

Frank Plumpton Ramsey. *Notes on Philosophy, Probability and Mathematics*. Edited by Maria Carla Galavotti, Bibliopolis, Naples, 1991. iii + 417 pp.

Reviewed by

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The publication of these notes promises to play a major role in the renewal of studies on the work of F. P. Ramsey which is exemplified, for instance, by the publication of [Sahlin 1990] (reviewed in *Modern Logic*, vol. 4, no. 1, pp. 87–89), and by the publication of a special issue of the journal *Theoria* devoted to the Philosophy of F. P. Ramsey (vol. LVII, 1991, no. 3, edited by the same editor of the volume we are reviewing here). The texts published in this volume are taken from the “Ramsey Collection” (RC) held in the Hillman Library of the University of Pittsburgh. As the editor informs us, the material to be found in the RC is varied, and covers almost ten years of Ramsey’s life.

Almost all the papers are published here for the first time, but only for a few of them has it been possible to indicate precisely when they were written. They span every field Ramsey’s curiosity and ingenuity went through, but in this review I will limit myself to the more logically based notes.

Even though it is difficult sometimes to discern precisely between what is a mere summary of other people’s papers from what does constitute (a sketch or a first draft of) an original elaboration, it surely can be said that besides being a source of information about the background literature to Ramsey’s thought, the material contained in this volume also contributes to our reconstruction of the debate that was going on among members of the philosophical community: Moore, Russell, Wittgenstein, and Johnson being quoted the most. To give the flavor of Ramsey’s theoretical dispositions it should be mentioned that sometimes these quotations are accompanied by some rude remarks, as

when he raises some perplexities about Hilbert's use of the transfinite functions  $\tau$ , defined as "sheer nonsense" (p. 181), or when we find a not very flattering judgment of Moritz Schlick whose book, *Allgemeine Erkenntnislehre*, is said to contain some "sad rubbish" (p. 345).

As a general remark, I completely agree with the editor's warning that "[t]he material forming this collection is to be seen as complementary to Ramsey's published papers" (p. 26). No radically new aspect of Ramsey's thought and development is contained in these notes, but they "will disclose valuable information on the various aspects of his thought, as well as its background and development" (p. 27). To this it could be added that only a (good) previous acquaintance with the published work of Ramsey allows the reader to put these notes in the right perspective. I mean, in particular, the proposals Ramsey devised to revise the system of *Principia Mathematica* (*PM*). It is well known that they concerned paradoxes and the Axiom of Reducibility, the Russellian notion of class, and the problems related to the notion of identity. The notes contained in this volume are, above all, relevant to the second and the third of these revision proposals.

As regards the notion of class, the main focus of Ramsey's remarks is the criticism of Russell's identification of the notion of "class" with that of "definable class" (*cfr.* for instance, note no. 51). The target of these remarks is the cumulative (platonistic) hierarchy of types attained through an extension of the linguistic apparatus of *PM*. (See, on this point, [Gandy 1977] and [Frascolla 1987].) Ramsey shares Russell's opinion that a level in the class hierarchy — except for the level of individuals — is nothing but the collection of the extensions of propositional functions expressed by open formulas of the language. The peculiar trait of Ramsey's variation on this Russellian theme consists in also admitting functions given by open formulas of infinite length. Relevant to this subject is the following statement, (note no. 15, p. 65): "We can think if not without any symbols, at least without sufficient symbols to give adequate expression to our thought. We do not put it into words till afterwards").

On the background of the *Tractatus*'s thesis which identifies every proposition with a truth-function of atomic propositions, Ramsey adopts Peirce's token-type distinction to emphasize that logic has to do with "proposition-as-types", so to speak, including both the types to which an instance (a token), as a matter of fact, has never corresponded, and the types which may not have an instance, because of the intrinsic finite (or "finitistic") nature of every human symbolic operation (that is to say, because of our inability to write propositions of infinite length).

A second major theme which Ramsey inherits from the *Tractatus* is the refusal to consider the identity sign "=" as an atomic relation expressing a factual relation between individuals, but rather as the assertion of a rule of intersubstitutability between names. On this basis, he developed his criticism of the Russellian reduction of identity to the Leibnizian notion of indiscernability. Ramsey focuses on the fact that the assertion of the existence of two individuals does not necessarily depend on some form of acquaintance with them, and hence on the possibility to distinguish between them. For the purpose of overcoming the cluster of problems linked to identity, Ramsey introduces the notion of a function of individuals given in extension, which amounts to the remarkable giving up of the logicistic tenet that functions always must be expressed by formulæ of the language (*cf.* notes nos. 47 and 49).

It is well known that this passage also was criticized by Wittgenstein in the late twenties when the Austrian philosopher came to link the sense of a proposition not with its truth conditions but with its verifiability or assertability conditions. In this new perspective, in which Wittgenstein rejects the extensionalistic conception of mathematical infinity, and, in particular, the linguistic extensionalism of Ramsey, the unmanageability of expressions of infinite length becomes a *logical*, and not a merely biological impossibility. Ramsey's later interests on finitism as illustrated by notes nos. 53 and 54 seem to suggest his willingness to follow the Austrian philosopher once more in this new turn.

I conclude with some short remarks. In note no. 53, entitled "Principles of Finitistic Mathematics", the major part of the comments refers to Skolem's and Weyl's work more than to Hilbert's. Reflecting on Hilbert's formalism, Ramsey distinguishes between consistency and the impossibility of proving a contradiction (p. 179), without fully explaining what kind of distinction he means. In general, Ramsey doesn't seem to fully appreciate the differences between finitistic and intuitionistic mathematics; on the other hand, this was an oversight with which many people at that time agreed. In note no. 54, "The Formal Structure of Intuitionistic Mathematics" (1929?), Ramsey puts forward perplexities of various kinds concerning the use of the notion of "Absurdity" introduced by Brouwer in order to give a satisfactory treatment of negation. In particular, Ramsey is suspicious of Brouwer's proof that  $AAA(p) \supset A(p)$  (where " $A(p)$ " means " $p$  is absurd"). What he emphasizes is that  $AA(p)$  can be derived not from  $p$ , but from  $P(p)$  (which means " $p$  can be proved"). In so doing, his notes echo and are relevant to themes extensively debated within intuitionistic mathematics between the late 'twenties and the early 'thirties by people like

Kolmogorov, Freudenthal, Griss, Heyting, etc. (See [Sundholm 1989] for a thorough presentation of the subject and for references). Relevant to this same point is also note no. 31, "The Nature of Propositions" (1921): the question disputed here concerns the possibility of analyzing such statements as " $p \supset q$ " in terms of belief. According to Ramsey, "I believe  $p$ " means "I have a belief with the reference or referential character  $p$ ". This tenet is supported by the theory that propositions are characters of beliefs. When I believe  $p$ , Ramsey means, my belief is multiply related to the things ordinarily called the constituents of  $p$ . " $p$  implies  $q$ ", " $p$  or  $q$ " assert relations between these referential properties of beliefs.

Lastly, I'd like to point out that the occurrence of the word "Mathematics" in the title chosen for the volume doesn't really match very well what the notes are about: no mention, for instance, is made there of Ramsey's theorem in combinatorics. A much more felicitous choice would have been "Logic and Foundations of Mathematics". It is obvious this is only a minor point, and in conclusion I wish to emphasize again the importance of this book for everybody interested in the history (and theory too) of philosophy, logic, and foundations of mathematics and probability.

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