \geq \mathrm{cat}(A)$.

Proof. This is more or less standard; for a reference, see [1]. Though it was not clearly stated there the proof of Lemma 6 in [1] works here. $\square$

Finally, we prove our Theorem 1.

Proof of Theorem 1. As was noted earlier, by Lemma 7 it suffices to show that $\mathrm{cat}(\psi^c) \geq 2$ for some $c \in [m, 2m)$.

First, applying Lemma 8 to $A = \psi_b^{m_b}$, we get an open covering $\{U_u\}_{u \in A/\mathbb{R}^2}$ satisfying (1)–(4) of Lemma 8. In particular, by Theorem 2,

$$\mathrm{cat}\Big(\bigcup_{u \in A/\mathbb{R}^2} \overline{U}_u\Big) = \mathrm{cat}(A) \geq 2.$$

Next, we claim we can choose $\varepsilon > 0$ such that

$$\varepsilon(1 + \gamma/m_b - \varepsilon_0) < \varepsilon_0 m_b \tag{16}$$

and

$$\psi_b^{m_b+\varepsilon} \subset \bigcup_{u \in A/\mathbb{R}^2} U_u, \tag{17}$$

where $\varepsilon_0 > 0$ is given in Lemma 9. To see (17) is true, assume not. Then there exist $\varepsilon_n \to 0$ and $u_n \in \psi_b^{m_b+\varepsilon_n}$ such that $u_n \notin \bigcup_{u \in A/\mathbb{R}^2} U_u$. Hence $\{u_n\}$ is a minimizing sequence for ψ_b on S. By Ekeland's variational principle (see e.g. [7]), we may assume $\{u_n\}$ is a $(\mathrm{PS})_{m_b}$ sequence for ψ_b. By Lemma 6, there exist $(x_n, y_n) \in \mathbb{R}^2$ such that $v_n(x, y) = u_n(x + x_n, y + y_n)$ converges in S to v_0, and $\psi_b'(v_0) = 0$, $\psi_b(v_0) = m_b$. That is, $v_0 \in A$. But because $\bigcup_{u \in A/\mathbb{R}^2} U_u$ is translation-invariant, $v_0 \notin \bigcup_{u \in A/\mathbb{R}^2} U_u$, a contradiction. Thus (17) holds.

Thus we have

$$A \subset \psi_b^{m_b+\varepsilon} \subset \bigcup_{u \in A/\mathbb{R}^2} \overline{U}_u$$

with A being a deformation retract of $\bigcup_{u \in A/\mathbb{R}^2} \overline{U}_u$. By Lemma 10, we get

$$\mathrm{cat}(\psi_b^{m_b+\varepsilon}) \geq \mathrm{cat}(A) \geq 2.$$

Finally, by (16), $\varepsilon' := \varepsilon_0(m_b + \varepsilon) - (1 + \gamma/m_b)\varepsilon > 0$. By Lemma 9,

$$\psi_b^{m_b+\varepsilon} \subset \psi^{m_b+\gamma-\varepsilon'} \subset \psi_b^{m_b+\gamma-\varepsilon'}.$$

By $(*)$ and property (C), $\psi_b^{m_b+\varepsilon}$ is a deformation retract of $\psi_b^{m_b+\gamma-\varepsilon'}$. By Lemma 10 again,

$$\mathrm{cat}(\psi^{m_b+\gamma-\varepsilon'}) \geq \mathrm{cat}(\psi_b^{m_b+\varepsilon}) \geq 2.$$

The proof is complete. $\square$

Remark. Inspecting our proof, we see that our arguments imply that ψ has as many geometrically distinct critical points on S as ψ_b does.

Acknowledgements. The first author thanks the members of the Institut de Mathématique Pure et Appliquée of the Université Catholique de Louvain for their invitation and their hospitality. The first author was partially supported by an NSF grant and a USU faculty summer grant.

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Manuscript received July 15, 1996

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