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ON HAUSDORFF COMPACTIFICATIONS

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Given a pair of spaces X and Y, a necessary and sufficient condition is found for Y to be homeomorphic to $\mathrm{cl}_{\alpha X}(\alpha X - X)$ for some compactification αX of X. From this follows a necessary and sufficient condition for Y to be homeomorphic to $\alpha X - X$ for some αX. As an application, a sufficient condition is found to insure the isomorphism of the upper semi-lattices of compactifications $K(X)$ and $K(Y)$ for arbitrary X and Y, and in consequence it appears that for every space X, there is a pseudocompact space Y with $K(X)$ isomorphic to $K(Y)$. A necessary condition for $K(X)$ to be isomorphic to $K(Y)$ is observed for arbitrary X and Y, and this leads to the consideration of spaces compactly generated at infinity. Examples are constructed.

All spaces considered are completely regular and Hausdorff. We consider the family of Hausdorff compactifications of X, each obtained by a quotient map on βX fixing X pointwise. It is known [3: 10.13] that this map, hereafter called the "Čech map" of the compactification, must be unique. Identify any two such compactifications if there is a homeomorphism between them which fixes X pointwise and let $K(X)$ be the family of equivalence classes partially ordered in the standard way: $\alpha_1 X \leqq \alpha_2 X$ if there is a continuous map from $\alpha_2 X$ onto $\alpha_1 X$ which fixes X pointwise. From [2], $K(X)$ is an upper semi-lattice which is a complete lattice if and only if X is locally compact. In [5] K. D. Magill, Jr. obtained the result which shall be referred to as *Magill's theorem*: For any two locally compact spaces X and Y, $K(X)$ is lattice-isomorphic to $K(Y)$ if and only if $\beta X - X$ is homeomorphic to $\beta Y - Y$.

In this paper, generalizations are obtained to each direction of Magill's theorem by dropping the requirement that X and Y be locally compact.

1. Compactifications.

LEMMA 1.0. *Let X be a compact Hausdorff space, Y be a compact T_1 space and $f: X \rightarrow Y$ be continuous and onto. The following are equivalent:*

(a) *Y is Hausdorff*

(b) *f is closed*

(c) *For every $p \in Y$ and for all open sets $U \subseteqq X$ such that $f^{\leftarrow}(p) \subseteqq U$, there is an open set $V \subseteqq Y$ with $p \in V$ and $f^{\leftarrow}[V] \subseteqq U$.*

For any space X, let $R(X)$ be the set of all points at which X is not locally compact. It follows that for any compactification αX of X, $R(X) = X \cap \mathrm{cl}_{\alpha X}(\alpha X - X)$.

THEOREM 1.1. *Given any two spaces X and Y, there is a compactification αX of X such that Y is homeomorphic to $\mathrm{cl}_{\alpha X}(\alpha X - X)$ if and only if there is a continuous map h from $\mathrm{cl}_{\beta X}(\beta X - X)$ onto Y such that h is one-to-one on $R(X)$.*

Proof. From the existence of the Čech map, the "only if" is trivial. Conversely with no loss of generality assume Y and $X - R(X)$ to be disjoint and define αX to be the set $Y \cup X - R(X)$. Let $f: \beta X \rightarrow \alpha Y$ be given by $f(x) = x$ for x in $X - R(X)$ and $f(x) = h(x)$ for x in $\mathrm{cl}_{\beta X}(\beta X - X)$. Place the quotient topology of f on αX, which is thus a compact T_1 space containing $[X - R(X)] \cup h[R(X)]$ densely. We need to show αX to be Hausdorff, and shall use part (c) of the Lemma to do this.

First suppose $p \in X - R(X)$ and U is an open set in βX such that $f^{\leftarrow}(p) = \{p\} \subseteqq U$. Let $V = [X - R(X)] \cap U$. Then V is a βX-open set and $f^{\leftarrow} \circ f[V] = V$. So $V = f[V]$ is open in αX, $p \in V$ and $f^{\leftarrow}[V] \subseteqq U$.

Now let $p \in \alpha X - [X - R(X)]$. Then $p \in Y$ and $f^{\leftarrow}(p) = h^{\leftarrow}(p)$ in $\mathrm{cl}_{\beta X}(\beta X - X)$. Let U be any βX-open neighborhood of $h^{\leftarrow}(p)$. Then $U \cap \mathrm{cl}_{\beta X}(\beta X - X)$ is an open set in $\mathrm{cl}_{\beta X}(\beta X - X)$ and contains $h^{\leftarrow}(p)$. Since h is a closed map, there exists a Y-open set A such that $h^{\leftarrow}(p) \subseteqq h^{\leftarrow}[A] \subseteqq U \cap \mathrm{cl}_{\beta X}(\beta X - X)$. But considering A as a set in $\alpha X - [X - R(X)]$, $f^{\leftarrow}[A] = h^{\leftarrow}[A]$ is open in $\mathrm{cl}_{\beta X}(\beta X - X)$. So there exists a βX-open set B such that $B \cap \mathrm{cl}_{\beta X}(\beta X - X) = f^{\leftarrow}[A]$. Let $G = B \cap U$, this is an open set in βX. Then $G \cap [X - R(X)] \subseteqq U \cap [X - R(X)]$ and $G \cap \mathrm{cl}_{\beta X}(\beta X - X) = f^{\leftarrow}[A]$. Whence if we set $V = A \cup [G \cap X - R(X)]$, we have $p \in V$ and

$$\begin{aligned} f^{\leftarrow}[V] &= f^{\leftarrow}[A \cup (G \cap X - R(X))] = f^{\leftarrow}[A] \cup f^{\leftarrow}[G \cap X - R(X)] \\ &= [G \cap \mathrm{cl}_{\beta X}(\beta X - X)] \cup [G \cap X - R(X)] = G\,. \end{aligned}$$

Thus V is open in αX and $f^{\leftarrow}[V] = G \subseteqq U$.

We conclude that αX is a compact Hausdorff space containing a dense homeomorphic image of X, and $f: \beta X \rightarrow \alpha X$ is its Čech map.

Finally, let $\tau: Y \rightarrow \mathrm{cl}_{\alpha X}(\alpha X - X)$ be given by $\tau(y) = f[h^{\leftarrow}(y)]$ for each $y \in Y$. Since $h^{\leftarrow}(y) \subseteqq \mathrm{cl}_{\beta X}(\beta X - X)$, for each point $q \in h^{\leftarrow}(y)$ we have $f(q) = h(q) = y$. So τ is well defined, and indeed it is a bijection. Moreover since f and h are closed maps, any set F of Y is closed if and only if $h^{\leftarrow}[F]$ is closed in $\mathrm{cl}_{\beta X}(\beta X - X)$, which is true if and only if $f(h^{\leftarrow}[F])$ is closed in $\mathrm{cl}_{\alpha X}(\alpha X - X)$. Thus τ is a homeomorphism from Y onto $\mathrm{cl}_{\alpha X}(\alpha X - X)$.

COROLLARY 1.2. *For any space X, the following are equivalent:*

(a) *X is locally compact.*

(b) *For every space Y: Y is homeomorphic to $\alpha X - X$ for some Hausdorff compactification αX of X if and only if Y is a continuous image of $\beta X - X$.*

Proof. For (b) implies (a), note that a map onto a single point is trivially continuous. For the converse, take $R(X) = \varnothing$ in the Theorem. The fact that (a) implies (b) was first observed in [4].

THEOREM 1.3. *Let X and Y be any two spaces. There is a compactification αX of X such that Y is homeomorphic to $\alpha X - X$ if and only if there is a compactification αY of Y and a continuous map h from $\mathrm{cl}_{\beta X}(\beta X - X)$ onto αY such that h carries $R(X)$ homeomorphically onto $\alpha Y - Y$.*

Proof. (If). By Theorem 1.1, there is a compactification αX of X such that $\mathrm{cl}_{\alpha X}(\alpha X - X)$ is homeomorphic to αY. Moreover if $f\colon \mathrm{cl}_{\beta X}(\beta X - X) \to \mathrm{cl}_{\alpha X}(\alpha X - X)$ is the restriction of the Čech map, we may choose the homeomorphism $\tau\colon \mathrm{cl}_{\alpha X}(\alpha X - X) \to \alpha Y$ by $\tau(X) = h[f^{\leftarrow}(X)]$ as in the final paragraph of Theorem 1.1. Since $\tau[R(X)] = \alpha Y - Y$, we see that τ carries $\alpha X - X$ homeomorphically onto Y.

(Only if) Suppose that $h\colon \alpha X - X \to Y$ is the given homeomorphism. Without loss of generality assume Y and $R(X)$ disjoint, and let αY be the set $Y \cup R(X)$. Define $k\colon \mathrm{cl}_{\alpha X}(\alpha X - X) \to \alpha Y$ by $k(p) = p$ if $p \in R(X)$ and $k(p) = h(p)$ if $p \in \alpha X - X$. Place the quotient topology with respect to k on αY, making αY into a compact T_1 space.

If F is any closed subset of $\mathrm{cl}_{\alpha X}(\alpha X - X)$, then since k is a bijection, $k^{\leftarrow} \circ k[F] = F$ and $k[F]$ is closed in the quotient topology on αY. Hence k is a homeomorphism between $\mathrm{cl}_{\alpha X}(\alpha X - X)$ and αY. So αY is Hausdorff and Y, being the image of a dense subset of $\mathrm{cl}_{\alpha X}(\alpha X - X)$ is dense in αY. Thus αY is a Hausdorff compactification of Y.

Let f be the restriction to $\mathrm{cl}_{\beta X}(\beta X - X)$ of the Čech map of αX. Then $k \circ f$ is continuous from $\mathrm{cl}_{\beta X}(\beta X - X)$ onto αY. But $k \circ f$ takes $\beta X - X$ onto Y and also takes $R(X)$ one-to-one onto $\alpha Y - Y$, so it is a homeomorphism from $R(X)$ onto $\alpha Y - Y$.

COROLLARY 1.4. *Let X and Y be any two spaces and h be a homeomorphism from $\mathrm{cl}_{\beta X}(\beta X - X)$ onto $\mathrm{cl}_{\beta Y}(\beta Y - Y)$ which carries $R(X)$ onto $R(Y)$. Let αX be any compactification of X and let f be the restriction of its Čech map to $\mathrm{cl}_{\beta X}(\beta X - X)$. Then there exists a unique (up to a homeomorphism preserving Y pointwise) compactification αY of Y, with Čech map g, such that $g(h(f^{\leftarrow}(x)))$ is a homeomor-*

phism from $\mathrm{cl}_{\alpha X}(\alpha X - X)$ *onto* $\mathrm{cl}_{\alpha Y}(\alpha Y - Y)$ *taking* $R(X)$ *onto* $R(Y)$.

2. The upper semi-lattice of compactifications. For each compactification αX of X, with Čech map f, define

$$\mathscr{F}(\alpha X) = \{f^{\leftarrow}(p): p \in \mathrm{cl}_{\alpha X}(\alpha X - X)\} .$$

This is a partition of $\mathrm{cl}_{\beta X}(\beta X - X)$ into compact subsets and coincides with Magill's terminology on locally compact spaces [5]. In particular, we retain his

LEMMA 2.1. $\alpha_1 X \leqq \alpha_2 X$ *if and only if* $\mathscr{F}(\alpha_2 X)$ *refines* $\mathscr{F}(\alpha_1 X)$. *Observe that in* $K(X)$, *the correspondence between compactifications and their decompositions is one-to-one.*

Let X and Y be any spaces and $K(X)$ and $K(Y)$ be their upper semi-lattices of compactifications. We say $K(X)$ is isomorphic to $K(Y)$ if there is a bijection between them which preserves order in both directions. Clearly an isomorphism preserves meets and joins wherever they exist.

THEOREM 2.2. *Let* X *and* Y *be any two spaces. If there is a homeomorphism from* $\mathrm{cl}_{\beta X}(\beta X - X)$ *onto* $\mathrm{cl}_{\beta Y}(\beta Y - Y)$ *which carries* $R(X)$ *onto* $R(Y)$, *then* $K(X)$ *is isomorphic to* $K(Y)$.

Proof. Let h be the given homeomorphism and $\Gamma: K(X) \rightarrow K(Y)$ the correspondence constructed in 1.4. By the symmetry of 1.4, Γ is a bijection. That Γ preserves order in both directions follows from the fact that $h[\mathscr{F}(\alpha X)] = \mathscr{F}[\Gamma(\alpha X)]$ and 2.1.

COROLLARY 2.3. *Let* X *and* Y *be two spaces with* $|R(X)| = |R(Y)| \leqq 1$. *If* $\beta X - X$ *is homeomorphic to* $\beta Y - Y$, *then* $K(X)$ *is isomorphic to* $K(Y)$.

Proof. In view of Magill's theorem, it suffices to consider $|R(X)| = |R(Y)| = 1$. Let $R(X) = \{p\}$ and $R(Y) = \{q\}$. Since $\mathrm{cl}_{\beta X}(\beta X - X)$ is the one point compactification of $\beta X - X$, open neighborhoods of p in $\mathrm{cl}_{\beta X}(\beta X - X)$ are the complements of compact sets in $\beta X - X$. If h is the given homomorphism, then h carries compact sets onto compact sets. So it carries neighborhoods of p onto neighborhoods of q and vice versa. Hence if we let $k: \mathrm{cl}_{\beta X}(\beta X - X) \rightarrow \mathrm{cl}_{\beta Y}(\beta Y - Y)$ extend h by $k(p) = q$, then k is a homeomorphism and $k[R(X)] = R(Y)$. The result now follows from 2.2.

The next result follows from a well known exercise [3: 9K].

LEMMA 2.4. *For any space Y and any compactification αY, there is a pseudo-compact space X such that Y is homeomorphic to $\beta X - X$ and $\alpha Y - Y$ is homeomorphic to $R(X)$.*

THEOREM 2.5. *For each space Y, there is a pseudocompact space X such that $K(Y)$ is isomorphic to $K(X)$.*

Proof. As in the construction for 2.4, let W be the ordinals less than the first uncountable ordinal ω_1 and W^* be its compactification. Set $X = [W^* \times \mathrm{cl}_{\beta Y}(\beta Y - Y)] - [\{\omega_1\} \times (\beta Y - Y)]$. Then X is pseudocompact, $R(X) = \{\omega_1\} \times R(Y)$ and $\beta X - X = \{\omega_1\} \times (\beta Y - Y)$. The result now follows from 2.2.

3. k-absolute spaces. A space is called compactly generated, or a k-space, if every set whose intersection with every compact set is compact is itself closed. To each space X we may associate a unique k-space $\mathscr{K}X$ with the same underlying set and the same compact sets by requiring that the closed sets be precisely those whose intersection with every compact set is compact. It follows that X is a k-space if and only if $X = \mathscr{K}X$.

DEFINITION 3.1. X is a k-absolute space if $\beta X - X$ is a k-space. This terminology is motivated by

THEOREM 3.2. *For any space X, the following are equivalent:*

(a) *$\beta X - X$ is a k-space.*

(b) *For every compactification αX, $\alpha X - X$ is a k-space.*

(c) *There exists a compactification αX such that $\alpha X - X$ is a k-space.*

Proof. Use the fact that the restriction to $\beta X - X$ of the Čech map of αX is perfect (i.e., closed, continuous, onto and the pre-image of each point is compact), and the fact that if $f: V \to W$ is a perfect map, then V is a k-space if and only if W is a k-space [1: Theorem 8].

k-absolute space include, but are not restricted to, locally compact spaces, realcompact spaces (N. Noble [6]) and spaces with compact $R(X)$. Some examples showing the independence of these classes are considered in §4.

THEOREM 3.3. *Let X and Y be any two spaces. If $\Gamma: K(X) \to K(Y)$ is an isomorphism, then there is a homeomorphism $f: \mathscr{K}(\beta X - X) \to \mathscr{K}(\beta Y - Y)$ such that for each αX in $K(X)$, $\mathscr{F}[\Gamma(\alpha X)] \cap (\beta Y - Y) = \{f[H]: H \in \mathscr{F}(\alpha X) \cap (\beta X - X)\}$. There are two such homeomorphisms if $|\beta X - X| = |\beta Y - Y| = 2$; otherwise the home-*

omorphism is unique.

Proof. $f: V \to W$ is a bijection which preserves compact sets in both directions if and only if $f: \mathscr{K}V \to \mathscr{K}W$ is a homeomorphism. The proof now, with only minor changes, is that of K. D. Magill [5: Theorem 1].

COROLLARY 3.4. *Let X and Y be any two k-absolute spaces. If $K(X)$ is isomorphic to $K(Y)$, then $\beta X - X$ is homeomorphic to $\beta Y - Y$.*

An example showing the converse of this corollary to be false is found in the following section. An example has been obtained by T. Thrivikraman [7] of a pair of spaces, one of which is k-absolute and the other is not, with $K(X)$ isomorphic to $K(Y)$, yet $\beta X - X$ not homeomorphic to $\beta Y - Y$.

4. **Examples.**

(A) k-absolute spaces.

(a) The rational numbers Q form a realcompact, thus k-absolute space which is nowhere locally compact. Hence $R(X) = Q$ is not compact.

(b) Let X be the ordinals $\leqq \omega_1$ with the discrete topology except at ω_1, which has a neighborhood base of tails. Then X is realcompact and $R(X) = \{\omega_1\}$ is compact.

(c) If W is the set of ordinals $< \omega_1$ with the interval topology and N is the positive integers, then $W \times N$ is locally compact, yet neither realcompact nor pseudocompact. (Not realcompact follows from the fact that closed subsets of realcompact spaces are realcompact, and $W \times N$ contains closed copies of W).

(d) To construct a class of k-absolute spaces which are neither locally compact nor realcompact, let Y be any k-space and as in 2.4, let $X = W^* \times \beta Y - \{\omega_1\} \times Y$. This is a k-absolute, pseudocompact and not compact, hence not realcompact space. $R(X)$ is homeomorphic to $\beta Y - Y$, hence it is compact if and only if Y is locally compact. NOTE: X is locally compact if and only if Y is compact.

(B) A pair of k-absolute spaces X and Y with $\beta X - X$ homeomorphic to $\beta Y - Y$, yet $K(X)$ and $K(Y)$ not isomorphic. Let $T = (0, 1)$ under its usual topology, T^* its one point compactification and T^{**} its two point compactification. Write $T^* - T = \{a\}$ and $T^{**} - T = \{b, c\}$.

Set $X = W^* \times T^* - \{\omega_1\} \times T$, so $R(X) = \{(\omega_1, a)\}$.

Set $Y = W^* \times T^{**} - \{\omega_1\} \times T$, so $R(Y) = \{(\omega_1, b), (\omega_1, c)\}$. So $|R(X)| \neq |R(Y)|$, yet $\beta X - X = \beta Y - Y = \{\omega_1\} \times T$, which is a k-space.

Place the following compact partition on $\beta X - X$: for each r, $0 < r < 1/2$, let $F_r = \{(\omega_1, r), (\omega_1, 1 - r)\}$; choose $t_r \notin \beta X$ and set $\alpha X = [\beta - \bigcup_r F_r] \cup \{t_r\colon 0 < r < 1/2\}$. Define the map $f\colon \beta X \to \alpha X$ by $f(x) = x$ if $x \in X$ and $f(x) = t_r$ if $x \in F_r$ and $f(\omega_1, 1/2) = (\omega_1, 1/2)$. If $G \subseteqq \beta X - X$, then $f^{\leftarrow} \circ f[G] = G$ if and only if G is symmetric with respect to $(\omega_1, 1/2)$. Place the quotient topology with respect to f on αX. To show αX is Hausdorff, we apply (c) of Lemma 1.0.

Let $x \in X - \{a\}$ and U be an open set of βX such that $x \in U$. Then set $V = U \cap X - \{a\}$. So $f^{\leftarrow} \circ f[V] = V$, which is an open set in βX, and $p \in V = f[V] \subseteqq U$.

If U is a βX-open neighborhood of a, then $U \cap \beta X - X \supseteqq \{\omega_1\} \times (0, 1) - \{\omega_1\} \times [d, e]$ for some $[d, e] \subseteqq (0, 1)$. Choose $\varepsilon > 0$ so that $[d, e] \subseteqq [\varepsilon, 1 - \varepsilon] \subseteqq (0, 1)$.

Then $\{\omega_1\} \times (0, 1) - \{\omega_1\} \times [\varepsilon, 1 - \varepsilon]$ is open in $\beta X - X$, so there exists a βX-open set H such that $H \cap \beta X - X$ equals this set. Let $V = f[U \cap H]$. Since $U \cap H \cap \beta X - X$ is symmetric with respect to $(\omega_1, 1/2)$ we see that

$$\begin{aligned} f^{\leftarrow}[V] &= f^{\leftarrow} \circ f[(U \cap H \cap \beta X - X) \cup (U \cap H \cap X)] \\ &= (U \cap H \cap \beta X - X) \cup f^{\leftarrow} \circ f(U \cap H \cap X) \\ &= U \cap H \subseteqq U\,. \end{aligned}$$

Therefore V is open in αX, $a \in V$ and $f^{\leftarrow}[V] \subseteqq U$.

If $t_r \in \alpha X - X$, then $f^{\leftarrow}(t_r) = F_r$. Let U be a βX-open neighborhood of F_r. Then $U \cap \beta X - X$ contains (ω_1, r), so there exists an $\varepsilon_1 > 0$ such that $\{\omega_1\} \times (r - \varepsilon_1, r + \varepsilon_1) \subseteqq U \cap \beta X - X$. In the same way, there exists an $\varepsilon_2 > 0$ such that $\{\omega_1\} \times (1 - r - \varepsilon_2, 1 - r + \varepsilon_2) \subseteqq U \cap \beta X - X$. Let $\varepsilon = \min(\varepsilon_1, \varepsilon_2)$. Then $[\{\omega_1\} \times (r - \varepsilon, r + \varepsilon)] \cup [\{\omega_1\} \times (1 - r - \varepsilon, 1 - r + \varepsilon)]$ is an open set in $\beta X - X$. So there exists a βX-open set H such that $H \cap \beta X - X$ is equal to this set. Let $V = f[U \cap H]$. Note $U \cap H \cap \beta X - X = H \cap \beta X - X$ is symmetric with respect to $(\omega_1, 1/2)$, $f^{\leftarrow} \circ f[U \cap H \cap \beta X - X] = U \cap H \cap \beta X - X$. Hence $f^{\leftarrow} \circ f[U \cap H] = U \cap H$ and V is open in αX. Since $F_r \subseteqq U \cap H$, we have $t_r \in V$ and $f[V] \subseteqq U$. So αX is Hausdorff and thus in $K(X)$.

Suppose $\Gamma\colon K(X) \to K(Y)$ is any isomorphism; then by 3.4 there is a homeomorphism $h\colon \beta X - X \to \beta Y - Y$ such that $\mathscr{F}[\Gamma(\alpha X)] \cap (\beta Y - Y) = \{h[H]\colon H \in \mathscr{F}(\alpha X) \cap (\beta Y - Y)\}$. Notice that any homeomorphism from $(0, 1)$ to $(0, 1)$ must be monotone: our argument is the same whether h is monotone increasing or monotone decreasing. So without loss of generality, suppose h monotone increasing.

Write $\Gamma(\alpha X) = \alpha Y$, where αX is the previously constructed compactification of X and let g be the restriction of the Čech map of αY to $\beta Y - Y$. Since $f\colon \beta X - X \to \alpha X - X$ is perfect, it follows that if $k = g \circ h \circ f^{\leftarrow}$, then k is a homeomorphism from $\alpha X - X$ onto $\alpha Y - Y$. Consider the sequence $t_n = (\omega_1, 1/n)$, $n \geqq 2$, in $\beta X - X$. The image of this sequence in $\alpha X - X$, which we may write as $p_n = f(t_n)$, $n \geqq 2$, has $\lim p_n = a$. So (p_n), $n \geqq 2$, is a converging sequence in $\mathrm{cl}_{\alpha Y}(\alpha Y - Y)$. But in $\mathrm{cl}_{\beta Y}(\beta Y - Y)$, $\lim h(\omega_1, 1/n) = b$ and $\lim (\omega_1, 1 - 1/n) = c$. Therefore $k(p_n)$, $n \geqq 2$, converges to both b and c in $\mathrm{cl}_{\alpha Y}(\alpha Y - Y)$. Since in a Hausdorff space, no sequence can converge to more than one point, $\Gamma(\alpha X)$ must not be Hausdorff. So Γ must not be an isomorphism and thus $K(X)$ and $K(Y)$ are not isomorphic.

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