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SOME EXAMPLES OF NON-TAUT SUBSPACES

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We obtain a necessary and sufficient condition for the tautness of each closed subspace of a Hausdorff space X w.r.t. the Alexander-Spanier cohomology functor $\overline{H}^{\circ}$. This is used to give an example of a normal Hausdorff space on which the concepts of an L-theory and a continuous cohomology theory (as defined by Spanier) are *not* equivalent. Finally, we provide examples of non-taut subspaces with respect to the classical cohomology theories which possess some further curious properties.

1. Introduction. Let X be a topological space, $A \subset X$ be a subspace and $\{H^p\}_{p\geq 0}$ be the cohomology functors of a cohomology theory for which all subspaces of X and inclusion maps among them are admissible. Since the set of all neighbourhoods of A in X is a directed set (directed downward by inclusion maps), we can form, for each $p \geq 0$ and each coefficient group G, the limit group

$$\varinjlim \{H^p(N, G) | N \text{ runs over all neighbourhoods of } A \text{ in } X\}$$

where the bonding homomorphisms are induced by inclusion maps. Also, for each N we have the inclusion maps $A \to N$ which induce a natural homomorphism

$$(*) \qquad \eta\colon \varinjlim_{N\supset A} H^p(N, G) \to H^p(A, G).$$

We say that A is *tautly embedded* in X w.r.t. the cohomology theory (H^p, δ) if for each coefficient group G and for each $p \geq 0$, the above map η is an isomorphism. (See [2], [3], [7], [9] for basic results.) In order to establish the existence of non-taut subspaces for the Alexander-Spanier cohomology, Spanier proved ([7], Theorem 2) the following necessary condition: *If each closed subspace of a space X is taut in X w.r.t. the zero-dimensional Alexander-Spanier cohomology functor $\overline{H}^{\circ}$, then X must be a normal space.* Our first result of this paper is to show that normality of X is not a sufficient condition for each closed subspace of X to be taut in X w.r.t. $\overline{H}^{\circ}$. In fact in §2, we prove that *a necessary and sufficient condition for each closed subspace of X to be tautly embedded in X w.r.t. $\overline{H}^{\circ}$ is that X be collectionwise* *normal*. In §3, we study the exact relationship between an L-theory as defined by Lawson in **[5]** and a non-negative continuous cohomology theory as defined by Spanier in **[8]**. We use the results of §2 to show that although these two theories are equivalent on paracompact Hausdorff spaces as proved in **[8]**, *they are really distinct on normal Hausdorff spaces.*

Concerning the definition of tautness, the following is a natural question frequently asked: *Give an example of a non-taut subspace w.r.t. a given cohomology theory* (H^p, δ) *such that if* N *runs over all neighbourhoods of* A *in* X, *then*

$$\varinjlim H^p(N, G) \approx H^p(A, G)$$

for all $p \geq 0$ *and all coefficients* G. In §4, we provide such curious examples for singular cohomology, Alexander-Spanier cohomology ($\approx$ Čech cohomology) and for sheaf cohomology (only partially).

All of our notations are standard. The symbols $\overline{H}^p$ always stand for the Alexander-Spanier cohomology functors as in **[9]**; sheaf theoretic functions are from **[1]**.

2. A necessary and sufficient condition. Recall ([**10**] p. 168) that a Hausdorff space X is called *collectionwise normal* if for any discrete collection of sets (or, equivalently, for any discrete collection of closed sets) $\{A_i | i \in I\}$ of X, there exists a discrete collection $\{U_i | i \in I\}$ of open sets of X such that $A_i \subset U_i$ for each $i \in I$. Every paracompact Hausdorff space is collectionwise normal **[4]** and every collectionwise normal space is evidently normal Hausdorff. However, it is well known that the converse of none of the above is true. A Hausdorff space X is said to be *completely collectionwise normal* if given a family $\{A_i\}$ of closed (or arbitrary) subsets of X which is discrete in its union (called relatively discrete), there exists a discrete family $\{U_i\}$ of open sets of X such that $A_i \subset U_i$ for each i. Evidently each completely collectionwise normal space is completely normal, but the converse is not ture. Also, note that every hereditarily paracompact space is completely collectionwise normal while the converse is again not ture.

The following result is due to Spanier.

2.1. THEOREM ([7] *Theorem* 2). *If* X *is a space such that every closed* (*respectively arbitrary*) *subspace of* X *is tautly embedded in* X *w.r.t.* $\overline{H}^{\circ}$, *then* X *is normal* (*respectively completely normal*).

In particular, this tells us that each subspace of a paracompact Hausdorff space X need not be taut in X unless X is hereditarily paracompact. For instance, consider the usual Tychonoff-Plank $X = [0, \Omega] \times [0, \omega]$ where Ω is the first uncountable ordinal and ω is the first infinite ordinal ([**10**], p. 106). We know that X is compact Hausdorff but not completely normal and so it has a subspace which is not taut w.r.t. $\overline{H}^\circ$; in fact, the deleted Tychonoff Plank $T = X - \{(\Omega, \omega)\}$ is not tautly embedded in X. Likewise, since T is not normal, there is a closed subspace of T which is not tautly embedded in T. In fact, the closed subspace $A = \{(\Omega, \alpha) | \alpha < \omega\} \cup \{(\alpha, \omega) | \alpha < \Omega\}$ of T is not tautly embedded in T.

When we use the full force of the arbitrariness of the coefficient group G for tautness, we obtain that *collectionwise normality* is indeed a necessary condition for each closed subspace of X to be taut in X. Then, interestingly enough, this necessary condition turns out to be also sufficient. We have

2.2. THEOREM. *Let X be a T_1-space. Then a necessary and sufficient condition for each closed (resp. arbitrary) subspace of X to be tautly embedded in X w.r.t. $\overline{H}^\circ$ is that X be collectionwise normal (resp. completely collectionwise normal).*

Proof. We will prove the necessity part of the first case—the necessity part of the other case is similar. Thus we now assume that each closed subspace of X is tautly embedded in X w.r.t. $\overline{H}^\circ$. Then from Theorem 2.1 it follows that X is Hausdorff. Let $\{A_\alpha | \alpha \in I\}$ be a discrete family of closed subsets of X. Then, clearly $A = \bigcup\{A_\alpha | \alpha \in I\}$ is closed in X. Hence the natural map

$$\eta\colon \varinjlim \{\overline{H}^\circ(U; G) | U \text{ is a neighbourhood of } A \text{ in } X\} \to \overline{H}^\circ(A)$$

is an isomorphism for each coefficient group G. Since any set can be given the structure of an abelian group, we can regard the indexing set I to be an abelian group. Now define a map $f\colon A \to I$ by setting $f(b) = \alpha$ if $b \in A_\alpha$. Then clearly f is a zero-cocycle on A. Since A is taut in X, this cocycle can be extended to a zero cocycle, say $\hat{f}$, on some open neighbourhood of A. Because a zero cocycle is simply a locally constant function, $\hat{f}\colon U \to I$ must be a locally constant function on U and so $U_\alpha = \hat{f}^{-1}(\{\alpha\})$, $\alpha \in I$, is a collection of mutually disjoint open sets of X which separate the family $\{A_\alpha | \alpha \in I\}$ of closed subsets of X. Thus X is collectionwise normal.

Next we prove the converse part of the second case—the converse of the first case is analogous, and is omitted. Assume that X is completely collectionwise normal and $A \subset X$. We have to prove that the natural map η is an isomorphism for each coefficient group G and for $p = 0$. It is obvious that η is always one-one. To prove that it is onto, let $f: A \to G$ be any zero cocycle on A, i.e., f is a locally constant function on A. Now decompose $A = \bigcup A_\alpha$ into mutually disjoint open sets A_α relative to A so that $f(A_\alpha) \in G$. This means $\{A_\alpha\}$ is a relatively discrete family of subsets of X. Since X is completely collectionwise normal, this family can be separated by mutually disjoint open sets $\{U_\alpha\}$ of X. Now define $\hat{f}: \bigcup U_\alpha \to G$ by $\hat{f}(U_\alpha) = f(A_\alpha)\ \forall_\alpha$. Then, clearly $\hat{f} \in \overline{H}^\circ(U, G)$ and $\hat{f}|A = f$. This proves that η is onto. □

2.3. Remark. The Hausdorff condition is needed in all tautness theorems for Alexander-Spanier cohomology because there are closed subsets of a compact T_1-space which are not taut (see Example (4.2)).

Now we give an example which shows that a closed subspace of even a completely normal Hausdorff space need not be tautly embedded w.r.t. Alexander-Spanier cohomology function $\overline{H}^*$.

2.4. Example. We consider the example given by Michael in ([**6**], p. 279, last paragraph) of a perfectly normal Hausdorff space X which is not collectionwise normal. Since a perfectly normal space is completely normal, X is completely normal Hausdorff. Therefore, by the above theorem there must be a closed subspace of X which is not tautly embedded in X w.r.t. Alexander-Spanier cohomology. In fact, there is a closed discrete subspace of X whose points cannot be separated by mutually disjoint open sets. Any such closed discrete subspace of X will not be taut in X.

Recall that a point subspace of *any* space X is tautly embedded in X [**2**] w.r.t. Alexander-Spanier cohomology. Since Michael's space mentioned above has a discrete family $\{x_\alpha\}$ of points which cannot be separated by disjoint open sets, we conclude that the union of a discrete family of taut subspaces of a space X need not be taut in X. However, we have the following result which will be needed later on.

2.5. Proposition. *Let X be a collectionwise normal space $\{A_\alpha | \alpha \in I\}$ be a discrete family of subspaces of X. Then, with respect to any* additive *cohomology theory $\{H^p\}$, the union $A = \bigcup\{A_\alpha | \alpha \in I\}$ is taut in X iff A_α is taut in X for each $\alpha \in I$.*

Proof. Since X is collectionwise normal, there exists a mutually disjoint family $\{N_\alpha|\alpha \in I\}$ of open sets of X such that $A_\alpha \subset N_\alpha$ for each $\alpha \in I$. This obviously means that if U^λ is any neighbourhood of A in X, then $\{U^\lambda \cap N_\alpha|\alpha \in I\}$ is a discrete family of open sets such that for each $\alpha \in I$, $A_\alpha \subset U^\lambda \cap N_\alpha$. It follows that the set of all such discrete neighbourhoods $\{\{N_\alpha^\lambda\}_\lambda|A_\alpha \subset N_\alpha^\lambda, \forall\alpha \in I\}$ of A form a cofinal system of neighbourhoods of A in X. Since A is taut in X, for each $p \geq 0$ and for each coefficient group G, the natural map

$$\varinjlim_{\lambda} H^p\left(\bigcup_{\alpha} N_\alpha^\lambda, G\right) \to H^p(A, G),$$

where $\bigcup N_\alpha^\lambda$ runs over all discrete family of neighbourhoods of A in X, is an isomorphism. By the additivity of H, we find that

$$\varinjlim_{\lambda} \prod_{\alpha} H^p N_\alpha^\lambda, G) \xrightarrow{\approx} \prod_{\alpha} H^p(A_\alpha, G)$$

via the natural map. In particular, this implies that the natural map

$$\varinjlim_{\lambda} H^p(N_\alpha^\lambda, G) \to H^p(A_\alpha, G)$$

is an isomorphism for each $p \geq 0$ and each coefficient group G; hence A_α is taut in X for each $\alpha \in I$.

Conversely, suppose each A_α is taut in X. If $\{U_\alpha^\lambda|\lambda \in \Lambda\}$ varies over all neighbourhoods of A_α in X which are contained in N_α, then the natural map

$$\varinjlim_{\lambda} H^p(U_\alpha^\lambda, G) \to H^p(A_\alpha, G)$$

is an isomorphism for each α. Hence we have

$$\begin{aligned}\varinjlim_{\lambda} H^p\left(\bigcup_{\alpha} U_\alpha^\lambda, G\right) &\approx \varinjlim_{\lambda} \prod_{\alpha} H^p(U_\alpha^\lambda, G)\\ &\approx \prod_{\alpha} \varinjlim_{\lambda} H^P(U_\alpha^\lambda, G)\\ &\approx \prod_{\alpha} H^P(A_\alpha, G) \approx H^P\left(\bigcup A_\alpha, G\right).\end{aligned}$$

Since $\{\bigcup U_\alpha^\lambda|\lambda \in \Lambda\}$ is a cofinal system of neighbourhoods of $\bigcup A_\alpha$ in X and since the composition of all of the above isomorphisms is the natural map η, $A = \bigcup A_\alpha$ is taut in X. □

3. Continuous cohomology theories and L-theories. In this section we will show that on a normal Hausdorff space the concepts of a continuous cohomology theory and an L-theory as defined in [8], in general, are *not* equivalent.

If X is a normal Hausdorff space, we consider the category $\mathscr{S}$ of all continuous cohomology theories on X—the objects are continuous cohomology theories on X and the morphisms are natural transformations between them commuting with δ and δ' up to sign. Similarly, we consider the category $\mathscr{L}$ of L-theories on X. Let $\mathscr{S}^+$ denote the subcategory of $\mathscr{S}$ consisting of all non-negative cohomology theories on X. Consider the two functors $L\colon S^+ \to \mathscr{L}$ and $S\colon \mathscr{L} \to S^+$ defined by Spanier ([8], Theorem (3.1)). We refer to L as the restriction functor and S as the extension functor. If $H \in S^+$, then for any $A \in \mathrm{cl}(X)$ we have

$$(S \circ L)(H)(A) = S(L(H)(A)) = S(H)(A) = H(A).$$

Thus $S \circ L = I_{S^+}$, the identity functor. Conversely, for any $H \in \mathscr{L}$, there is a natural map $(L \circ S)(H)(A) = S(H)(A) = \varinjlim \{H(N) | N$ a closed neighbourhood of A in $X\} \to H(A)$ induced by the restriction homomorphisms. Clearly, the above is a homomorphism from the L-theory $(L \circ S)(H)$ into H. If X is paracompact and H is additive, then it is shown in [8] that $(L \circ S)(H)$ is also additive. Thus $(L \circ S)(H)$ and H are two additive L-theories on a paracompact Hausdorff space X which are isomorphic for point subspaces $x \in X$. Therefore, by Lawson's Theorem [5], the above map is an isomorphism for all $A \in \mathrm{cl}(X)$. Thus, we have

3.1. THEOREM (*Spanier*). *If X is a normal Hausdorff space and $L\colon S^+ \to \mathscr{L}$, $S\colon \mathscr{L} \to S^+$ are the two functors above, then $S \circ L = I_{S^+}$. Furthermore, if X is paracompact Hausdorff, then for any additive L-theory $H \in \mathscr{L}$, $(L \circ S)(H) = H$.*

We now give an example to show that on a normal Hausdorff space the two concepts are really different.

3.2. EXAMPLE. Consider the normal Hausdorff space X constructed by Michael [6] which is not collectionwise normal. Let $(\overline{H}, \delta)$ denote the Alexander-Spanier cohomology theory in the sense of Eilenberg-Steenrod. Since each point $x \in X$ is tautly embedded in any closed set A of X containing x (in fact points are always taut in any space w.r.t. $\overline{H}^*$, see [2] for a more general result), $(\overline{H}, \delta)$ is an L-theory on X. Then, $(S(\overline{H}), S(\delta))$ is a continuous cohomology theory on X. Since X is not collectionwise normal, there is a closed subspace A_0 of X by Theorem 2.2, which is not taut in X w.r.t. $\overline{H}$. If $L \circ S = I_{\mathscr{L}}$, then the natural map $L \circ S(\overline{H})(A) \to \overline{H}(A)$ must be an isomorphism for each $A \in \mathrm{Cl}(X)$. But $(L \circ S)(\overline{H})(A) = L(S(\overline{H}))(A) = S(\overline{H})(A) = \varinjlim \{\overline{H}(N) | N$ is a closed neighbourhood of A in $X\}$ This contradicts the fact that A_0 is not taut in X w.r.t. $\overline{H}$. Thus $L \circ S \not\approx I_{\mathscr{L}}$ and we have the following:

3.4. THEOREM. *Let X be any normal Hausdorff space which is not collectionwise normal. Then there exists an additive L-theory on X which cannot be obtained from its extension to a continuous cohomology theory on X by the restriction functor. In particular, the Alexander-Spanier cohomology is* not *a continuous cohomology theory on such a normal Hausdorff space X.*

It is worth noting that the extension functor S preserves additivity on a collectionwise normal space; for paracompact Hausdorff spaces this was proved in [8] and the same method works in the case of collectionwise normal spaces yielding the following

3.5. PROPOSITION. *Let X be a collectionwise normal space and H, Δ be an additive L-theory on X. Then SH, $S\Delta$ is an additive cohomology theory on X.*

3.6. COROLLARY. *In Example* (3.4) *of* [8] *if H is an additive functor, then the function $\overline{H}$ defined there is also additive on any collectionwise normal space.*

4. Examples of non-taut subspaces. In this section we give examples for various theories $\{H^p\}$ to show that there are spaces X having a closed subspace A such that

$$\varinjlim \{H^p(U, G) | U \text{ is a neighbourhood of } A \text{ in } X\} \approx H^p(A, G)$$

for each $p \geq 0$ and each coefficient group G, but A is not tautly embedded in X w.r.t. $\{H^p\}$. This implies that in the definition of tautness of a subspace in a space X, we must insist that the natural map η (see $(*)$ in the Introduction) is an isomorphism for all p and all coefficient groups G.

4.1. EXAMPLE (Singular cohomology). Let $X = \mathbb{R}^2$ and A_0 be the topologist's sine curve in X. Then we know that A_0 is not taut in X w.r.t. singular cohomology ([**9**], p. 290). Now let us define point subspaces A_n of X by

$$A_n = \{(-n, 0)\}$$

for $n = 1, 2 \dots$. Consider the set $A = \bigcup\{A_n | n = 0, 1, 2, \dots\}$. Then, evidently A is the union of a discrete family of closed subsets of X and X itself is collectionwise normal. Since A_0 is not taut in X, A cannot be taut in X by Proposition 2.6. However, we assert that for any coefficient group G and for all $p \geq 0$,

$$\varinjlim \{H^p(U, G) | U \text{ is a neighbourhood of } A \text{ in } X\} \approx H^p(A, G).$$

For $p = 0$, both of the above groups are isomorphic to $\prod_1^\infty G$. For $p > 0$, note that A_0 consists of two path components, one homeomorphic to a closed interval I and the other homeomorphic to an open interval J. Since I and J both are contractible, all positive dimensional integral singular homologies of A_0 vanish and so by the Universal Coefficient Theorem $H^p(A_0, G) = 0$, $\forall p > 0$. On the other hand if U is any neighbourhood of A_0, then it can be seen that U contains a contractible open neighbourhood of A_0. Consequently, the set of all discrete collections $\{\{U_n | n = 0, 1, 2, \dots\}\}$ where U_n is a contractible open set containing A_n, forms a cofinal family of open neighbourhoods of A in X. This means, for all $p > 0$,

$$\varinjlim \{H^p(U, G) | U \text{ is a neighbourhood of } A \text{ in } X\} = 0.$$

4.2. EXAMPLE. (Alexander-Spanier Cohomology): Because of several tautness theorems of very general nature for this cohomology [**7**], no such examples can be found in any hereditarily paracompact Hausdorff space. The following would seem to be the simplest example where cohomologies can be easily computed: Let $Y = \{0, 1, 2, \dots, \omega\}$ be given the cofinite topology, where ω is the first countably infinite ordinal. Then Y is clearly a compact T_1-space in which any two open sets intersect. In fact all the open sets are homeomorphic to Y which is clearly connected. Let $B = \{0, \omega\}$ be the closed subset of Y and note that B is not taut in Y w.r.t. Alexander-Spanier cohomology. In fact $\overline{H}^\circ(B, \mathbb{Z}) \approx \mathbb{Z} \oplus \mathbb{Z}$ whereas if $U_1 \supset U_2 \supset B$ are any two open sets containing B, then the restriction homomorphism $\overline{H}^\circ(U_1, \mathbb{Z}) \to \overline{H}^\circ(U_2, \mathbb{Z})$ is an isomorphism, each being isomorphic to $\mathbb{Z}$, i.e.

$$\varinjlim \{\overline{H}^{\circ}(U, \mathbb{Z}) | U \text{ is a neighbourhood of } B \text{ in } Y\} \approx \mathbb{Z}.$$

Now, let $X = \bigcup_{n=1}^{\infty} Y_n$ be the topological sum of countable number of copies of Y, and let $A = \bigcup_{n=1}^{\infty} B_n$ where $B_1 = \{0, \omega\}$, $B_n = \{0\}$ for $n > 1$ with $B_n \subset Y_n$. We claim that $\forall p \geq 0$ and for each coefficient group G,

$$\varinjlim \{\overline{H}^{p}(U, G) | U \text{ is a neighbourhood of } A \text{ in } X\} \approx \overline{H}^{p}(A, G).$$

For $p = 0$, both groups are isomorphic to $\prod_1^{\infty} G$. For $p > 0$, clearly $\overline{H}^p(A, G) = 0$. On the other hand, let us show that for $p > 0$, $\overline{H}^p(Y, G) = 0$. To see this recall that Alexander-Spanier cohomology $\overline{H}$ of any space is naturally isomorphic to the Čech cohomology $\check{H}$ of that space. Now if $U = \{U_1, U_2, \dots, U_n\}$ is any finite open cover of Y, then since their intersection is nonempty, the simplicial complex $K(U)$, i.e., the nerve of U is in fact an n-simplex. Hence $\forall p > 0$, the simplicial cohomology groups $H^p(K(U), G) = 0$. Since $\check{H}^p(Y, G) = \varinjlim H^p(K(U), G)$ where U runs over all open covers of Y and since Y is compact, we find that $\forall p > 0$, $\check{H}^p(Y, G) = 0$. Now, by additivity of $\overline{H}^p$, it follows clearly that if U runs over all open neighbourhoods of A in X, then $\forall p > 0$ and for all coefficient groups G

$$\varinjlim \overline{H}^p(U, G) = 0.$$

4.3. Example (Sheaf cohomology). In this case our problem is (cf. [3] for tautness in sheaf cohomology) to give an example of a space X, a closed subspace A of X and a family φ of supports on X such that A is not φ-taut in X and

$$\varinjlim \{H^p_{\varphi \cap N}(N, \mathcal{A}/N) | N \text{ is a neighbourhood of } A \text{ in } X\}$$
$$\approx H^p_{\varphi \cap A}(A, \mathcal{A}/A)$$

for all $p > 0$ and all sheaves $\mathcal{A}$ of abelian groups on X. The requirement that the two groups be isomorphic for all sheaves on X seems quite formidable and we have no such example. However, if we require only that the above groups are isomorphic for all *constant sheaves* on X then the needed example, similar to Example 4.1, is as follows: Let $X = \mathbb{R}^2$, $A = \{(n, 0) | n \in \mathbb{Z}\}$ and $\varphi = \{(0, 0\} \cup \operatorname{cl} d | H_-$, where H_- is the open half plane on the left of y-axis. Note that $A_0 = \{(0, 0)\}$ is not *φ-taut* in X because for the constant sheaf $\mathcal{A} = \mathbb{Z}$ any nonzero section $s \in \overline{H}^{\circ}(A_0, \mathbb{Z})$ can never be extended to an element s' in $H^0_{\varphi \cap U}(U, \mathbb{Z})$ for any open neighbourhood U of A_0. Since $\mathbb{R}^2$ is collectionwise normal and A is the union of a discrete family of point subspaces A_0, A cannot be φ-taut in X. However, now one can easily verify that for any constant sheaf G on X

$$H^p_{\varphi \cap A}(A, G) \approx \varinjlim_{U \supset A} H^p_{\varphi \cap U}(U, G) \approx \begin{cases} \prod_1^\infty G, & p = 0, \\ 0, & p \neq 0. \end{cases}$$

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