ORIGIN OF RUSSELL'S EARLY THEORY OF LOGICAL TRUTH AS PURELY GENERAL TRUTH: BOLZANO, PEIRCE, FREGE, VENN, OR MACCOLL?

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Who most likely influenced Russell's early theory of logical truth as purely general truth: Bolzano, Peirce, Frege, Venn, or MacColl?

Russell's theory of logical truth as invariance, or as truth under any interpretation, anticipates Alfred Tarski (1936), J. C. C. McKinsey (1945), Rudolf Carnap (1947), Saul Kripke (1980), Joseph Almog (1989), and John Etchemendy (1990), and has antecedents in Bernard Bolzano and John Venn. Russell's theory resembles Bolzano's substitutional account of logical truth, which Etchemendy compares to Tarski's satisfactional account of logical truth (Etchemendy [14, pp. 27-33])¹.

Tarski himself compares his closely related definition of logical consequence to Carnap and to Bolzano (Tarski [52, pp. 413-18, 417 n.+] following H. Scholtz). Russell's theory of what is possible as what is sometimes the case is close to McKinsey [30, p. 83] and Venn [55, p. 40], with roots in Diodorus Cronus and Parmenides.

Russell's fundamental paper on modality, "Necessity and Possibility" (Russell [51]), which Russell read to the Oxford Philosophical Society on October 22, 1905 (Urquhart [54, p. 507]), was not published during his lifetime. It appeared in a volume of the *Collected Papers of Bertrand Russell* only in 1994 (Russell [34]). Had it been published in 1905 as the companion piece to "On Denoting" I believe it was, the course of modal logic — not to mention the course of Russell studies — might well have been different².

¹Etchemendy discusses Bolzano's theory of satisfiability and analytic propositions (Bolzano [5, pp. 193-99]). Bolzano's theory of necessity and possibility is different (Bolzano [5, pp. 255-59]).

²Saul A. Kripke, for example, says in *Naming and Necessity* that not only did Russell have a theory "plainly incompatible with our direct intuitions of rigidity" (Kripke [25, p. 14]), but that one reason for this was that Russell "did not consider modal questions" (Kripke [25, p. 14]). Nicholas Rescher goes further in his article, "Russell and Modal Logic." There he holds that Russell, with his "massive influence" and "deliberately held negative views toward modal conceptions," was almost single-handedly responsible for "the stunted development of modal logic [for]...two © 2001 *Modern Logic*

Russell's idea is simple: to use notions of ordinary quantificational logic to define and analyze away modal notions. Modal notions are eliminated across the board. The individual ("existential") and universal quantifiers are used to simulate and replace modal notions. These quantifiers are interpreted as functioning as if they had modal meanings-in-use. They do not in fact have modal meanings-in-use. Literally speaking, Russell has banished modality from logic. Yet functionally speaking, Russell's idea leads to a modal logic based on a rich and sophisticated theory of modality. And all this without having to assume any modal entities or even modal notions. The modern moral is that a modal logic is as a modal logic does. This is modal logic without modal metaphysics.

The three definitions on which the implied modal logic is based are:

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F(x) is necessary with respect to x = Df F(x) is always true
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(true universal generalization)

F(x) is possible with respect to x = Df F(x) is not always false

(true "existential" generalization)

F(x) is impossible with respect to x = Df F(x) is always false

(true universal generalization over the negation of F)

I call this group of definitions "MDL".

MDL occurs in at least nine works over a period of at least thirty-six years (1905-40):

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c. 1903-05 "Necessity and Possibility" (Russell [51, p. 518])
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1906 Review of MacColl's *Symbolic Logic and Its Applications* (Russell [49, p. 257])

1908 "'If' and 'Imply', A Reply to Mr. MacColl" (Russell [47, p. 301]);

1908 "Mathematical Logic as Based on the Theory of Types" (Russell [38, p. 66n])

1913 "On the Notion of Cause," in Mysticism and Logic (Russell [43, p. 176])

generations" (Rescher [32, pp. 146-48]). More recently, Raymond Bradley, in *The Nature of All Being*, has eagerly accepted Rescher's "fascinating, and revealing, account of Russell's 'baneful influence' on the development, and recognition, of modal logic" without any reservations (Bradley [7, p. 63]). These three philosophers are only the tip of the iceberg of popular philosophical opinion.

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1918 "The Philosophy of Logical Atomism" (Russell [40, pp. 231, 232, 233, 240, 242, 254-55])
1919 Introduction to Mathematical Philosophy (Russell [46, p. 165])
1927 The Analysis of Matter (Russell [44, p. 170])
1940 An Inquiry Into Meaning and Truth (Russell [35, p. 37])
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MDL is the stepping-stone from which Russell's implicit modal logics start. The first such logic analyzes logically necessary truths as fully general true propositions, where a fully general proposition is a proposition which contains only logical constants and bound individual and predicate variables. I call this first modal logic "FG-MDL". Russell's idea is that a fully general true proposition is necessary (i.e. always true) not just with respect to some one of its variables, but is necessary (i.e. always true) with respect to all of its variables. Such a proposition may be called fully necessary. In his 1903-05 "Necessity and Possibility," Russell defines "analytic propositions" as fully general true propositions (Russell [34, p. 519]), equating analytic truth with fully generalized truth. I accept Gregory Landini's formalization of this. Where A is a statement and the definiens is A's universalization,

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"Analytically true(A) = Df (F_1 	ldots F_m, x_1 	ldots x_n)A_{F_1 	ldots F_m, x_1 	ldots x_n}" (Landini [28]; see Dejnožka [11, pp. ix, 3]).
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Thus for example, " $(x)(F)(Fx \vee \neg Fx)$ " belongs to FG-MDL, while " $(x)(\operatorname{Red}(x) \vee \neg \operatorname{Red}(x))$ " does not, since "Red()" is a descriptive constant. We today might wish to include the second of these statements as a logical truth, since it is an instance of the first statement. But for Russell, full generality is always a requirement of logical truth because "pure logic" is universal; it contains only bound variables and logical constants [40, pp. 139, 240-41]). Logicians should not be interested in empirical questions of what the real world contains (Russell [40, p. 199]). Even those timeless universals which can only be empirically known, such as the color red, are not part of pure logic.

Russell eventually finds that full generality is not a sufficient condition, but only a necessary condition, of logically necessary truth. The 1914-19 Russell therefore adds a second requirement to that of full generality, namely, that a logically true proposition be true in virtue of logical form, or tautologous. This implies Russell's second and more mature modal logic, which I call "FG-MDL*" (pronounced FG-MDL star). In 1913 FG-MDL* is characterized only in terms of logical form,

where a form is some sort of special entity. By 1919 FG-MDL* is in effect characterized alternatively in terms of logical form and in terms of tautology, as if these features were not significantly different (Russell [46, pp. 199-205]). This is so even though Russell finds that he can easily define "form" in terms of what remains the same in a proposition through replacements of its descriptive constants:

The "form" of a proposition is that, in it, that remains unchanged when every constituent of the proposition is replaced by another. ([46, p. 199]; see [40, p. 238])

but does not know how to define "tautologous" ([40, p. 205]), and can only say it means what used to be intended by the old term "analytic" ([40, pp. 203-4]). It seems clear that Russell would find a proposition tautologously true if and only if it is true in virtue of its form. Evidently he wishes to use truth in virtue of form as a technical notion to explain the intuitive notion of tautology. In any case, FG-MDL* is Russell's mature implicit modal logic. Russell never expressly describes a modal logic. But logical necessity is just logical truth for him. Thus "logically true" is his implied necessity operator on statements.

FG-MDL* as described in the last block-indented quotation is still extremely close to being a full generality theory. Likewise in Russell's 1918 "The Philosophy of Logical Atomism," where Russell says that logical truths are true in virtue of logical form and are tautologies as well ([40, pp. 240-41]), but defines form merely as "that which is in common between any two propositions of which the one can be obtained from the other by substituting other constituents for the original ones" ([40, p. 238]), and warns against admitting a "too substantial" sort of logical form ([40, p. 239]).

Russell diagnoses the problem with FG-MDL as being that some purely general truths are not logical truths. His example is "[t]he proposition that there are exactly 30,000 things in the world" (Russell [40, p. 240]). Russell's cure is to add the new requirement of truth in virtue of logical form. Russell's diagnosis and cure are debatable³. But it is time to move to our historical question.

³In 1904, Russell claimed to prove there are infinitely many numbers (Russell [50]). In 1906, he claimed to prove there are infinitely many propositions, as complex but single entities (Russell [48]; see Landini [29, p. 373]; Cocchiarella [10, pp. 90, 112 n.9]). Either proof would also be a proof of the axiom of infinity, a proof of the existence of at least one thing (the infamous "existence assumption" of *Principia*), and a disproof of the proposition that there are exactly 30,000 things. But Russell came to abandon such proofs, and came to believe that the axiom of infinity is logically contingent. Landini believes that the logical contingency of all individuals in ramified theory of types is why Russell moved from FG-MDL to FG-MDL* (Landini [28]; see Dejnožka [11, pp. ix, 11]).

Would any infinite set of necessary beings remove Russell's 30,000 things counterexample to FG-MDL? Russell admits infinitely many purely logical universals as timeless beings from 1903 until at least 1912 (Russell [36, p. 109]), but it is highly controversial whether he would allow his *Principia* individual quantifier to range over universals as type 0 order 0 individuals. Curiously enough, Landini counts universals among Russell's individuals, undermining his own explanation of why Russell must abandon FG-MDL (Landini [26, p. 292]; [27, pp. 293-94]; [29, pp. 386-87]). Landini argues that all *Principia* "objects" are individuals (Landini [26, p. 292]). Now, Russell does count qualities and relations as objects in *Principia* (Russell [45, vol. 1, p. 43]). But he also speaks of "[t]he division of objects into types" (Russell [45, vol. 1, p. 161]).

Consider the counterexample, "There are exactly 30,000 logically contingent things," where the logical contingency operator is a logical constant. There can be exactly 30,000 logically contingent beings even if there are infinitely many necessary beings.

Gustav Bergmann notes in effect a second problem with FG-MDL. Evidently, some purely general truths are not true in virtue of their logical form, but instead true in virtue of the fact that in an ideal language, any attempt to deny them would be ill-formed. This is Bergmann's distinction between what is impossible 1 (marked by ill-formed expressions), impossible 2 (analytically false), and impossible 3 (a priori false) (Bergmann [4, pp. 23-24]). Assuming that standard notation is an ideal language, it is "possible1 for several ordinary things to have exactly the same properties" (Bergmann [4, p. 23]), since " $(\exists x)(\exists y)(F)[(x \neq y)\&(Fx \equiv Fy)]$ " is well-formed. It is also possible 1 for some particulars to have no properties, e.g., Bergmann's bare particulars taken seriously as mere individuators, since " $(\exists x)(F)\neg Fx$ " is well-formed. But it is necessary 1 that particulars are not properties of particulars, since "ab" is ill-formed.

Bergmann's problem continues to exist in FG-MDL*, so Russell's cure fails to solve it. The problem is based on the saying-showing distinction in Wittgenstein's *Tractatus*, which in turn has roots in Frege's famous thesis that you cannot refer to the concept *horse* by means of the singular definite article I apparently just used to do so. Perhaps necessary 1 truths, though shown not said in an ideal language, are true in virtue of form in a negative sense. But is it logical form or categorial, *i.e.* metaphysical, form? Are such truths logical truths? Are they analytic or synthetic?

I raise a third problem with FG-MDL. Namely, how purely general are truths when logical subjects are still distinguished from logical predicates? This third problem continues to exist in FG-MDL*, so Russell's cure fails here as well. The problem seems connected to Bergmann's "ab" problem, since the "ab" problem is that both "a" and "b" are logical subjects.

Fourth, pace Bergmann, fully general " $(x)(\exists F)Fx$ " seems logically true. But it universalizes to false "(x)(F)Fx." It seems logically true that any object has some property, but it is false that any object has any property. Thus true full generalization and true full universalizability seem incompatible as necessary and sufficient conditions of logical truth. These two theories of logical truth seem reconciled here only if you admit bare particulars with Bergmann, so that it is false that any object has some property. Russell's "There are 30,000 things" is fully general, but is not fully universalized, as the "There are" indicates. It is a counterexample to FG-MDL only if its full universalization is true.

MDL is not a modal logic in itself. But it is the core of FG-MDL, and its origin may be fairly said to be the origin of FG-MDL. I propose, then, to investigate the origin of MDL.

Leila Haaparanta sees MDL as possibly originating from Frege's equating necessity with generality in Begriffsschrift sect. 4 (Haaparanta [19, pp. 38-40, 46 n.26]). The citation is correct, but there are three uses of "necessary" in sect. 4, and only the third is like MDL. These uses are as follows. (a) Frege equates "necessary" or, more accurately, "apodictic" (absolutely demonstrable or certain) propositions with propositions derivable from true analytic general axioms. derived proposition need not be general; it can be an instantiation. Frege is saying that formally marking a proposition as apodictic adds nothing to showing it can be derived from such axioms. His point is very much like Russell's point that calling a proposition logically necessary adds nothing to saying it is logically true. Frege then says that to call a proposition possible is either (b) to say it is not known to be demonstrably false or (c) to say that the universalization of its negation is false. On (c), it is possible that this crow is white, if not all crows are nonwhite. This implies that it is necessary that this crow is white, if all crows are white. Clearly (c) is very close to, if not the same as, MDL. But Haaparanta missed the best text. In Grundgesetze der Arithmetik, Frege holds that there is no such thing as a merely possible object (Frege [16, p. 222]). Thus the only possible objects are actual objects. This implies that true universal propositions concern all possible objects, and are therefore necessary. This view would be a very Russellian motive for MDL. Thus a Fregean influence on Russell is not only possible but likely.

Simo Knuuttila compares MDL to Peirce's 1901 Baldwin's Dictionary entry "Modality" (Knuuttila [23, pp. ix-x]). There Peirce says, "The simplest account of necessity is the scholastic, according to which the necessary... proposition is a sort of universal proposition; the possible...proposition, a sort of particular proposition." (Peirce [31]). A Peirceian influence is theoretically possible. Frege is, of course, more likely than Peirce to have influenced Russell. Whitehead and Russell say, "In all questions of logical analysis, our chief debt is to Frege" ([45, p. viii]). Still, Haaparanta and Knuuttila show that the idea was already available when Russell wrote: Frege and Peirce had just published versions of it. I might add that Russell cites Peirce several times

The basic intuition behind theories of logical truth as purely general truth is simply, What else could purely general truth be, and what else is really needed for logical truth? Russell might be the first to question this intuition, but he will not be the last.

in *Principles* and easily could have read Peirce's entry in *Baldwin's Dictionary* by 1905.

Bolzano is another possible influence. Russell's early theory of logical truth as fully generalized truth resembles Bolzano's substitutional theory of logical truth (Bolzano [5, pp. 193-99]). Russell cites Bolzano's Paradoxes of the Infinite four times in Principles. Bolzano anticipates Frege and Russell by suggesting in 1810 that "a discussion of mathematical method is basically nothing but logic" (Bolzano [5, p. xxvi]). Bolzano deems "5 + 7 = 12" analytic (Bolzano [6, p. 19]), and gives an "all but logistic definition of... 'similar to' " (Bolzano [6, p. 20]). Bolzano was the major transitional figure between Leibniz and Frege (Kluge [22, pp. 231-80]) and arguably influenced Frege's sense-reference distinction (Grossmann [18]). Bolzano counts some logical truths as synthetic (Berg [3, p. 102]); Russell deems logic synthetic in the strict Kantian sense (Russell [41, p. 457]). But Bolzano's theory of logical truth is not in *Paradoxes* (Bolzano [6]). Kenneth Blackwell, founding chief archivist of the Bertrand Russell Archives, finds no evidence in the Archives that Russell ever read Bolzano's *Theory of Science*.

Venn is a possibility merely by being the chief English logician just before Russell (Venn [55, p. 40]). But I know of no evidence to show his specific influence on Russell here.

My view is that MDL is a modification of MacColl's theory of a "certainty," an "impossibility," and a "variable." The chief reason is that Russell expressly says so:

Mr. MacColl speaks of 'propositions' as divided into the three classes of certain, variable, and impossible. We may accept this division as applying to propositional functions. A function which can be asserted is certain, once which can be denied is impossible, and all others are (in Mr. MacColl's sense) variable. (Russell [38, p. 66])

Russell's earliest published statements of MDL are generally replies to MacColl, and generally use 'certain' in place of 'necessary' for a propositional function's being always true, following MacColl. See Ivor Grattan-Guinness ([17, pp. 118-19]). Russell's innovation is to predicate modalities of propositional functions instead of propositions as MacColl does.

To sum up, it might seem that the safest answer to our question is simply that the idea was in the air. But that general truth is trumped by the "smoking gun" text specifically citing MacColl. Whether MacColl was influenced in turn, e.g. by Venn or Bolzano, I leave to others to discover⁴.

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 $^{^4}$ Dejnožka ([11]) discusses Russell's implicit modal logics in depth. Dejnožka ([12, p. 290 n.6]) assays the strength of FG-MDL and FG-MDL* as S5 + I. Dejnožka ([13]) describes three levels of Russell's theory of modality.

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